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An Infinite Capacity Single Server Markovian Queueing System With Discouraged Arrivals Retention Of Reneged Customers and Controllable Arrival Rates With Feedback

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Abstract

Objectives: The purpose of this study is to (i) introduce an infinite capacity of interdependent queueing model and retention of reneged customers with feedback controllable arrival rates, (ii) compute the mean quantity of clients within the system and determining the anticipated waiting time of those clients, (iii)deal with the model descriptions, steady-state equations, and characteristics, which are expressed in terms and (iv) analyze the probabilities of the queueing system and its characteristics with numerical verification of the obtained results. **Methods :** The Poisson process is used to manage the arrival rates through quicker and slower arrival rates while delivering the input. The service additionally offers an exponential distribution. The service is supplied by the server using FCFS. In this article, two types of models are used which are the system's conditions, where represents the number of units present in the queue in which their probability and all probabilities are distributed based on the speed of advent using this concept. Then, the steady-state probabilities are computed using a recursive approach. **Findings:** This paper discovers the number of clients using the system on average and the expected number of clients in the system using the probability of the steady-state calculation. Little's formula is used to derive the expected waiting period of the clients in the system. **Novelty:** There are articles connected to the finite capacity of failed service in functioning and malfunctioning, but this takes the initiative to provide a link in connection with the rates of the controllable arrivals and inter dependency in the arrival and service processes. Mathematics Subject allocation: 60K25, 68M20, 90B22.

Keywords: Infinite Capacity; Interdependent Controllable; Arrival and Service rates; Reverse balking; Single Server; Bivariate Poisson process and feedback

1 Introduction

Queueing models are used in a variety of real-world contexts to facilitate the effective design, analysis, and assessment of various technological systems and their behaviour, including client waiting times, estimated customer counts, etc. Queueing models are more practical for analysing situations in real life. These explanations assist researchers in carefully choosing the conditions and maintaining an appropriate estimate to accurately determine the stream.

When a consumer enters a line and later chooses to depart it without being served is called reneging. A customer who may choose to come again to the queue until the service is completed is said to be feedback. A reneged customer has a chance of being induced to remain in the queue for more services with a probability of q_1, and he has a chance of leaving the queue without obtaining the service with a probability of p_1. A customer again goes to the queue due to unsatisfied previous service with probability q_2 and he is satisfied by having a chance of acquiring the service p_2.

Lines filled with dejected newcomers have computer programs that process jobs. When the system is utilized often, job submissions are discouraged, and arrivals are represented as a Poisson process with a state-dependent arrival rate. Morse considered dismay in which, based on a declining arrival rate the inverse exponential law. Haight[1] was the first to examine queuing with reneging. He waited in line and focused on how to make a decision that would have the least number of negative consequences. Ancker and Gafarian have studied "M/M/1/N queueing systems with balking and reneging and performing its steady state analysis".

Srinivasa Rao. K and Shobha. They discussed "the M/M/1 interdependent queueing model with a controllable arrival rate". Srinivasan and Thiagarajan have analyzed "the M/M/1/K interdependent queueing model with controllable arrival rates and M/M/C/K/N interdependent queueing model with controllable arrival rates, balking, reneging and spares".Kapodistriais educated in"a single server markovian queue with impatient customersand considered the situations, where customers abandoned the system concurrently".

Kumar and Sharma have utilised" the M/M/1/N queueing system with retention ofreneged customers". Rakesh Kumar and S K Sharma have analysed "a single server Markovian queueing system with discouraged arrivals and retention of reneged customers".Thiagarajan and Premalatha have elaborated on "a single server Markovianqueueingsystem with discouraged arrivals retention of reneged customers and controllable arrival rates".

Nithya, N., Anbazhagan, N., Amutha, S., Jeganathan, K., Park, G. C., Joshi, G. P., & Cho, W. (2023)[2] from this paper I have studied how the controlled arrivals on the retrial queueing inventory system which are essential interruption for vacation server.Sasikala, S., and V. Abinaya[3], I studied about "The M/M/C/N Interdependent Inter Arrival Queueing Model with Controllable Arrival Rates, Reverse Balking and Impatient Customers" by using the old method for Reverse Balking. Sindhu, S., AchyuthaKrishnamoorthy, and Dmitry Kozyrev[4] "On Queues with Working Vacation and Interdependence in Arrival and Service Processes" how vacation works an independence in process of arrival and service. Yue, Rui, Guangchuan Yang, YichenZheng, and ZongTian[5] "Capacity estimation at all-way stop-controlled intersections considering pedestrian crossing effects" used to know about pedestrian crossing effect.

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2 Methodology

2.1 The model's description

For this model I take one server with an infinite capacity of the system retention of reneged customers with discouraged arrivals.Customer arrivals are independent, identically, and exponentially distributed with the rate according to the Poisson flow of rate $\lambda_0$and λ_1, which relies on the number of customers in the system and service timeμ.

According to the arrival order, customers are waiting in the queue. Each consumer must wait a specific amount of time before receiving service after joining the queue. If it doesn't start by then, he will get anxious and either leave the line without receiving service with probabilityp_1 or stay in line to receive with probability q_1.

The consumer can either exit the system with probability p_2 or join it as a feedback customer with probability q_2 after receiving the service. The reneging time with feedback follows an exponential distribution with parameter ξ.

$$P[X_1(t) = x_1, X_2(t) = x_2$$

$$= e^{-(\lambda_i+\mu-\epsilon)t} \sum_{j=0}^{min(x_1,x_2)} \frac{(\epsilon t)^j ((\lambda_i-\epsilon)t]^{x_1-j} ((\mu-\epsilon)t]^{x_2-j}}{j!(x_1-j)!(x_2-j)!}$$

where$x_1, x_2 = 0,1,2,\ldots$ $\lambda_i > 0$, $i = 0,1$; $\mu > 0$, $0 \leq \epsilon < min(\lambda_i, \mu)$, $i = 0,1$

$X_1(t), X_2(t)$represents the arrival process and the service process respectively.

3 Results and Discussion

3.1 Steady State Equations

We observe that $P_n(0)$occurs when$0 \leq n \leq r-1$; both $P_n(0)$ and $P_n(1)$ exists when $r+1 \leq n \leq R-1$: only $P_n(1)$occurs when$R \leq n \leq \infty$.

$$-(\lambda_0-\epsilon)P_0(0) + p_2(\mu-\epsilon)P_1(0) = 0 \tag{1}$$

$$\begin{aligned} -\left[\frac{\lambda_0-\epsilon}{n+1} + p_2(\mu-\epsilon) + (n-1)\xi p_1\right] P_n(0) + (p_2(\mu-\epsilon) + n\xi p_1] P_{n+1}(0) \\ + \left(\frac{\lambda_0-\epsilon}{n}\right) P_{n-1}(0) = 0,\ 1 \leq n \leq r-1 \end{aligned} \tag{2}$$

$$\begin{aligned} -\left[\frac{\lambda_0-\epsilon}{r+1} + p_2(\mu-\epsilon) + (r-1)\xi p_1\right] P_r(0) + (p_2(\mu-\epsilon) + r\xi p_1] P_{r+1}(0) \\ + \left(\frac{\lambda_0-\epsilon}{r}\right) P_{r-1}(0) + (p_2(\mu-\epsilon) + r\xi p_1] P_{r+1}(1) = 0 \end{aligned} \tag{3}$$

$$-\left[\frac{\lambda_0-\epsilon}{n+1}+p_2(\mu-\epsilon)+(n-1)\xi p_1\right]P_n(0)+(p_2(\mu-\epsilon)+n\xi p_1]P_{n+1}(0) + \left(\frac{\lambda_0-\epsilon}{n}\right)P_{n-1}(0)=0,\quad r+1\leq n\leq R-2 \tag{4}$$

$$-\left[\frac{\lambda_0-\epsilon}{R}+p_2(\mu-\epsilon)+(R-2)\xi p_1\right]P_{R-1}(0)+\left(\frac{\lambda_0-\epsilon}{R-1}\right)P_{R-2}(0)=0 \tag{5}$$

$$-\left[\frac{\lambda_1-\epsilon}{r+2}+p_2(\mu-\epsilon)+r\xi p_1\right]P_{r+1}(1)+(p_2(\mu-\epsilon)+(r+1)\xi p_1]P_{r+2}(1)=0 \tag{6}$$

$$-\left[\frac{\lambda_1-\epsilon}{n+1}+p_2(\mu-\epsilon)+(n-1)\xi p_1\right]P_n(1)+(p_2(\mu-\epsilon)+n\xi p_1]P_{n+1}(1) + \left(\frac{\lambda_1-\epsilon}{n}\right)P_{n-1}(1)=0, n=r+2,r+3,...,R-1 \tag{7}$$

$$-\left[\frac{\lambda_1-\epsilon}{R+1}+p_2(\mu-\epsilon)+(R-1)\xi p_1\right]P_R(1)+(p_2(\mu-\epsilon)+R\xi p_1]P_{R+1}(1) + \left(\frac{\lambda_1-\epsilon}{R}\right)P_{R-1}(1)+\left(\frac{\lambda_0-\epsilon}{R}\right)P_{R-1}(0)=0 \tag{8}$$

$$-\left[\frac{\lambda_1-\epsilon}{n+1}+p_2(\mu-\epsilon)+(n-1)\xi p_1\right]P_n(1)+(p_2(\mu-\epsilon)+n\xi p_1]P_{n+1}(1) + \left(\frac{\lambda_1-\epsilon}{n}\right)P_{n-1}(1)=0,\quad R+1\leq n\leq\infty \tag{9}$$

let $S_1=\frac{\lambda_0-\epsilon}{p_2(\mu-\epsilon)}, T_1=\frac{\lambda_1-\epsilon}{p_2(\mu-\epsilon)}$ and $E_1=\frac{\xi p_1}{p_2(\mu-\epsilon)}$

From (1) and (2) we get,

$$P_n(0)=\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{S_1}{1+(k-1)E_1}\right]P_0(0), n=1,2,...,r \tag{11}$$

Using (11) in (3) then we get,

$$P_{r+1}(0)=\left[\frac{1}{(r+1)!}\prod_{k=1}^{r+1}\frac{S_1}{1+(k-1)E_1}\right]P_0(0)-P_{r+1}(1)$$

Using the above result in (4) we get,

$$P_n(0)=\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{S_1}{1+(k-1)E_1}\right]P_0(0)-\frac{P_{r+1}(1)}{\prod_{l=r+1}^{n-1}[1+lE_1]}.A_r \tag{12}$$

where $A_r=\frac{{S_1}^{n-(r+1)}}{nP_{n-(r+1)}}+\frac{{S_1}^{n-(r+2)}}{nP_{n-(r+2)}}[1+rE_1]+...+[1+rE_1][1+(r+1)E_1]...[1+(n-3)E_1][1+(n-2)E_1], n=r+1,r+2,...R-1$

Using (12) in (5) we get,

$$P_{r+1}(1)=\left[\frac{\frac{S_1^R}{R!}\prod_{k=1}^{r+1}\frac{1}{1+(k-1)E_1}}{B_r}\right]P_0(0) \tag{13}$$

Where $B_r = \frac{S_1{}^{R-(r+1)}}{RP_{R-(r+1)}} + \frac{S_1{}^{R-(r+2)}}{RP_{R-(R+2)}}(1+rE_1]+...+(1+rE_1](1+(r+1)E_1]...[1+(R-2)E_1]$

Using (13) in (6) we get,

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^{n-1}[1+lE_1]}.C_r \tag{14}$$

where $C_r = \frac{T_1{}^{n-(r+1)}}{nP_{n-(r+1)}} + \frac{T_1{}^{n-(r+2)}}{nP_{n-(r+2)}}[1+rE_1]+...+[1+rE_1][1+(r+1)E_1]...[1+(n-2)E_1], n = r+1, r+2, ..., R-1, R$

Using (12) and (14) in (8)

$$\begin{aligned} P_{R+1}(1) &= \frac{P_{r+1}(1)}{\prod_{l=r+1}^{R}(1+lE_1]}\{\frac{T_1{}^{R-r}}{(R+1)P_{R-r}} + \frac{T_1{}^{R-(r+1)}}{(R+1)P_{R-(r+1)}}[1+rE_1]+... \\ &+ \frac{T_1}{(R+1)P_1}[1+rE_1]...[1+(R-2)E_1]\} \end{aligned} \tag{15}$$

Using (14) and (15) in (9) we get,

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^{n-1}[1+lE_1]}D_r \tag{16}$$

where $D_r = \frac{T_1{}^{n-(r+1)}}{nP_{n-(r+1)}} + \frac{T_1{}^{n-(r+2)}}{nP_{n-(r+2)}}[1+rE_1]+...+\frac{T_1{}^{n-R}}{nP_{n-R}}[1+rE_1][1+(r+1)E_1]...[1+(R-2)E_1], n = R+1, R+2, ..., \infty.$

3.2 Characteristics of the Model

The probability of quicker arrival time in the system is

$$\begin{aligned} P(0) &= \sum_{n=0}^{\infty} P_n(0) \\ &= \sum_{n=o}^{r} P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \end{aligned} \tag{17}$$

$P_n(0)$occurs only when"$n = 0, 1, 2, ..., r-1, r, r+1, ..., R-2, R-1$"

From (11),(12) and (13) we get

$$\begin{aligned} P(0) &= \{\sum_{n=0}^{R-1}\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{S_1}{1+(k-1)E_1}\right] \\ &-\sum_{n=r+1}^{R-1}\frac{A_r}{B_r}\left[\frac{S_1^R}{R!}\prod_{k=1}^{n-1}\frac{1}{1+(k-1)E_1}\right]\}P_0(0) \end{aligned} \tag{18}$$

The probability of slower rate of arrival in the system is

$$\begin{aligned} P(1) &= \sum_{n=r+1}^{R} P_n(1) + \sum_{n=R+1}^{n(1)} \\ &= \{\sum_{n=r+1}^{R} C_r\left[\frac{S_1^R}{R!}\prod_{l=1}^{n-1}\frac{1}{1+lE_1}\right]\frac{1}{B_r} + \\ &\sum_{n=R+1}^{r}\left[\frac{S_1^R}{R!}\prod_{l=1}^{n-1}\frac{1}{1+lE_1}\right]\frac{1}{B_r}\}P_0(0) \end{aligned} \tag{19}$$

From the normalising state"$P(0)+P(1) = 1$", it is possible to determine the probability $P_0(0)$ the system is clear.

From (18) and (19) we get

$$P_0(0)=\frac{1}{\left\{\begin{array}{l}1+\sum_{n=1}^{R-1}\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{S_1}{1+(k-1)E_1}\right]\\ -\sum_{n=r+1}^{R-1}A_r\frac{S_1R}{R!}\prod_{k=1}^{n-1}\left(\frac{1}{1+(k-1)E_1}\right)\frac{1}{B_r}+\\ \sum_{n=r+1}^{R}C_r\frac{s_1R}{R!}\prod_{l=1}^{n-1}\left(\frac{1}{1+lE_1}\right)\frac{1}{B_r}+\\ \sum_{n=R+1}^{\infty}D_r\frac{S_1R}{R!}\prod_{l=1}^{n-1}\left(\frac{1}{1+lE_1}\right)\frac{1}{B_r}\end{array}\right\}} \quad (20)$$

The system's typical customer count is provided by

$$L_s=L_{s_0}+L_{s_1}$$

where
$L_{s_0}=\sum_{n=0}^{r}nP_n(0)+\sum_{n=r+1}^{R-1}nP_n(0)$ and $L_{s_1}=\sum_{n=r+1}^{R-1}nP_n(1)+\sum_{n=R}^{\infty}nP_n(1)$

$$L_{s_0}=\left\{\sum_{n=0}^{R-1}\left[n\frac{1}{n!}\prod_{k=1}^{n}\frac{S_1}{1+(k-1)E_1}\right]-\sum_{n=r+1}^{R-1}A_r\left[n\frac{S_1{}^R}{R!}\prod_{k=1}^{n-1}\frac{1}{1+(k-1)E_1}\right]\frac{1}{B_r}\right\}P_0(0)$$

$$L_{s_1}=\left\{\sum_{n=r+1}^{R}C_r\left[n\frac{S_1^R}{R!}\prod_{l=1}^{n-1}\frac{1}{1+lE_1}\right]\frac{1}{B_r}+\sum_{n=R+1}^{\infty}D_r\left[n\frac{S_1{}^R}{R!}\prod_{l=1}^{n-1}\frac{1}{1+lE_1}\right]\frac{1}{B_r}\right\}P_0(0)$$

Using Little's Formula, the system's anticipated customer wait time is $W_s=\frac{L_s}{\overline{\lambda}}$
where $\overline{\lambda}=\lambda_0P(0)+\lambda_1P(1)$

3.3 Numerical Illustrations

For different values of$\lambda_0,\lambda_1,\mu,\epsilon$, the fixed values of ξ,r,R,K,p_1,p_2 and the estimate of $P_0(0),P(0),P(1),L_s$ and $W_s$are calculated, and the results are arranged. Let us take $r=3,\ R=6,p_1=1,p_2=1\,and\,\xi=1.$

Table 1. M/M/1/∞ interdependent queueing model retention of reneged customer and controllable arrival rates with feedback

λ_0	λ_1	μ	ϵ	$P_{0,0}(0)$	$P(0)$	$P(1)$	L_s	W_s
10	5	5	0	0.4021	0.97078	0.02922	0.93591	0.19158
9	5	5	0	0.4699	0.98815	0.01185	0.74053	0.18687
7	5	5	0	0.4579	0.95885	0.04115	0.33523	1.62018
8	5	5	0	0.6362	0.99545	0.00455	0.44981	0.11262
6	2	5	0.5	0.3259	0.95357	0.04643	1.12821	0.19547
6	3	5	0.5	0.5583	0.99253	0.00747	0.52092	0.13056
6	4	5	0.5	0.5623	0.98129	0.01871	0.58721	0.14749
6	5	5	0.5	0.5881	0.99813	0.00187	0.51212	0.17097
7	4	4	0	0.3258	0.92827	0.07173	1.36863	0.24233
7	4	7	0	0.5452	0.99581	0.00419	0.59473	0.14954
7	4	5	0	0.4469	0.98662	0.01338	1.05718	0.26518
7	4	2	0	0.7229	0.99892	0.00108	0.31544	0.15775
9	6	7	0.3	0.3275	0.99227	0.00773	0.70773	0.17809
9	6	7	0.5	0.4695	0.98828	0.01172	0.73700	0.18567
9	6	7	0.7	0.4832	0.99313	0.00687	0.78192	0.17823
9	6	7	0.9	0.5217	0.99912	0.00088	0.81924	0.10753

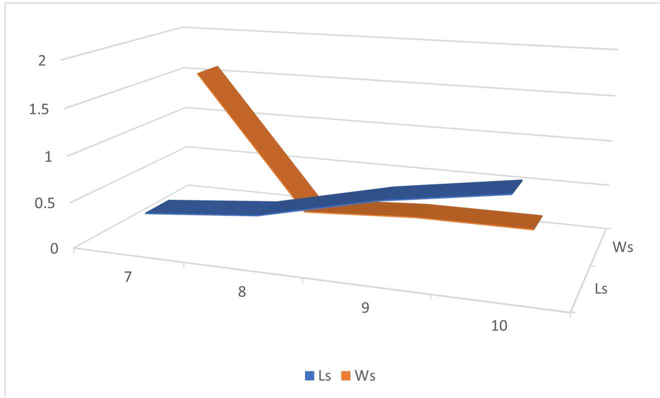

Fig 1. Number of consumers using the system on average L_s and the anticipated length of the customer's wait W_s by varying faster rate of arrival λ_0 and other parameters kept fixed

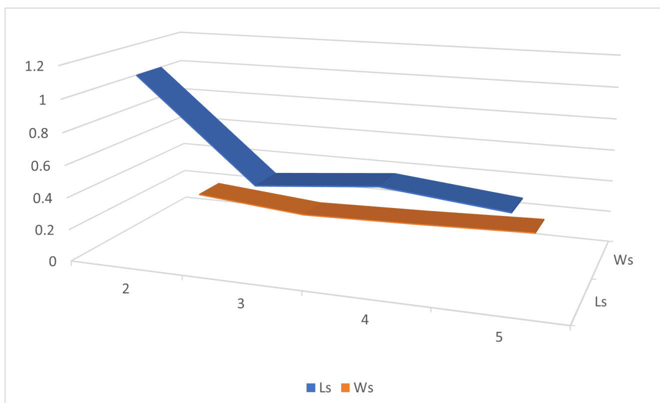

Fig 2. Number of consumers using the system on average L_s and the anticipated length of the customer's wait W_s by varying faster rate of arrival λ_1 and other parameters kept fixed.

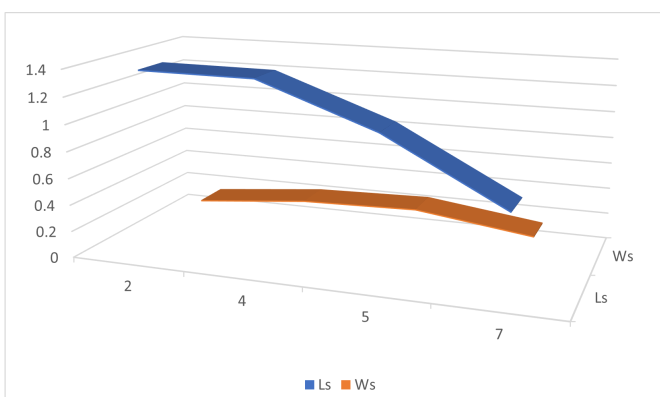

Fig 3. Number of consumers using the system on average L_s and the anticipated length of the customer's wait W_s by varying faster rate of arrival μ and other parameters kept fixed

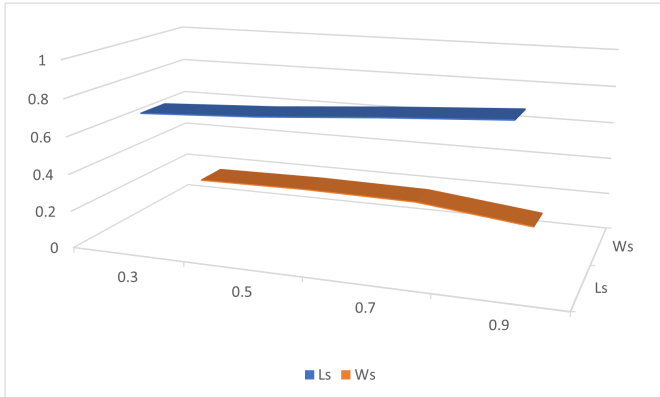

Fig 4. Number of consumers using the system on average L_s and the anticipated length of the customer's wait W_s by varying faster rate of arrival ϵ and other parameters kept fixed

4 Conclusion

This study demonstrates that the arrival rates in a feedback service may be managed using a novel strategy that incorporates both faster and slower rates. A steady-state probability analysis is used to examine the average number of clients in the system and determine the average waiting time for clients in the system. The queueing system's probability and features are examined, along with a numerical validation of the obtained data and graphs. These models can be used to analyse real-world scenarios in phone centres with a limited number of waiting rooms, banks, hospital emergency rooms with a limited number of beds, and computer systems with a limited amount of buffers. This innovative study offers a special technique for managing different arrival rates in these breakdown service scenarios, including both quicker and slower rates.

The example table shows that L_s and W_s drop when the mean dependency rate rises while the other parameters remain constant. As soon as the service rate rises while the other parameters remain constant, $P_0(0)$ and $P(0)$ rise while $P(1)$, L_s and W_s fall. When the pace of arrival goes down, $P_0(0)$, $P(0)$,L_s and W_s go up.

This model consists of the previous models as clear instances. For example, when $\xi = 0$, this model narrows down to"$M/M/1/K$ interdependent queueing model with controllable arrival rates"[7]. When λ_0 tends to λ_1 and $\epsilon = 0$, this model reduces to the "$M/M/1/K$ queueing model with retention of reneged customers"[10]. When $\lambda_0$prone to$\lambda_1, \epsilon = 0$this model condenses to the standard$M/M/1/K$ queueing model.

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