INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 12-06-2024
Accepted: 20-07-2024
Published: 14-08-2024

Citation: Delphi AM (2024) A New Sandwich Theorem Relating to the Generalized Integral Operator for Univalent Analytic Functions. Indian Journal of Science and Technology 17(32): 3344-3349. https://doi.org/10.17485/IJST/v17i32.1877

* **Corresponding author**.

ali.m@csw.uobaghdad.edu.iq, alimathdelphi@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

A New Sandwich Theorem Relating to the Generalized Integral Operator for Univalent Analytic Functions

Ali Musaddak Delphi[1]*

1 Department of Mathematics, College of Science for Women, University of Baghdad, Iraq

Abstract

Objective: To determine subordination, inclusion, superordination and sandwich theorems for new subclass of analytic univalent functions relating to the generalized integral operator, within a unit disk V that is open. **Methods:** The procedures of the proof these theorems which used to find new results for this subject depend on the Lemma1.2 and Lemma1.3 mentioned inside this paper. **Finding:** The inclusion theorem for new subclass of univalent analytic function convolution with a generalized integral operator and the sandwich theorem were obtained using new findings for the subordination and superordination theorems. **Novelty:** This study is contributing to the existing literature on the topic of the convolution between univalent analytic function and generalized integral operator.

Keywords: Univalent functions; Analytic functions; Sandwich theorems; Subordination theorems; Superordination theorems

1 Introduction

The study of differential inequalities of real-valued functions was expanded to include complex-valued functions defined in the unit disk by Miller and Mocanu [1]. They created a brand-new approach to geometric function theory called the admissible functions or differential subordination technique. Sandwich theorem is a new topic that was introduced together with many classes of analytic functions using this highly effective approach of obtaining new conclusions, it is expressed and proven mainly for functions defined on the unit disk. there are two research gaps for this subject the first one how to estimate the coefficient bound of sandwich theorem and this lead to get geometric properties of univalent functions, such as distortion, covering and boundary behavior, exploring these properties in greater depth can lead to a deeper understanding of the theorem and the second research gap extending the results to more general domains, such as unit ball of C^n, is an active area of research. These research gaps are still open questions and challenges that need to be addressed. The specific needs and techniques for such extensions depend on the context and the particular goals of the research.

Let H use the form to represent the class of functions

$$\eta(s) = s + \sum_{k=2}^{\infty} e_k s^k, e_k \geq 0 \tag{1.1}$$

in $V = (s \in C : (s| < 1\}$, that are analytic and univalent.

The definition of the convolution of $\eta \& \psi \in H$ is

$$(\eta * \psi)(s) = z + \sum_{k=2}^{\infty} e_k h_k s^k = (\psi * \eta)(s),$$

And η is subordinate to ψ represented by $\eta \prec \psi$, if analytic function $y(s)$ meets the conditions $(y(z)| < 1$ and $y(0) = 0$, such that $\eta(z) = \psi(y(s))$. In other words, if $\eta(V)$ is contained in $\psi(V)$ with $\eta(0) = \psi(0)$ then $\eta \prec \psi$ (see[2]).

Defined M as the collection of each analytic and injective functions η on $\bar{V} \backslash L(\eta)$, thus

$$L(\eta) = \left\{c \in \partial V : \lim_{s \to c} \eta(s) = \infty\right\},$$

If $c \in \partial V \backslash L(\eta)$ then $\eta'(c) \neq 0$, (see[3])

Let $B[e,n] = \{\eta \in B : \eta(s) = e + e_n s^n + e_{n+1} s^{n+1} + \ldots, n \in z^+ \& e \in C\}$, and the analytic functions class in V is denoted by $B(V)$.

Miller and Mocanu in[4] take into consideration the problem of finding prerequisites to the admissible functions $\chi : C^3 \times V \to C$, for b is univalent in V with $\mu \in M$ and $\lambda \in B[e,n]$ that satisfy

$$\chi\left(\lambda(s), s\lambda'(s), s^2\lambda''(s); s\right) \prec b \tag{1.2}$$

this is leading to $\lambda(s) \prec \mu(s)$.

Also, in[5] take into consideration the problem of founding prerequisites to the admissible functions $\bar{y} : C^3 \times V \to C$, for $b \in B$, $\lambda \in M$ and $\mu \in B[e,n]$ that satisfy

$$b(s) \prec \bar{y}\left(\lambda(s), s\lambda'(s), s^2\lambda''(s); s\right) \tag{1.3}$$

In[6], the following definition is given for the integral operator $T^m(r,\epsilon)\eta(s)\,(r > 0; \epsilon \geq 0; m \in N_0)$ of a function $\eta \in H; s \in V$:

$$T^0(r,\epsilon)\eta(s) = \eta(s),$$

$$T^1(r,\epsilon)\eta(s) = \left(\frac{1+\epsilon}{r}\right) s^{1-\left(\frac{1+\epsilon}{r}\right)} \int_0^s t^{\left(\frac{1+\epsilon}{r}\right)-2} \eta(t)dt \quad ,$$

$$T^2(r,\epsilon)\eta(s) = \left(\frac{1+\epsilon}{r}\right) s^{1-\left(\frac{1+\epsilon}{r}\right)} \int_0^s t^{\left(\frac{1+\epsilon}{r}\right)-2} T^1(r,\epsilon)\eta(t)dt \quad ,$$

and, in general,

$$T^m(r,\epsilon)\eta(s) = \left(\frac{1+\epsilon}{r}\right) s^{1-\left(\frac{1+\epsilon}{r}\right)} \int_0^s t^{\left(\frac{1+\epsilon}{r}\right)-2} T^{m-1}(r,\epsilon)\eta(t)dt \quad ,$$

$$= T^1(r,\epsilon)\left(\frac{s}{1-s}\right) * T^1(r,\epsilon)\left(\frac{s}{1-s}\right) * \ldots * T^1(r,\epsilon)\left(\frac{s}{1-s}\right) * \eta(s)$$

$$(- - - - - - - - - - m - times - - - - - - - - - -] \tag{1.4}$$

FromEquation (1.1) & Equation (1.4) we deduce the following for $\eta(s) \in H$:

$$T^m(r,\epsilon)\eta(s) = s + \sum_{k=2}^{\infty}\left[\frac{1+\epsilon}{1+\epsilon+r(k-1)}\right]^m e_k s^k \tag{1.5}$$

It is simple to confirm from Equation (1.5) that

$$rs\left(T^{m+1}(r,\epsilon)\eta(s)\right)' = (1+\epsilon)T^m(r,\epsilon)\eta(s) - (1+\epsilon-r)T^{m+1}(r,\epsilon)\eta(s) \quad (r>0) \tag{1.6}$$

It should be noted that the sandwich theorem, for convolution of univalent analytic functions with other known operators on the unit disk, has been investigated recently in[7–16].

2 Methodology

Our subordination and superordination conclusion were established using the following lemmas as the methodology:

Lemma 2.1[5]

Let μ be univalent in V, $\gamma \in C/(0\}$, and

$$Re\left(1+\frac{s\mu''(s)}{\mu'(s)}\right) > max\left\{0, -Re\left(\frac{1}{\gamma}\right)\right\}.$$

If λ is analytic, $\lambda(0)=\mu(0)$ with

$$\lambda(s)+\gamma s\lambda'(s) \prec \mu(s)+\gamma s\mu'(s)$$

then $\lambda(s) \prec \mu(s)$.

Lemma 2.2[17]

Let μ be convex univalent & $\lambda(s)+\gamma s\lambda'(s)$ is univalent in V,$\mu(0)=e$, $Re(\gamma)>0$ and $\gamma \in C$. If $\lambda \in B[e,1] \cap M$ with

$$\mu(s)+\gamma s\mu'(s) \prec \lambda(s)+\gamma s\lambda'(s)$$

then $\mu(s) \prec \lambda(s)$.

3 Results and discussion

Theorem 3 .1.

Let μ be univalent in V, and $Re\left\{1+\frac{s\mu''(s)}{\mu'(s)}\right\} > max\{0, -Re(\frac{t}{\vartheta})\}$
with $\mu(0)=1, \vartheta \in C\setminus(0\}, t, \sigma > 0$, if $\eta \in H$ satisfies

$$\begin{gathered}\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^m(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^t \\ \left(1+\frac{\vartheta(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^m(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^m(r,\epsilon)\eta(s)}\right\}\right) \prec \mu(s)+\frac{\vartheta}{t}s\mu'(s),\end{gathered} \tag{2.1}$$

then

$$\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^m(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^t \prec \mu(s)$$

Proof: Let

$$\lambda(s) = \left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^m(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^t, (s \in V)$$

The logarithmic derivatives of λ are calculated, and the result is

$$\frac{s\lambda^{'}(s)}{\lambda(s)}=t\left[\frac{(1-\sigma)s\left(T^{m+1}(r,\epsilon)\eta(s)\right)^{'}+2\sigma s\left(T^{m}(r,\epsilon)\eta(s)\right)^{'}-(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)-2\sigma T^{m}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right] \tag{2.2}$$

We obtain the following subordination in as a result of Equation (1.6)

$$\frac{s\lambda^{'}(s)}{\lambda(s)}=\frac{t(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}$$

Therefore,

$$\frac{s\lambda^{'}(s)}{t}=\frac{(1+\epsilon)}{r}\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}$$
$$\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}$$

The hypothesis subordination Equation (2.1) is transformed into

$$\lambda(s)+\frac{\vartheta}{t}s\lambda^{'}(s)\prec\mu(s)+\frac{\vartheta}{t}s\mu^{'}(s)$$

Applying **Lemma 2.1** with $\gamma=\frac{\vartheta}{t}$, we now arrive the proof of our theorem.

Making use of the principle of subordination between analytic functions, we introduce the subclass $F_t^m\left(T,\eta,r,\epsilon,\sigma\right)$ of the class H which is defined by

$$F(m,\ T,\eta,r,\epsilon,\sigma)=\left\{\eta(s)\in H:\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}\prec\mu(s),\ s\in V\right\}$$

In the following theorem, we investigate the inclusion relations of the subclass $F(m,\ T,\eta,r,\epsilon,\sigma)$ associated with the integral operator $T^{m}(r,\epsilon)\eta(s)$.

Theorem 3.2

Let μ be univalent in V, and $Re\left\{1+\frac{s\mu^{''}(s)}{\mu^{'}(s)}\right\}>max\{0,-Re(\frac{t}{\vartheta})\}$

with $(0)=1,\vartheta\in C\setminus\{0\},t,\sigma>0$, and if $\eta\in H$, then

$$F(m,T,\eta,r,\epsilon,\sigma)\subset F(m+1,T,\eta,r,\epsilon,\sigma)$$

Proof: Let $\eta\in F(m,\ T,\eta,r,\epsilon,\sigma)$ and put

$$\lambda(s)=\left(\frac{(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)+2\sigma T^{m+1}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t},\ (s\in V)$$

The logarithmic derivatives of λ are calculated, and the result is

$$\frac{s\lambda^{'}(s)}{\lambda(s)}=t\left[\frac{(1-\sigma)s\left(T^{m+2}(r,\epsilon)\eta(s)\right)^{'}+2\sigma s\left(T^{m+1}(r,\epsilon)\eta(s)\right)^{'}-(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)-2\sigma T^{m+1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)+2\sigma T^{m+1}(r,\epsilon)\eta(s)}\right] \tag{2.3}$$

We obtain the following subordination as a result of Equation (1.6)

$$\frac{s\lambda^{'}(s)}{\lambda(s)}=\frac{t(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m+1}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+2}(r,\epsilon)\eta(s)\ +2\sigma T^{m}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)+2\sigma T^{m+1}(r,\epsilon)\eta(s)}\right\}$$

Therefore,

$$\frac{s\lambda^{'}(s)}{t}=\frac{(1+\epsilon)}{r}\left(\frac{(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)+2\sigma T^{m+1}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}$$
$$\left\{\frac{(1-3\sigma)T^{m+1}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+2}(r,\epsilon)\eta(s)\ +2\sigma T^{m}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+2}(r,\epsilon)\eta(s)+2\sigma T^{m+1}(r,\epsilon)\eta(s)}\right\}$$

The hypothesis subordination Equation (2.1) is transformed into

$$\lambda(s)+\frac{\vartheta}{t}s\lambda^{'}(s) \prec \mu(s)+\frac{\vartheta}{t}s\mu^{'}(s)$$

Applying **Lemma 2.1** with $\gamma=\frac{\vartheta}{t}$, we see that $\lambda(s)\prec\mu(s)$, that is $\eta\in F(m+1,\ T,\eta,r,\epsilon,\sigma)$ and so $F(m,\ T,\eta,r,\epsilon,\sigma)\subset F(m+1,\ T,\eta,r,\epsilon,\sigma)$

Theorem 3.3

Let μ be convex univalent in, $\mu(0)=1,\vartheta\in C$, $Re(\vartheta)>0$ and $t>0$.

If $\eta\in H$ and

$$\mu(s)+\frac{\vartheta}{t}s\mu^{'}(s)\prec\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t} \left(1+\frac{\vartheta(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}\right) \tag{2.3}$$

with

$$\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t} \left(1+\frac{\vartheta(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}\right)$$

Be univalent in V, and

$$\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}\in B[\mu(0),1]\cap M,$$

then

$$\mu(s)\prec\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}$$

Proof: Let $\lambda(s)=\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t},\ \ (s\in V)$

If we follow the method used to prove **Theorem 3.1.**, the next form for the subordination Equation (2.3) is

$$\mu(s)+\frac{\vartheta}{t}s\mu^{'}(s) \prec\lambda(s)+\frac{\vartheta}{t}s\lambda^{'}(s),$$

To complete the proof use **Lemma 2.2** with $\gamma=\frac{\vartheta}{t}$

Next, by using the previous result, we obtain the following Sandwich theorem

Theorem 3.4.

Let μ_1,μ_2 be convex univalent in $V,\mu_1(0)=\mu_2(0)=1,\vartheta\in C$, $Re(\vartheta)>0$

and $t>0$. If $\eta\in H$ and

$$\mu_1(s)+\frac{\vartheta}{t}s\mu_1^{'}(s)\prec\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t} \left(1+\frac{\vartheta(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\ +2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}\right)\prec\mu_2(s)+\frac{\vartheta}{t}s\mu_2^{'}(s) \tag{2.4}$$

with

$$\left(\frac{(1-\sigma)\ T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}\in B[\mu(0),1]\cap M$$

and

$$\left(\frac{(1-\sigma)\,T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}$$
$$\left(1+\frac{\vartheta(1+\epsilon)}{r}\left\{\frac{(1-3\sigma)T^{m}(r,\epsilon)\eta(s)-(1-\sigma)T^{m+1}(r,\epsilon)\eta(s)\;+2\sigma T^{m-1}(r,\epsilon)\eta(s)}{(1-\sigma)\,T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}\right\}\right)$$

be univalent in V, then

$$\mu_1(s)\prec\left(\frac{(1-\sigma)\,T^{m+1}(r,\epsilon)\eta(s)+2\sigma T^{m}(r,\epsilon)\eta(s)}{(1+\sigma)s}\right)^{t}\prec\mu_2(s)$$

Proof: By combining the proof of **Theorem 3.1.** and **Theorem 3.3.**, we complete this proof.

4 Conclusion

A new subclass of univalent analytic function convolution with a generalized integral operator on an open unit disk is derived in the current study. Additionally, the sandwich and inclusion theorems are also derived for this new subclass. In the future, these results can be employed to derive coefficient estimates and geometric function theory properties.

References

1) Miller S, Mocanu PT. differential subordinations and univalent functions. *Michigan Mathematical Journal*. 1981;28(2):157–172. Available from: https://doi.org/10.1307/mmj/1029002507.
2) Bulbocaco T. Differential subordinations and superordinations. Cluj-napoca. 2005. Available from: https://scholar.google.com/citations?view_op=view_citation&hl=en&user=U9eU6v4AAAAJ&citation_for_view=U9eU6v4AAAAJ:d1gkVwhDpl0C.
3) Miller S, Mocanu PT. Subordinants of differential superordinations. Complex Variables, Theory and Application. *An International Journal*. 2003;48(10):815–826. Available from: https://doi.org/10.1080/02781070310001599322.
4) Miller S, Mocanu PT. Differential subordinations: Theory and Applications. *Differential subordinations: Theory and Applications, Series on Monographs and Text Books in Pure and Applied Math, 225Marcel Dekker , New Yourk and Basel*. 2000. Available from: https://doi.org/10.1201/9781482289817.
5) Miller S, Mocanu PT, T P. Briot-Bouquet differential superordinations and sandwich therorems. *Journal of Mathematical Analysis and Applications*. 2007;329(1):327–335. Available from: https://doi.org/10.1016/j.jmaa.2006.05.080.
6) El-Ashwah R, Aouf MK. Differential subordination and superordination for certain subclasses of analytic functions involving an extended integral operator. *Differential subordination and superordination for certain subclasses of analytic functions involving an extended integral operator Acta Universitatis Apulensis*. 2011;28:341–350. Available from: https://www.emis.de/journals/AUA/acta28/Paper30-Acta28-2011.pdf.
7) El-Deeb S, Bulbocaco T. Differential sandwich-type results for symmetric functions connected with a q-analog integral operator. *Mathematics*. 1185;7:1–17. Available from: https://doi:10.3390/math7121185.
8) Cotirla LI, Catas A, A. A Differential sandwich theorem for analytic functions defined by an integral operator. *Acta Universitatis Apulensis: Mathematics-Informatics*. 2020. Available from: https://doi.org/10.17114/j.aua.2020.62.02.
9) Al-Ameedee SA, Atshan WG, Al-Maamori FA. On sandwich results of univalent functions defined by a linear operator. *Journal of Interdisciplinary Mathematics*. 2020;23(4):803–809. Available from: https://doi:10.1080/09720502.2020.1727613.
10) Atshan WG, Ali AAR. On sandwich theorems results for certain univalent functions defined by generalized operators. *Iraqi Journal of Science*. 2021;62(7):2376–2383. Available from: https://doi:10.24996/ijs.2021.62.7.27.
11) Bakheet SR, Atshan WG. Third-order sandwich results for analytic univalent functions defined by integral operator. *Advance in Mechancics*. 2022;10:1178–1197. Available from: https://advancesinmechanics.com/pdf/1178.pdf.
12) Catas A, Borsa E, Lambor L. Best dominants and subordinants for certain sandwich-type theorems. *Symmetry*. 2022;4(62):1–12. Available from: https://doi.org/10.3390/sym14010062.
13) Delphi A, Jassim KA. Results of differential sandwich theorem of the univalent functions associated with generalized salageon integro-differential operator. *Journal of Interdisciplinary Mathematics*. 2022;25(8):2573–2577. Available from: https://doi.org/10.1080/09720502.2022.2094095.
14) Çetinkaya A, Cotirla LI. Briot-Bouquet differential subordinations for analytic functions involving the struve function. *Fractal and fraction*. 2022;6:1–12. Available from: https://doi.org/10.3390/fractalfract6100540.
15) Mihsin BK, Atshan WG, Alhily SS. On new sandwich results of univalent functions defined by alinear operators. *Iraqi Journal of Science*. 2023;63(12):5467–5475. Available from: https://doi:10.24996/ijs.2022.63.12.32.
16) Jasim H, Atshan WG. Some differential subordinations and superordinations results for analytic univalent function using Theyab-Atshan-Lupas-Abdullah operator. *Internation Journal of nonlinear Analysis and Applications*. 2023;14(8):45–54. Available from: http://dx.doi.org/10.22075/ijnaa.2023.21505.4244.
17) Alhily SS, Lupas AA. Sandwich theorems involving fractional integrals applied to the q-analogue of the multiplier transformation. *AIMS Mathematics*. 2024;9(3):5850–5862. Available from: https://doi.org/10.3934/math.2024284.