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RESEARCH ARTICLE

# A Stratification of Integer Solutions to the Triple Equations $x + y = z^2$, $2x + y = 2z^2 + w^2$, $x + 2y = 10p^3$

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## Abstract

**Objectives:** The purpose of this research article is to find non-zero distinct integer solutions to the system of triple equations with five unknowns represented by $x + y = z^2$, $2x + y = 2z^2 + w^2$, $x + 2y = 10p^3$. Different sets of distinct integer solutions are presented. **Methods:** Various substitution strategies and factorization techniques are employed for finding non-zero distinct integer solutions. **Findings:** Four different choices of substitutions are utilized to determine many non-zero distinct integer solutions to the given system of triple equations with five unknowns. **Novelty:** In this analysis, the given system of triple equations with five unknowns is reduced to a single non-homogeneous cubic equation for which solutions can be found elegantly.

**Keywords:** System of triple equations; Triple equations with five unknowns; Non-homogeneous cubic Equation; Ternary cubic equation; Integer solutions

## 1 Introduction

It is well-known that the Diophantine problems offer an unlimited field for research by reason of their variety. The successful completion of exhibiting all integers satisfying their requirements set forth in the problems adds to the further progress of number theory. Systems of indeterminate quadratic equations of the form $ax + c = a^2, bx + d = b^2$ where $a, b, c, d$ are non-zero distinct constants, have been investigated for integer solutions by several authors, and with a few possible exceptions, most of them were primarily concerned with rational solutions. Even those existing works, wherein integral solutions have been attempted, deal essentially with specific cases only and do not exhibit methods of finding integral solutions in the general form.

In [1–4], the following four systems of Double Equations are studied:

i. $x + y = z + w, y + z = (x + w)^2$

ii. $x + y = u^2, \frac{x}{D} + y = v^2$

iii. $N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$

iv. $x - y = u^2, \frac{x}{D} - y = v^2$

Also, in [5–8], the following four systems of Triple Equations are studied:

v. $x + y = 2a^2, 2x + y = 5a^2 + b^2, x + 2y = c^3$

vi. $x + y = a^2, 2x + y = a^2 + 3b^2, x + 2y = a^2 + c^2$

vii. $x+y=a^2, 2x+y=a^2+b^2, x+2y=a^2+5c^2$
viii. $x+y=2a^2, 2x+y=5a^2-b^2, x+2y=5c^3$

This communication exhibits different sets of non-zero distinct integer solutions for the system of triple equations with five unknowns given by $x+y=z^2, 2x+y=2z^2+w^2, x+2y=10p^3$.

## 2 Methodology

The system of triple equations with five unknowns $x, y, z, w$ and $p$ to be solved is

$$x+y=z^2 \tag{1}$$

$$2x+y=2z^2+w^2 \tag{2}$$

$$x+2y=10p^3 \tag{3}$$

Solving Equation (1) & Equation (2) for $x, y$ we have

$$x=z^2+w^2 \tag{4}$$

$$y=-w^2 \tag{5}$$

Substituting Equation (4) & Equation (5) in Equation (3), the resulting equation is

$$z^2=w^2+10p^3 \tag{6}$$

The process of solving Equation (6) to obtain the values of $z, w$ and $p$ through different methods is illustrated below. In view of Equation (4) and Equation (5), the corresponding values of $x$ and $y$ are obtained.

## 3 Results and Discussions

### 3.1 Technical procedure-I

It is noted that the substitutions given by

$$p=20\alpha^{2s}, w=100\alpha^{3s} \tag{7}$$

in Equation (6) leads to

$$z=300\alpha^{3s} \tag{8}$$

Using the above values of $w$ and $z$ in Equation (4) and Equation (5) one obtains

$$x=10^5\,\alpha^{6s}, w=-10^4\,\alpha^{6s} \tag{9}$$

Note Equations (7) and (8) and Equation (9) satisfy the system of triple Equations (1), (2) and (3).

### 3.2 Technical procedure-II

The choice

$$z=9w \tag{10}$$

in Equation (6) gives

$$8w^2 = p^3 \tag{11}$$

which is satisfied by

$$p = 2\alpha^{2s},\, w = \alpha^{3s} \tag{12}$$

From Equations (12), (10) and (5) and Equation (4), it is seen that

$$x = 82\alpha^{6s},\, y = -\alpha^{6s},\, z = 9\alpha^{3s} \tag{13}$$

Thus, Equation (12) and Equation (13) represent the integer solutions to the system of equations Equations (1), (2) and (3).

### 3.3 Technical procedure-III

Consider

$$z = (10k-1)\,w, k > 1 \tag{14}$$

Using Equation (14) in Equation (6), note that it gives

$$(10k^2 - 2k)\,w^2 = p^3 \tag{15}$$

which is satisfied by

$$w = (10k^2 - 2k)\,\alpha^{3s},\, p = (10k^2 - 2k)\,\alpha^{2s} \tag{16}$$

In view of Equations (4) and (5) and Equation (14), one obtains

$$\left.\begin{array}{c} x = 4k^2(5k-1)^2(100k^2-20k+2)\alpha^{6s} \\ y = -4k^2(5k-1)^2\alpha^{6s} \\ z = 2k(5k-1)(10k-1)\alpha^{3s} \end{array}\right\} \tag{17}$$

Thus, Equation (16) and Equation (17) represent the integer solutions to the system of equations Equations (1), (2) and (3).

### 3.4 Technical procedure-IV

Write Equation (6) as the system of double equations as given in Table 1 below:

**Table 1. System of double equations**

| **System** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $z+w$ | $p^3$ | $5p^3$ | $5p^2$ | $2p^2$ | $p^2$ |
| $z-w$ | 10 | 2 | 2 p | 5 p | 10 p |

Solving each of the above system of equations, the values of z, w and p are obtained. In view ofEquation (4) and Equation (5), the corresponding values of x and y are found. For simplicity, we present below the solutions for each of the above systems:

Solutions for System 1:

$x = 32k^6 + 50, y = -16k^6 + 40k^3 - 25$

$z = 4k^3 + 5,\ w = 4k^3 - 5,\ p = 2k$

Solutions for System 2:

$x = 800k^6 + 2, y = -400k^6 + 40k^3 - 1$

$z = 20k^3 + 1, w = 20k^3 - 1, p = 2k$

Solutions for System 3:

$x = k^2\left(200k^2 + 8\right), y = k^2\left(-100k^2 + 40k - 4\right)$

$z = 10k^2 + 2k, w = 10k^2 - 2k, p = 2k$

Solutions for System 4:

$x = k^2\left(32k^2 + 50\right), y = k^2\left(-16k^2 + 40k - 25\right)$

$z = 4k^2 + 5k, w = 4k^2 - 5k, p = 2k$

Solutions for System 5:

$x = k^2\left(8k^2 + 200\right), y = k^2\left(-4k^2 + 40k - 100\right)$

$z = 2k^2 + 10k, w = 2k^2 - 10k, p = 2k$

### 3.5 Technical procedure-V

Taking

$$p = 10\alpha^2 P, w = 10^2 \alpha^3 W, z = 10^2 \alpha^3 Z \tag{18}$$

in Equation (6), we have

$$Z^2 = W^2 + P^3 \tag{19}$$

After some algebra, it is seen that Equation (19) is satisfied by

$$\begin{gathered} Z = 4s^3 + 6s^2 + 3s + 1 \\ W = 4s^3 + 6s^2 + 3s \\ P = 2s + 1 \end{gathered}$$

In view of Equations (4) and (5) & Equation (18), the corresponding integer solutions to the system of equations Equation (1)Equations (2) and (3) are given by

$$\begin{gathered} x = 10^4\alpha^6 \left[\left(4s^3 + 6s^2 + 3s + 1\right)^2 + \left(4s^3 + 6s^2 + 3s\right)^2\right] \\ y = -10^4\alpha^6 \left(4s^3 + 6s^2 + 3s\right)^2 \\ z = 10^2\alpha^3 \left(4s^3 + 6s^2 + 3s + 1\right) \\ w = 10^2\alpha^3 \left(4s^3 + 6s^2 + 3s\right) \\ p = 10\alpha^2 (2s + 1) \end{gathered}$$

**Note 3.1**

It is worth to mention that Equation (19) is also satisfied by

$$Z = \frac{P(P+1)}{2}, W = \frac{P(P-1)}{2}$$

In view of Equations (4) and (5) & Equation (18), the corresponding integer solutions to the system of equations Equations (1), (2) and (3) are given by

$$\begin{gathered} x = 10^4\alpha^6 \left[\frac{P^2(P^2+1)}{2}\right] \\ y = -10^4\alpha^6 \left[\frac{P(P-1)}{2}\right]^2 \\ z = 10^2\alpha^3 \left[\frac{P(P+1)}{2}\right] \\ w = 10^2\alpha^3 \left[\frac{P(P-1)}{2}\right] \\ p = 10\alpha^2 P \end{gathered}$$

Note that the values of the variables $x, y, z, w$ are linear combinations of triangular numbers of different ranks.

## 4 Conclusion

In this paper, varieties of integer solutions to the system of non-linear triple equations with five unknowns are obtained by reducing the system of triple equations to a single non-homogeneous cubic equation with three unknowns for which integer solutions can be found successfully through a suitable substitution strategy. Further, the system of non-linear triple equations with five unknowns is reduced to a non-homogeneous cubic equation with two unknowns whose integer solutions are obtained with clarity and brevity. It is worth mentioning that the system of non-linear triple equations with multiple variables can be reduced to a solvable equation with fewer variables.

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