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Discrete Model of Two Species Neutralism with Limited Resources

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Abstract

Objectives: To study the discrete model of two-species neutralism in ecology with natural growth rates and death rates. **Method:** The system comprises of two neural species S_1 and S_2 with limited resources. This model consists of two first-order non-linear coupled ordinary difference equations as its core equations. The system's biological viability is examined. **Findings:** Using the model equations, all potential equilibrium locations are found, and the stability requirements for each are explored. If the absolute value of each characteristic equation eigenvalue is less than one, the model is stable; if not, it is unstable. **Novelty:** Neutralism is the absence of interaction between the constituent parts of a mixed organism. The species do not know one another, do not harm one another, and do not gain anything from one another even if they coexist. As an example from real life, frogs and bunnies live side by side in a grassland with little interaction between them.

Keywords: Characteristic equation; Eigen value; Equilibrium point; Stable; Unstable

1 Introduction

Ecology is the study of the interactions between organisms and their environment. The organisms include animals and plants, the environment includes the surroundings of animals. The study of living things (plants and animals) in connection to their environments and habits is known as ecology. This discipline of knowledge is a branch of evolutionary biology purported to explain how or to what extent the living beings are regulated in nature. Allied to the problem of population regulation is the problem of species distribution- prey-predator, competition and so on. The subject of ecology can be broadly sub-divided as auto-ecology (the study of single species populations) and synecology (the study of two or more communities). Synecological studies lead to the concept of the eco-system. This concept is a direct outcome of the intensive work of several life scientists/biologists and botanists of many generations. An eco-system may be considered as a unit that includes animals, plants and the physical environment in which these live. This area of knowledge seeks to explain how many different kinds of plants and animals can live together in the same place for many generations.

Animals and plants share the same habitat. Sometimes they can only share for so long before some locally go extinct, but there are other circumstances when many different kinds persist in a habitat indefinitely. As such, ecology may also be referred as the study of distribution and abundance of species under habitat availing the same resources. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation and so on. The dynamics of a diffusive three-species Lotka–Volterra system with delays were investigated in[1]. A partial differential equation system was approximated mathematically using delay differential equations and the Galerkin Method, which yields semi-analytical solutions.[2] put out a fractional-order predator-prey model that takes into account predator species' anti-predator and prey refuge behaviours. He provides evidence for the existence, singularity, non-negativity, and infinite nature of solutions. The evolution of discrete time models from the Malthus model to contemporary population models that consider a wide range of variables impacting the dynamics and structure is discussed. The authors' most significant and intriguing findings from their investigation of biological systems using recurrent equations[3]. The proposed fractional order system's equilibrium points' existence and stability criteria are ascertained by analysing a fractional order eco-epidemiological system with infected prey, as researched by the author[4]. And[5] offers a thorough overview of the state of the art at the moment for using mathematical models to analyse how population pressure and human population growth affect wildlife that depends on forests and forest biomass.

The mathematical examination of specific aspects of numerous disciplines is known as mathematical modelling, and it is a noteworthy illustration of an interdisciplinary activity. Biology, epidemiology, physiology, ecology, sociology, immunology, bio-economics, genetics, and pharmacokinetics are a few of these disciplines. Since its founding, it has donated its excellent services to the growth of science and technology in general and engineering in particular. These days, it forms the basis for new developments in contemporary science. Each branch of mathematics has a unique relevance, and its influence spans multiple dimensions. In recent years, mathematical modelling has reached its pinnacle, permeated every aspect of society, and captured the interest of all. Since the beginning of the previous century, statisticians and mathematicians have been drawn to all biological populations that exist in natural environments. A practical analysis of these populations results in mathematical modelling of various species residing in various environments. In these cases, physical and environmental parameters are crucial in producing suitable models for mathematical expressions such as differential and difference equations, which are typically non-linear for various animal and microbial population categories. Using a fractional order prey predator model with toxic harvesting, the author[6] investigates the dynamics of a fishery in an aquatic setting with two zones: an open access fishery zone and a restricted area for prey where fishing is strictly forbidden.[7] examined a scenario in which two rival predators fight over a single prey, and the predators' fear of the prey affects the prey's ability to reproduce. In multi-species communities, distinct biotic interactions are a pervasive force in the natural ecosystem that play a crucial role in determining the stability of the community and the results of species coexistence[8]. Further,[9] introduced the mathematical modeling of the Ecological Systems Algorithm, a novel evolutionary search algorithm influenced by biology. A mathematical model for analyzing obesity and overweight in a population and how it affects the rise in the number of diabetes is presented by the author[10]. He considers social issues and the relationships between various societal elements when building the model.

***Notation Adopted*:**

$N_i(t)$: The population strength of species S_i, t: Time instant, a_i: Natural growth rates of S_i, a_{ii}: Self-inhibition coefficient of S_i, d_i: Natural death rates of S_i, i = 1, 2. Further, the variables N_1, and N_2 are non-negative, and the model parameters a_1, a_2, a_{11}, d_1, a_{22}, and d_2 are assumed to be non-negative constants.

1.1. Model I: Both the Species with Natural Growth Rates:

Saxena[11] refers to a broad category of both interacting and non-interacting, infinitely numerous living things on Earth's surface that share fundamental characteristics including growth and reproduction, decay and death, and migration between and across domains.[12] Investigates the processes of birth and death in interacting random settings where there is a mutual dependence between the dynamics of the environment's state and the rates of birth and death.

Consider the non-linear autonomous system of discrete difference equations

$$N_i(t) - N_i(t-1) = a_i N_i(t-1) - a_{ii} N_i^2(t-1),\ i = 1,2 \tag{1}$$

The above equations can be written in terms of recurrence relations as

$$N_i(t) = \alpha_i N_i(t-1) - a_{ii} N_i^2(t-1);\ where\, \alpha_i = (a_i + 1) \geq 1\,,\ i = 1,2$$

The equilibrium states for the given system are obtained by solving the equations at

$$N_i(t+1) = N_i(t)\,,\ i = 1,2$$

Fully washed out state:

$$E_1 : \overline{N}_1 = 0, \overline{N}_2 = 0$$

The first species washed out state:

$$E_2 : \overline{N}_1 = 0, \overline{N}_2 = a_2/a_{22}$$

The second species washed out state:

$$E_3 : \overline{N}_1 = a_1/a_{11}, \overline{N}_2 = 0$$

Normal steady state:

$$E_4 : \overline{N}_1 = a_1/a_{11}, \overline{N}_2 = a_2/a_{22}$$

1.2. Model II: Both the Species with Natural Death Rates:

Consider the non-linear autonomous system of discrete difference equations

$$N_i(t) - N_i(t-1) = -d_i N_i(t-1) - a_{ii} N_i^2(t-1),\ i = 1,2 \tag{2}$$

The Equations (1) and (2) can be written in terms of recurrence relations as

$$N_i(t) = \alpha_i N_i(t-1) - a_{ii} N_i^2(t-1);\ where\ \alpha_i = 1 - d_i,\ i = 1,2$$

The equilibrium states for the given system are obtained by solving the equations at

$$N_i(t+1) = N_i(t)\ ,\ i = 1,2$$

Fully washed out state:

$$E_1 : \overline{N}_1 = 0, \overline{N}_2 = 0$$

The second species survives:

$$E_2 : \overline{N}_1 = 0, \overline{N}_2 = -d_2/a_{22}$$

The first species survives:

$$E_3 : \overline{N}_1 = -d_1/a_{11}, \overline{N}_2 = 0$$

Normal steady state:

$$E_4 : \overline{N}_1 = -d_1/a_{11}, \overline{N}_2 = -d_2/a_{22}$$

2 Methodology

2.1. Stability Analysis

2.1.1. Stability of the Equilibrium States for Model I:

Mukherjee[7] examined a model in which two rival predators share a common prey, and the fear the predators instill in the prey affects the prey's ability to reproduce.[13] uses both analytical techniques to investigate the dynamic behaviour of a certain class of discrete predator-prey system featuring both refuge and fear effect.

The basic equations can be linearized about the equilibrium point $E(\overline{N}_1, \overline{N}_2)$

We get, in the matrix notation as $N(t) = A.N(t-1)$ where $A = diagonal\left[1 + a_1 - 2a_{11}\overline{N}_1, 1 + a_2 - 2a_{22}\overline{N}_2\right]$ and $N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$. The characteristic equation for the system is det $[A - \lambda I] = 0$.

The equilibrium point $E(\overline{N}_1, \overline{N}_2)$ is stable, if absolute value of each of the eigen values of the characteristic equation is less than one.

2.1.2. Stability of the Equilibrium States for Model II:

The basic equations can be linearized about the equilibrium point $E(\overline{N}_1, \overline{N}_2)$We have, in the matrix notation as $N(t) = B.N(t-1)$ where $B = diagonal\left[1-d_1-2a_{11}\overline{N}_1, 1-d_2-2a_{22}\overline{N}_2\right]$ The characteristic equation for the system is det $[B-\lambda I]=0$

3 Results and Discussion

3.1. Stability Analysis of Model I:

The eigen values of the characteristic equation of E_1 are a_1+1, a_2+1.Since, the absolute values of these both are greater than or is equal to one. Hence, the fully washed out state is **unstable**.

The absolute value of a_1+1 in the characteristic equation of E_2 is greater than or is equal to one. Hence, the first species washed out state is **unstable**.

The absolute value of a_2+1 in the characteristic equation of E_3 is greater than or is equal to one. Hence, the second species washed out state is **unstable**.

The absolute value of both eigen values is less than one in the characteristic equation of E_4 only when $a_1, a_2 < 2$. Therefore, the normal steady state is **stable**, only when $a_1, a_2 < 2$.

3.2. Stability Analysis of Model II:

The eigen values of the characteristic equation of E_1 are $1-d_1$, $1-d_2$.Since, the absolute values of these both are less than one only when $d_1, d_2 < 2$. Hence, the fully washed out state is **stable**.

The absolute values of $1+d_2$ and $1+d_1$in the characteristic equation of E_2 and E_3 are greater than one. Hence, the second and third equilibrium states **unstable**.

The eigen values of the characteristic equation of E_1 are $1+d_1$, $1+d_2$.Since, the absolute values of these both are greater than. Hence, the normal steady state is **unstable**.

4 Conclusion

The two discrete models of two interacting species neutralism in ecology with natural growth and mortality rates are the subject of this research. We established every potential equilibrium state for two models in this work. It is concluded that in model I, only the fully washed out condition is stable in all four equilibrium stages, but in model II, only the typical steady state is stable. Examination of a few relationships between species, including those involving competition, mutualism, agonistic relationships, neutralism, commensalism, ammensalism, and predation in relation to population dynamics and the additional response function types, combination harvesting, selective harvesting, and variable rate harvesting, in addition to constant rate harvesting, can all be added to these models. Further research will be undertaken in these directions.

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