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Soham Transform in Fractional Differential Equations

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Abstract

Objectives: Soham transforms is one of the appropriate tools for solving fractional differential equations that are flexible enough to adapt to different purposes. **Methods**: Integral transform methods help to simplify fractional differential equations into algebraic equations. Enable the use of classical methods to solve fractional differential equations. **Findings:** In this paper, the Soham transform can solve linear homogeneous and non-homogeneous Fractional Differential Equations with constant coefficients. Finally, we use this integral transform to obtain the analytical solution of non-homogeneous fractional differential equations. **Novelty:** The Soham transform method is a suitable and very effective tool for obtaining analytical solutions of fractional differential equations with constant coefficients. Soham Transform is more multipurpose as the Laplace transform is limited to fractional differential equations. Soham Transform is in the development stage.

Keywords: Soham Transform; Fractional Differential Equations; Integral transforms; Reimann Liouville Fractional Integral; Caputo Fractional Derivative

1 Introduction

Fractional calculus relates to a classical standard result from differential and integral calculus. It deals with the study and applications of differential equations, and methods of solution of differential equations of arbitrary non-integer order. Soham transform is described in various fields. It has been widely used in engineering, control engineering, mechanics, science, bioengineering, image processing, visco-elasticity, etc. Integral transform was first developed by Pierre-Simmon Laplace in 1780. Since then, many researchers developed a lot of integral transforms of the Laplace type like Laplace, Sumudu, Elazaki, Kamal, Sawi, Snehu, Mahgoub, and Aboodh transforms. These are the essential mathematical tools for solving fractional differential equations. To find the analytical solution of fractional differential equations, Integral transforms are the most useful techniques of mathematics. A comprehensive study is required to explore the connection between the Soham transform and the Laplace transform and their potential applications. The need for more examples and applications of integral transform in solving fractional differential equations, such as in physics, engineering, and signal processing. Use integral transforms to solve different types of fractional differential equations, including those with non-constant coefficients and nonlinear terms.

Recently Patil Dinkar, Wagh Shivali Nanasaheb, and Bachhav Tejas Popat used the Solution of Abel's Integral Equation by using the Soham Transform [1]. Further Application of the Sawi transform of the error function is used by D. P. Patil for evaluating improper integrals for the error function [2]. Soham integral transform was introduced by S. S. Khakale and D. P. Patil. [3] to solve the ordinary differential equations. Recently Patil D. P. Rathi S. [4] used Soham transform to solve the system of ordinary differential equations of first order and first degree. Patil D., Suryawanshi Y., and Nehete M. [5] used the Soham transform to obtain the solution of the linear Volterra integral equation of the first kind. Fractional differential equations and methods of solution were well explained by I. Podlubny [6] in 1988. Mohamed Elarbi Benattia, and Kacem Belghaba [7] introduced the Application of the Aboodh Transform for Solving Fractional Delay Differential Equations. Aruldoss R. and Devi R. A. used the Aboodh transform for solving fractional differential equations [8]. Applications of "Emad-Sara Transform" for the Solution of System of Differential Equations introduced by D. P. Patil, A. V. Gavit, D. S. Gharate [9]. D. P. Patil, D. S. Shirsath, V. S. Gangurde, [10] introduced by Application of Soham Transform in Newton's Law of Cooling. D. P. Patil, P. S. Wagh, Pratiksha Wagh [11] introduced by Applications of Soham Transform in Chemical Sciences. D. P. Patil, R. K. Godage [12] introduced by Applications of Soham transform in Laminar Flow between Parallel Plates.

This paper includes some basic definitions and related methods to fractional calculus and also demonstrates the Soham transform method for solving fractional differential equations to obtain exact solutions with Caputo derivatives.

2 Methodology

Material and Methods

2.1 Basic Definitions:

In this section, we introduce some fundamental definitions and properties of fractional calculus and Soham transform.

Definition 1. Caputo Fractional Derivative

For $\alpha \epsilon \mathbb{R},\ \alpha > 0$, then Caputo fractional derivative of order $\alpha\, of\ f(x)$ is denoted by ${}^{C}D^{\alpha}\, f(x)$ and is defined by

$$^{C}D^{\alpha}\, f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^m(t)\, dt\ , \quad x > 0 \tag{1}$$

Where $m-1 \le \alpha \le m,\ m \in N^+\ and\, \Gamma(.)$ denotes the Gamma function.

Definition 2. Reimann-Liouville Fractional Integral

For $f(x) \in C_\mu,\ \mu \ge -1$ then the Reimann Liouville fractional integral I of $f(x)$ of order $\alpha \in \mathbb{R},\ \alpha > 0$ is denoted by $I^\alpha f(x)$ and is defined as

$$I_x^\alpha [f(x)] = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\, dt, \tag{2}$$

Where $x > 0\ and\ I^0 f(x) = f(x)$

Definition 3. Mittag-Leffler function of one parameter

For parameters $\alpha, \beta > 0$ the Mittag-Leffler function of one parameter α is denoted by $E_\alpha(x)$ and is defined as

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+1)}\ , \ \alpha > 0,\ x \in C \tag{3}$$

and $E_{\alpha,\beta}(x)$ is the Mittag-Leffler function with two parameters $\alpha\ and\ \beta$ is defined as

$$E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+\beta)},\ \ \alpha > 0,\ Re(\alpha),\ Re(\beta) > 0 \tag{4}$$

Where C is the set of complex numbers and the direct generalization of exponential series with $\beta = 1, \quad we\ get,\ E_{\alpha,1}(x) = E_\alpha(x)$.

Definition 4. Soham Transform

The Soham transform is defined as a function of exponential order. We consider functions in the set B defined by

$$B = \left\{ f(t) : \exists M,\ k_1,\ k_2 > 0,\ |f(t)| < Me^{|t}{}_j,\ if\, t \in (-1)^j \times [0,\infty) \right\} \tag{5}$$

For a given function in the set B, the constant M must be a finite number, k_1, k_2 may be finite or infinite.
Soham transform denoted by the operator $\delta(.)$ defined by the integral equations

$$\delta\left[f\left(t\right)\right] = P\left(v\right) = \frac{1}{v}\int_0^\infty f\left(t\right)e^{-v^\alpha t}dt \tag{6}$$

Where α is nonzero real numbers $t \geq 0, \quad k_1 \leq v \leq k_2$

Table 1. Soham Transform of Some Functions

Sr. No.	F(x)	$\delta\left(f\left(x\right)\right\} = P(v)$
1	1	$\frac{1}{v^{\alpha+1}}$
2	x	$\frac{1}{v^{2\alpha+1}}$
3	x^2	$\frac{1}{v^{3\alpha+1}}$
4	x^3	$\frac{1}{v^{4\alpha+1}}$
5	$x^m,\, m \geq 0$	$\frac{\Gamma(m+1)}{v^{m\alpha+\alpha+1}}$
6	e^{ax}	$\frac{1}{v(v^{\alpha}+a)}$
7	$sin(ax)$	$\frac{a}{v(v^{2\alpha}+a^2)}$
8	$cos(ax)$	$\frac{v^\alpha}{v(v^{2\alpha}+a^2)}$
9	$sinh(ax)$	$\frac{av}{(v^{2\alpha}-a^2)}$
10	$cosh(ax)$	$\frac{v^\alpha}{(v^{2\alpha}-a^2)}$

2.2 Soham Transform for Derivatives of Function *f(x)*

Let $\delta\left(f\left(x\right)\right\} = P\left(v\right)$ are:

$$(i)\ \delta\left\{f'\left(x\right)\right\} = v^\alpha P\left(v\right) - \frac{1}{v}f(0) \tag{7}$$

$$(ii)\ \delta\left\{f^{''(x)}\right\} = v^{2\alpha}P\left(v\right) - v^{\alpha-1}f\left(0\right) - \frac{1}{v}f^{'}\left(0\right) \tag{8}$$

$$(iii)\ \delta\left\{f^n\left(x\right)\right\} = v^{n\alpha}P\left(v\right) - \frac{1}{v}\sum_{k=0}^{n-1}v^{\alpha(n-1-k)}f^k\left(0\right) \tag{9}$$

2.3 Inverse Theorem for Soham Transform

Inverse Soham transform of $f(t)$ is $P(v)$ and δ^{-1} is called the inverse Soham transform operator then inverse Soham transform is defined as

$$\delta^{-1}\left[P\left(v\right)\right] = f\left(t\right) \tag{10}$$

2.4 Lemma 1: Convolution Theorem for Laplace Transform

If $l\left\{f\left(t\right)\right\} = f\left(s\right)$ and ℓ\{g(t)\}=g(s), then
$l\left\{f\left(t\right)*g(t)\right\} = l\left\{t\left(t\right)\right\}l\left\{g\left(t\right)\right\} = f\left(s\right)g\left(s\right)$

2.5 Lemma 2: Convolution Theorem for Soham Transform[5]

Let $f\left(x\right)\ and\ g(x)$ be two functions and $\delta\left\{f\left(x\right)\right\} = F(v)$and $\delta\left\{g\left(x\right)\right\} = G\left(v\right)$. The convolution of $f\ and\ g$ denoted by $f*g$ is the function on $u \geq 0$ given by

$$f*g\left(x\right) = \int_{u=0}^{x} f\left(u\right)g\left(x-u\right)du\ then\ \delta\left[f*g\left(x\right)\right] = v\delta\left[f\left(x\right)\right]\delta\left[g\left(x\right)\right] = vF\left(v\right)G\left(v\right) \tag{11}$$

3 Results and Discussion

3.1 Theorem 1

For $m-1<\alpha\le m,\ m\in N$ and if $P(v)$ is the Soham transform of $f(x)$, then the Soham transform of Riemann-Liouville fractional integral of order α is

$$\delta\left[I^{\alpha}f(x)\right]=\frac{1}{v^{n\alpha}}p(v) \tag{12}$$

Proof: Suppose the Riemann-Liouville fractional integral of function $f(x)$

$I_x^{\alpha}[f(x)]=\frac{1}{\Gamma(\alpha)}\int_0^x(x-\tau)^{\alpha-1}f(\tau)d\tau$

$I_x^{\alpha}[f(x)]=\frac{1}{\Gamma(\alpha)}[x^{\alpha-1}*f(x)]$

Applying the Soham transform to the above equation on both sides, we get,

$[I_x^{\alpha}f(x)]=\delta\left\{\frac{1}{\Gamma(\alpha)}[x^{\alpha-1}*f(x)]\right\}$

$=\frac{1}{\Gamma(\alpha)}\delta[x^{\alpha-1}*f(x)]$

By using the convolution theorem for the Soham transform of the above equation, we get,

$=\frac{1}{\Gamma(\alpha)}v\,\delta\left[x^{\alpha-1}\right]\delta[f(x)]$

$=\frac{1}{\Gamma(\alpha)}v\frac{\Gamma(\alpha-1+1)}{v^{n\alpha-n+n+1}}P(v)$

$=\frac{1}{\Gamma(\alpha)}\frac{v\,\Gamma(\alpha)}{v^{n\alpha+1}}P(v)$

$\delta\left[I^{\alpha}f(x)\right]=\frac{v}{v^{n\alpha+1}}p(v)=\frac{1}{v^{n\alpha}}P(v)$

Therefore,

$\delta\left[I^{\alpha}f(x)\right]=\frac{1}{v^{n\alpha}}P(v)$

The proof is completed.

3.2 Theorem 2

For $m-1<\alpha\le m,\ m\in N$ and if $P(v)$ is Soham transform of $f(x)$, then Soham transform of Caputo α fractional derivative of function $f(x)$ is

$$\delta\left[{}^{C}D^{\alpha}f(x)\right]=v^{n\alpha}P(v)-\frac{v^{n\alpha}}{v^{nm+1}}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0) \tag{13}$$

Proof: Suppose the Caputo fractional derivative of function $f(x)$ is

${}^{C}D^{\alpha}f(x)=I^{m-\alpha}D^{m}(x)=\frac{1}{\Gamma(m-\alpha)}\int_0^x(x-t)^{m-\alpha-1}f^{m}(t)\,dt$

Applying the Soham transform of the above equation on both sides, we get,

$\delta\left[{}^{C}D^{\alpha}f(x)\right]=\frac{1}{\Gamma(m-\alpha)}\delta\left[x^{m-\alpha-1}*f^{m}(t)\right]$

$=\frac{1}{\Gamma(m-\alpha)}v\,\delta\left[x^{m-\alpha-1}\right]*\left[f^{m}(t)\right]$

By using Equation (11), we get,

$=\frac{1}{\Gamma(m-\alpha)}v\frac{(m-\alpha-1+1)}{v^{n(m-\alpha-1)+n+1}}\left[v^{nm}P(v)-\frac{1}{v}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)\right]$

$=\frac{v}{v^{nm-n\alpha-n+n+1}}\left[v^{nm}P(v)-\frac{1}{v}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)\right]$

$=\frac{v}{v^{n(m-\alpha)+1}}\left[v^{nm}P(v)-\frac{1}{v}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)\right]$

$=\frac{1}{v^{(nm-n\alpha)}}\left[v^{nm}P(v)-\frac{1}{v}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)\right]$

$=\frac{1}{v^{(nm-n\alpha)}}v^{nm}P(v)-\frac{1}{v^{(nm-n\alpha)}}\frac{1}{v}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)$

$=v^{n\alpha}P(v)-\frac{v^{n\alpha}}{v^{nm+1}}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)$

Therefore,

$\delta\left[{}^{C}D^{\alpha}f(x)\right]=v^{n\alpha}P(v)-\frac{v^{n\alpha}}{v^{nm+1}}\sum_{k=0}^{m-1}v^{n(m-1-k)}f^{k}(0)$

The proof is completed.

3.3 Theorem 3

If $\alpha,\ \beta > 0$ and the Mittag- Leffler function with two parameters α and β is $E_{\alpha,\beta}(.)$ then we have inverse Soham transform formula is given by

$$\delta\left[x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha})\right] = \frac{1}{v}\int_0^{\infty} x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha})e^{-v^{\alpha}x}dx \tag{14}$$

Proof: By using the definition of the Soham transform of the function $x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha})$, we have

$\delta\left[x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha})\right] = \sum_{k=0}^{\infty}\frac{a^k x^{\alpha k}}{\Gamma(\alpha k+\beta)}\frac{1}{v}\int_0^{\infty} x^{\beta-1}e^{-v^{\alpha x}}dx$

$= \sum_{k=0}^{\infty}\frac{a^k}{\Gamma(\alpha k+\beta)}\,\delta\left[x^{\alpha k+\beta-1}\right]$

$= \sum_{k=0}^{\infty}\frac{a^k}{\Gamma(\alpha k+\beta)}\,\frac{\Gamma(\alpha k+\beta)}{v^{n\alpha k+n\beta-n+n+1}}$

$= \sum_{k=0}^{\infty}\frac{a^k}{v^{n\alpha k+n\beta+1}}$

$= \frac{1}{v^{n\beta+1}}\sum_{k=0}^{\infty}\left(\frac{a}{v^{n\alpha+n}}\right)^k \quad \left(\because\sum_{k=0}^{\infty}a^k = \frac{1}{1-a}\right)$

$= \frac{1}{v^{n\beta+1}}\frac{1}{1-\frac{a}{v^{n\alpha}}}$

$= \frac{v^{n\alpha}}{v^{n\beta+1}(v^{n\alpha}-a)}$

Therefore,

$\delta\left[x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha})\right] = \frac{v^{n\alpha}}{v^{n\beta+1}(v^{n\alpha}-a)}$

Applying inverse Soham transform on both sides, we get,

$x^{\beta-1}E_{\alpha,\beta}(ax^{\alpha}) = \delta^{-1}\left[\frac{v^{n\alpha}}{v^{n\beta+1}(v^{n\alpha}-a)}\right]$

The proof is completed.

3.4 Applications

In this section, we demonstrate the Soham transform method for solving some fractional differential equations to obtain exact solutions with Caputo derivatives.

Example 1. Consider the following linear inhomogeneous equation

$$^{C}D^{\alpha}y(x) + y(x) = \frac{2x^{2-\alpha}}{\Gamma(3-\alpha)} - \frac{x^{1-\alpha}}{\Gamma(2-\alpha)} + x^2 - x \tag{15}$$

Subject to the initial condition $y(0) = 0$, where $0 \le \alpha \le 1$.

Solution: Applying the Soham transform to the above Equation (15) on both sides, we get,

$$\delta[^{C}D^{\alpha}y(x) + y(x)] = \delta\left[\frac{2x^{2-\alpha}}{\Gamma(3-\alpha)}\right] - \delta\left[\frac{x^{1-\alpha}}{\Gamma(2-\alpha)}\right] + \delta\left[x^2\right] - \delta[x]$$

By using Equation (11) to the above equation, we get,

$(v^{n\alpha})P(v) + P(v) = \frac{2}{v^{\alpha(2-n)+\alpha+1}} - \frac{1}{v^{\alpha(1-n)+\alpha+1}} + \frac{2}{v^{3\alpha+1}} - \frac{1}{v^{2\alpha+1}}$

$= 2\left(\frac{1}{v^{\alpha(2-n)+\alpha+1}} + \frac{1}{v^{3\alpha+1}}\right) - \left(\frac{1}{v^{\alpha(1-n)+\alpha+1}} + \frac{1}{v^{2\alpha+1}}\right)$

$= 2\left(\frac{v^{n\alpha}}{v^{3\alpha+1}} + \frac{1}{v^{3\alpha+1}}\right) - \left(\frac{v^{n\alpha}}{v^{2\alpha+1}} + \frac{1}{v^{2\alpha+1}}\right)$

$(v^{n\alpha}+1)P(v) = 2\left(\frac{v^{n\alpha}}{v^{3\alpha+1}} + \frac{1}{v^{3\alpha+1}}\right) - \left(\frac{v^{n\alpha}}{v^{2\alpha+1}} + \frac{1}{v^{2\alpha+1}}\right)$

$(v^{n\alpha}+1)P(v) = 2\left(\frac{v^{n\alpha}+1}{v^{3\alpha+1}}\right) - \left(\frac{v^{n\alpha}+1}{v^{2\alpha+1}}\right)$

$P(v) = \frac{2}{v^{3\alpha+1}} - \frac{1}{v^{2\alpha+1}}$

Taking the inverse of the Soham transform, we get,

$\delta^{-1}(P(v)] = \delta^{-1}\left[\frac{2}{v^{3\alpha+1}}\right] - \delta^{-1}\left[\frac{1}{v^{2\alpha+1}}\right]$

$y(x) = x^2 - x$

Example 2. Consider the initial value problem of the non-homogeneous Bagley-Torvik equation

$$D^2y(x) + {}^{C}D^{\frac{3}{2}}y(x) + y(x) = 1 + x \tag{16}$$

Subject to the initial conditions $y(0) = y'(0) = 1$.

Solution: Applying the Soham transform of Equation (16) on both sides, we get,

$\delta\left[D^2 y(x)\right]+\delta\left[{}^C D^{\frac{3}{2}} y(x)\right]+\delta[y(x)]=\delta[1+x]$

Using theorem (2) we get,

$\left(v^{2\beta}P(v)-v^{\beta-1}y(0)-y^{'}(0)\right)+v^{\frac{3}{2}\beta}P(v)-\frac{v^{\frac{3}{2}\beta}}{v^{\beta m+1}}\left(v^{\beta(m-1)}y(0)+v^{\beta(m-2)}y^{'}(0)\right)+P(v)$
$=\frac{1}{v^{\beta+1}}+\frac{1}{v^{2\beta+1}}$

$\left(v^{2\beta}+v^{\frac{3}{2}\beta}+1\right)P(v)-\left(v^{\beta-1}+\frac{1}{v}+\frac{v^{\left(\frac{3}{2}\right)\beta}}{v^{\beta+1}}+\frac{v^{\left(\frac{3}{2}\right)\beta}}{v^{2\beta+2}}\right)=\frac{1}{v^{\beta+1}}+\frac{1}{v^{2\beta+1}}$

$\left(v^{2\beta}+v^{\frac{3}{2}\beta}+1\right)P(v)=\left(\frac{1}{v^{\beta+1}}+v^{\beta-1}+\frac{v^{\left(\frac{3}{2}\right)\beta}}{v^{\beta+1}}\right)+\left(\frac{1}{v^{2\beta+1}}+\frac{1}{v}+\frac{v^{\left(\frac{3}{2}\right)\beta}}{v^{2\beta+2}}\right)$

$\therefore P(v)=\frac{1}{v^{\beta+1}}+\frac{1}{v^{2\beta+1}}$

Applying the inverse Soham transform of the above equation on both sides, we get,

$\delta^{-1}[P(v)]=\delta^{-1}\left[\frac{1}{v^{\beta+1}}\right]-\delta^{-1}\left[\frac{1}{v^{2\beta+1}}\right]$

we get, the analytical solution of the above equation is,

$y(x)=1+x$

3.5 Discussion

Integral transforms are among the most useful techniques in mathematics, employed to find solutions to differential equations, partial differential equations, integro-differential equations, delay differential equations, and population growth. Integral transform methods are highly effective techniques in mathematics, often used to obtain the analytical solution of fractional differential equations. Many researchers have shown great interest in finding numerical solutions to both linear and non-linear fractional differential equations. The connection of the Soham transform with the Laplace transform goes much deeper. Soham transform might be a generalization of Laplace transform, but this needs to be rigorously proven and both transforms might share similar properties or applications, but this requires further investigation. In this paper, we also discuss the Soham transform of fractional integrals and derivatives and its application of the Soham transform to find the solution of fractional differential equation with Caputo sense.

4 Conclusion

This study solves some linear homogeneous and non-homogeneous fractional differential equations with constant coefficients using the Soham transform method. This study proved three theorems related to Soham transform method and some examples as the exact solution of fractional differential equations. The Soham transform method is a very effective tool for solving analytical solutions of fractional differential equations with constant coefficients.

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