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RESEARCH ARTICLE

Isolate Domination Decomposition of Tensor Product of Cycle Related Graphs

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Abstract

Objectives: This study deals about tensor product of some cycle related graphs which admits an IDD. **Methods:** We have a decomposed a graph into parts in such a way that the isolate domination number of partitions ranges from 1 to m. We have used basic terms and propositions of isolate domination over the graph in order to obtain the results. **Findings:** We have introduced Isolate Domination Decomposition (IDD) of Graphs[1] and is defined as a collection $\psi = \{G_1, G_2, \ldots, G_m\}$ of subgraphs of G such that every edge of G belongs to exactly one G_i, each G_i is connected and it contains atleast one edge and $\gamma_o(G_i) = i, 1 \leq i \leq m$. Also we have found the range of vertices for a graph under which the conditions of IDD are satisfied along with the converse part. **Novelty:** Domination and Decomposition are widely used in networking, block design, coding theory and many fields. Motivated by the concept of ascending pendant domination and decomposition[2,3], we have used here the isolate domination combined with decomposition to characterize the graphs which admits this new parameter and to investigate their vertex bounds.

Keywords: Dominating Set; Domination Number; Isolate Dominating Set; Decomposition; Isolate Domination Decomposition; Tensor Product; Cycle Related Graph

1 Introduction

Let $G = (V, E)$ be a simple connected graph where n and q denote the number of vertices and edges of a graph G respectively. All the graphs considered here are finite and undirected. The concept of isolate domination was introduced by I. Sahul Hamid and S. Balamuruganand further studied by Benjier H. Arriola. Many concepts related to domination have introduced and studies related to the bounds of domination number are existing so far. So many works have done under different types of domination and the characterization of vertices and edges are analyzed. Decomposition also plays a vital role in such a way graph is decomposed into subgraphswith every edge of G belongs to exactly one G_i. The theories under the combination of domination and decomposition[4,5] developed recently but not much applied in graphs. So we planned to work with the combination of isolate domination and decomposition and introduced a new concept of Isolate Domination Decomposition(IDD) of Graphs [1]. We have already worked with this concept in standard graphs, comb, crown and star related graphs. On our motive to deal with product graphs, we have obtained here that tensor product of some cycle related graphs admits an IDD and their vertex bounds.

1.1 Definition

A dominating set for a graph G is a subset D of V such that every vertex not in D is adjacent to atleast one vertex in D. A dominating set D is said to be a minimal dominating set if no proper subset of D is a dominating set. The cardinality of a minimal dominating set of a graph G is called the domination number of G and is denoted by $\gamma(G)$.

1.2 Definition

A decomposition of a graph G is a collection ψ of connected edge disjoint subgraphs $G_1, G_2, \ldots, G_m$ of G such that every edge of G belongs to exactly one G_i.

1.3 Definition [6,7]

A dominating set S of a graph G is said to be an isolate dominating set of G if $\langle S \rangle$ has atleast one isolated vertex. An isolate dominating set S is said to be a minimal isolate dominating set if no proper subset of S is an isolate dominating set. The cardinality of a minimal isolate dominating set of G is called the isolate domination number of G and is denoted by $\gamma_0(G)$.

1.4 Definition

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The tensor product $G = G_1 \wedge G_2$ is defined as a graph with vertex set $V_1 \times V_2$. Edge set is defined as follows: $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are two vertices of G with $u_i \epsilon V_1$ and $v_i \epsilon V_2 (i = 1, 2)$ then $w_1 w_2 \epsilon E(G)$ iff $u_1 u_2 \epsilon E_1$ and $v_1 v_2 \epsilon E_2$.

1.5 Proposition

For the paths P_n and the cycles C_n, we have $\gamma_0(P_n) = \gamma_0(C_n) = \lceil \frac{n}{3} \rceil$. For a complete k- partite graph $G = K_{m_1, m_2, \ldots, m_k}$, we have $\gamma_0(G) = Min\{m_1, m_2, \ldots, m_k\}$. In particular, $\gamma_0(K_n) = 1$.

1.6 Definition [1]

An Isolate Domination Decomposition(IDD) of a graph G is a collection $\psi = \{G_1, G_2, \ldots, G_m\}$ of subgraphs of G such that

(i) Every edge of G belongs to exactly one G_i.
(ii) Each G_i is connected and it contains atleast one edge.
(iii) $\gamma_o(G_i) = i, 1 \le i \le m$.

1.7 Theorem [1]

A Cycle C_n admits an IDD $\psi = \{G_1, G_2, \ldots, G_m\}$ iff $\frac{3m^2-3m+2}{2} \le n \le \frac{3m^2+m}{2}, m \ge 2$. In particular C_n admits an IDD $\psi = \{G\}$ iff $n = 3$

2 Methodology

1. The graph is decomposed into m- partitions in such a way every edge is in exactly one partition.
2. Each partition must be connected and it contains no isolated vertices
3. Finally isolate domination number of partitions is from 1to m. It can be derived by using 1.5. proposition.
4. If so a graph admits these above conditions, we conclude that the graphs admits an IDD.
5. Also here we have found the bounds of vertices by using $q = \sum_{i=1}^{m} q(G_i)$

3 Results and Discussion

3. 1 Theorem

For a graph $C_n \wedge K_2, n \geq 3$ and $n \equiv 1\,(mod\ 2)$admitsan IDD as m- parts iff

$\frac{3m^2-3m+2}{4} \leq n \leq \frac{3m^2+m}{4}, m \geq 2.$

Proof Obviously, $C_n \wedge K_2 \cong C_{2n}$for $n \equiv 1\,(mod\ 2)$. Since a cycle $C_{2n}, n \geq 3$ admits an IDD as m- parts if f$\frac{3m^2-3m+2}{4} \leq n \leq \frac{3m^2+m}{4}, m \geq 2$ by 1.7. Theorem. Hence the theorem follows.

Illustration For $m = 3$, $C_7 \wedge K_2$ admits an IDD as 3- parts. The following Figures 2, 3 and 4 represents an IDD of $C_7 \wedge K_2$. Here $\gamma_0\,(G_i) = i, 1 \leq i \leq 3$.

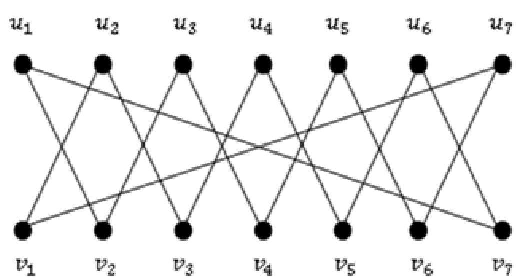

Fig 1. $C_7 \wedge K_2$

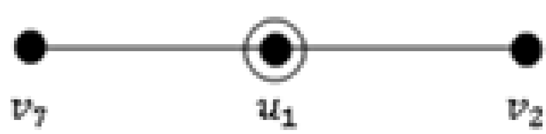

Fig 2. G_1

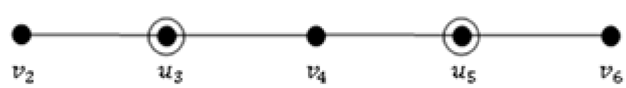

Fig 3. G_2

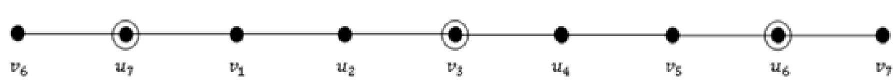

Fig 4. G_3

3.2 Theorem

For a graph $C_n \wedge K_2, n \geq 3$ admits an IDD as $4m$- parts iff

$12m^2 - 3m + 1 \leq n \leq 12m^2 + m,\ m \geq 1.$

Proof Let $C_n = v_1 v_2 \ldots v_n v_1$ be a cycle with $q = n$ edges. On taking tensor product of C_n with K_2, we get $C_n \wedge K_2$ with $q = 2n$ edges.

Suppose $n = 12m^2 - 3m + 1, m \geq 1$ be the minimum possible number of vertices.

To prove $C_n \wedge K_2$ admits an IDD as $4m$- parts.

It is enough to prove that $\gamma_0 (G_i) = i,\ 1 \leq i \leq 4m.$

Let $V (C_n \wedge K_2) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

Define the decomposition $G_i (1 \leq i \leq 4m)$ as follows:

$$G_1 = P_2$$

$$G_2 = P_5$$

$$G_{4m-3} = P_{12m-11}, m \geq 2$$

$$G_{4m-2} = P_{12m-8}, m \geq 2$$

$$G_{4m-1} = P_{12m-5}, m \geq 1$$

$$G_{4m} = P_{12m-2}, m \geq 1$$

Clearly, each $G_i (1 \leq i \leq 4m)$ is connected and it contains at least one edge.

Each $C_n \wedge K_2$ can be decomposed into $4m$- paths as

$$(P_2, P_5, P_7, P_{10}, P_{13}, P_{16}, P_{19}, P_{22}, \ldots, P_{12m-11}, P_{12m-8}, P_{12m-5}, P_{12m-2}\},\ m \geq 2$$

By using 1.5. Proposition, we have

$\gamma_0 (G_1) = \gamma_0 (P_2) = \left\lceil \frac{2}{3} \right\rceil = 1.$

$\gamma_0 (G_2) = \gamma_0 (P_5) = \left\lceil \frac{5}{3} \right\rceil = 2.$

$\gamma_0 (G_{4m-3}) = \gamma_0 (P_{12m-11}) = \left\lceil \frac{12m-11}{3} \right\rceil = 4m - \left\lfloor \frac{11}{3} \right\rfloor = 4m - 3, m \geq 2.$

$\gamma_0 (G_{4m-2}) = \gamma_0 (P_{12m-8}) = \left\lceil \frac{12m-8}{3} \right\rceil = 4m - \left\lfloor \frac{8}{3} \right\rfloor = 4m - 2, m \geq 2.$

$\gamma_0 (G_{4m-1}) = \gamma_0 (P_{12m-5}) = \left\lceil \frac{12m-5}{3} \right\rceil = 4m - \left\lfloor \frac{5}{3} \right\rfloor = 4m - 1, m \geq 1.$

$\gamma_0 (G_{4m}) = \gamma_0 (P_{12m-2}) = \left\lceil \frac{12m-2}{3} \right\rceil = 4m - \left\lfloor \frac{2}{3} \right\rfloor = 4m, m \geq 1.$

It is clear that $\gamma_0 (G_i) = i,\ 1 \leq i \leq 4m.$

Hence, $C_n \wedge K_2$ admits an IDD as $4m$- parts.

Suppose $n = 12m^2 + m, m \geq 1$ be the maximum possible number of vertices.

To prove $C_n \wedge K_2$ admits an IDD as $4m$- parts.

It is enough to prove that $\gamma_0 (G_i) = i,\ 1 \leq i \leq 4m.$

Let $V (C_n \wedge K_2) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

Define the decomposition $G_i(1 \leq i \leq 4m)$ as follows:

$$G_{4m-3} = P_{12m-9}, m \geq 1$$

$$G_{4m-2} = P_{12m-6}, m \geq 1$$

$$G_{4m-1} = P_{12m-3}, m \geq 1$$

$$G_{4m} = P_{12m}, m \geq 1$$

Clearly, each $G_i(1 \leq i \leq 4m)$ is connected and it contains atleast one edge.

Each $C_n \wedge K_2$can be decomposed into $4m$- paths as

$$\{P_3, P_6, P_9, P_{12}, P_{15}, P_{18}, P_{21}, P_{24}, \ldots, P_{12m-9}, P_{12m-6}, P_{12m-3}, P_{12m}\}, m \geq 1$$

By using 1.5. Proposition, we have

$\gamma_0(G_{4m-3}) = \gamma_0(P_{12m-9}) = \left\lceil \frac{12m-9}{3} \right\rceil = 4m-3, m \geq 1$

$\gamma_0(G_{4m-2}) = \gamma_0(P_{12m-6}) = \left\lceil \frac{12m-6}{3} \right\rceil = 4m-2, m \geq 1$

$\gamma_0(G_{4m-1}) = \gamma_0(P_{12m-3}) = \left\lceil \frac{12m-3}{3} \right\rceil = 4m-1, m \geq 1.$

$\gamma_0(G_{4m}) = \gamma_0(P_{12m}) = \left\lceil \frac{12m}{3} \right\rceil = 4m, m \geq 1.$

It is clear that $\gamma_0(G_i) = i,\ 1 \leq i \leq 4m$.

Hence, $C_n \wedge K_2$ admits an IDD as $4m$- parts.

Conversely, Suppose that $C_n \wedge K_2$ admits an IDD as $4m$- parts. Then$\gamma_0(G_i) = i,\ 1 \leq i \leq 4m$. Also each $G_i(1 \leq i \leq 4m)$ is connected and it contains atleast one edge.

To prove$12m^2 - 3m + 1 \leq n \leq 12m^2 + m, m \geq 1$.

For$i = 1$, we find the possible connected non-isomorphic subgraphs$G_1$of$C_n \wedge K_2$such that $\gamma_0(G_1) = 1$. Hence $G_1$is anyone of $\{P_2, P_3\}$.

For$i = 2$, we find the possible connected non-isomorphic subgraphs$G_2$of$C_n \wedge K_2$ such that $\gamma_0(G_2) = 2$. Hence G_2 is anyone of $\{P_4, P_5, P_6\}$.

For$i = 3$, we find the possible connected non-isomorphic subgraphs$G_3$of$C_n \wedge K_2$such that $\gamma_0(G_3) = 3$. Hence G_3 is anyone of $\{P_7, P_8, P_9\}$.

For$i = 4$, we find the possible connected non-isomorphic subgraphs$G_4$of$C_n \wedge K_2$such that $\gamma_0(G_4) = 4$and $G_1 \cup G_2 \cup G_3 \cup G_4isC_n \wedge K_2$.Hence G_4 is anyone of $\{P_{10}, P_{11}, P_{12}\}$.

On continuing in this way,

In general, we can conclude the following for each $G_i(2 \leq i \leq 4m)$,

$G_{4m-3}$is anyone of $\{P_{12m-11}, P_{12m-10}, P_{12m-9}\}$, $m \geq 2$

$G_{4m-2}$is anyone of $\{P_{12m-8}, P_{12m-7}, P_{12m-6}\}$, $m \geq 1$

$G_{4m-1}$is anyone of $\{P_{12m-5}, P_{12m-4}, P_{12m-3}\}$, $m \geq 1$

$G_{4m}$is anyone of $\{P_{12m-2}, P_{12m-1}, P_{12m}\}$, $m \geq 1$.

On taking the minimum possibility of paths as $G_i(1 \leq i \leq 4m)$, we construct the following table.

Since $G_i(1 \leq i \leq 4m)$are decomposition of$C_n \wedge K_2$, we have $q = \sum_{i=1}^{4m} q(G_i)$.

$$q = (1+4+6+9) + 12 + \cdots + 12m - 12 + 12m - 9 + 12m - 6 + 12m - 3, m \geq 2$$

$$2n = 20 + 3(4 + 5 + 6 + \ldots + 4m - 1)$$

$G_2 = P_5$	$G_3 = P_7$	$G_6 = P_{16}$	$G_7 = P_{19}$	$\cdots$	$G_{4m-2} = P_{12m-8}$	$G_{4m-1} = P_{12m-5}$
$G_1 = P_2$	$G_4 = P_{10}$	$G_5 = P_{13}$	$G_8 = P_{22}$		$G_{4m-3} = P_{12m-11}$	$G_{4m} = P_{12m-2}$

$$n = 12m^2 - 3m + 1, m \geq 1$$

On taking the maximum possibility of paths as $G_i(1 \leq i \leq 4m)$, we construct the following table.

$G_2 = P_6$	$G_3 = P_9$	$G_6 = P_{18}$	$G_7 = P_{21}$	$\cdots$	$G_{4m-2} = P_{12m-6}$	$G_{4m-1} = P_{12m-3}$
$G_1 = P_3$	$G_4 = P_{12}$	$G_5 = P_{15}$	$G_8 = P_{24}$		$G_{4m-3} = P_{12m-9}$	$G_{4m} = P_{12m}$

Since $G_i(1 \leq i \leq 4m)$are decomposition of$C_n \wedge K_2$, we have $q = \sum_{i=1}^{4m} q(G_i)$.

$$q = 2+5+8+11+14+\cdots+12m-10+12m-7+12m-4+12m-1, m \geq 1$$

$$2n = 2m(4+(4m-1)3)$$

$$n = 12m^2 + m, m \geq 1$$

Hence, $12m^2 - 3m + 1 \leq n \leq 12m^2 + m, m \geq 1$.

Illustration For $m = 1$, $C_{12} \wedge K_2$ admits an IDD as 4- parts. The following Figures 6, 7, 8 and 9 represents an IDD of $C_{12} \wedge K_2$.

Here $\gamma_0(G_i) = i, 1 \leq i \leq 4$.

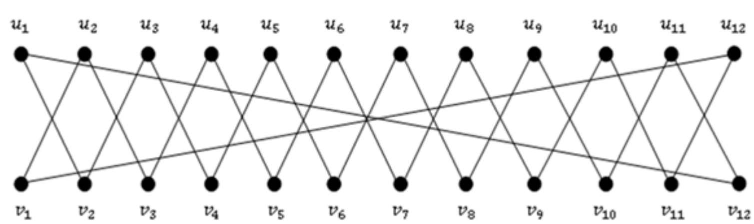

Fig 5. $C_{12} \wedge K_2$

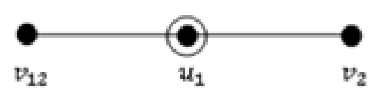

Fig 6. G_1

Fig 7. G_2

Fig 8. G_3

Fig 9. G_4

3.3 Theorem

For a graph $W_{n+1} \wedge K_2, n \geq 3$ admits an IDD as $4m+2$- parts if $12m^2+9m+1 \leq n \leq 12m^2+3m,\ m \geq 1$.

Proof Let W_{n+1} be a wheel with $q=2n$ edges. On taking tensor product of W_{n+1} with K_2, we get $W_{n+1} \wedge K_2$ with $q=4n$ edges.

Suppose $n=12m^2+9m+1, m \geq 1$ be the minimum possible number of vertices.

To prove $W_{n+1} \wedge K_2$ admits an IDD as $4m+2$- parts.

It is enough to prove that $\gamma_0\left(H_j\right)=j,\ 1 \leq j \leq 2$and $\gamma_0\left(G_i\right)=i+2,\ 1 \leq i \leq 4m$.

Let $V\left(W_{n+1} \wedge K_2\right)=\left\{\left(u, v_j\right)_{j=1,2}, \left(u_i, v_j\right)/1 \leq i\ \leq m \text{ and } j=1,2\right\}$

Define the decomposition $H_j(1 \leq j \leq 2)$ and $G_i(1 \leq i \leq 4m)$ as follows:

$$H_1=\langle N[(u, v_1)]\rangle$$

$$H_2=\langle N\left[\left(u, v_2\right)\right]\rangle \cup\left(\left(u_1, v_2\right),\left(u_n, v_1\right)\right) \text{in } W_{n+1} \wedge K_2$$

Set $H=(W_{n+1} \wedge K_2) \backslash (H_1 \bigcup H_2)$

We define $G_1=P_8$

$$G_{4m-3}=P_{12m-5}, m \geq 2$$

$$G_{4m-2}=P_{12m-2}, m \geq 1$$

$$G_{4m-1}=P_{12m+1}, m \geq 1$$

$$G_{4m}=P_{12m+4}, m \geq 1 \text{in} H$$

Clearly, each $H_j(1 \leq j \leq 2)$ and $G_i(1 \leq i \leq 4m)$ are connected and it contains atleast one edge.

Each $W_{n+1} \wedge K_2$can be decomposed into $4m+2$- parts as

$$\{H_1, H_2, P_8, P_{10}, P_{13}, P_{16}, P_{19}, P_{22}, P_{25}, P_{28} \ldots, P_{12m-5}, P_{12m-2}, P_{12m+1}, P_{12m+4}\}, m \geq 2$$

By using 1.5. Proposition, we have

$\gamma_0(H_1) = \gamma_0(K_{1,n}) = \min\{1, n\} = 1, n \geq 3$

The minimal isolate dominating set of H_2 is $\{(u, v_2), (u_1, v_2)\} = D_2$ (say)

Thus $\gamma_0(H_2) = |D_2| = 2$.

$\gamma_0(G_1) = \gamma_0(P_8) = \left\lceil \frac{8}{3} \right\rceil = 3.$

$\gamma_0(G_{4m-3}) = \gamma_0(P_{12m-5}) = \left\lceil \frac{12m-5}{3} \right\rceil = 4m - \left\lfloor \frac{5}{3} \right\rfloor = 4m-1, m \geq 2.$

$\gamma_0(G_{4m-2}) = \gamma_0(P_{12m-2}) = \left\lceil \frac{12m-2}{3} \right\rceil = 4m - \left\lfloor \frac{2}{3} \right\rfloor = 4m, m \geq 1.$

$\gamma_0(G_{4m-1}) = \gamma_0(P_{12m+1}) = \left\lceil \frac{12m+1}{3} \right\rceil = 4m + \left\lceil \frac{1}{3} \right\rceil = 4m+1, m \geq 1.$

$\gamma_0(G_{4m}) = \gamma_0(P_{12m+4}) = \left\lceil \frac{12m+4}{3} \right\rceil = 4m + \left\lceil \frac{4}{3} \right\rceil = 4m+2, m \geq 1.$

It is clear that $\gamma_0(H_j) = j,\ 1 \leq j \leq 2$and $\gamma_0(G_i) = i+2,\ 1 \leq i \leq 4m$.

Hence, $W_{n+1} \wedge K_2$ admits an IDD as $4m+2$- parts.

Suppose $n = 12m^2 + 13m, m \geq 1$ be the maximum possible number of vertices.

To prove $W_{n+1} \wedge K_2$ admits an IDD as $4m+2$- parts.

It is enough to prove that $\gamma_0(H_j) = j,\ 1 \leq j \leq 2$and $\gamma_0(G_i) = i+2,\ 1 \leq i \leq 4m$.

Let $V(W_{n+1} \wedge K_2) = \{(u, v_j)_{j=1,2}, (u_i, v_j)/1 \leq i \leq m\, and j = 1, 2\}$

Define the decomposition $H_j(1 \leq j \leq 2)$ and $G_i(1 \leq i \leq 4m)$ as follows:

$$H_1 = \langle N[(u, v_1)] \rangle$$

$$H_2 = \langle N[(u, v_2)] \rangle \cup ((u_1, v_2), (u_n, v_1)) \text{ in } W_{n+1} \wedge K_2$$

$$\text{Set } H = (W_{n+1} \wedge K_2) \setminus (H_1 \cup H_2)$$

We define

$$G_{4m-3} = P_{12m-3}, m \geq 1$$

$$G_{4m-2} = P_{12m}, m \geq 1$$

$$G_{4m-1} = P_{12m+3}, m \geq 1$$

$$G_{4m} = P_{12m+6}, m \geq 1 \text{ in } H.$$

Clearly, each $H_j(1 \leq j \leq 2)$ and $G_i(1 \leq i \leq 4m)$ are connected and it contains atleast one edge.

Each $W_{n+1} \wedge K_2$can be decomposed into $4m+2$- parts as

$$\{H_1, H_2, P_9, P_{12}, P_{15}, P_{18}, P_{21}, P_{24}, P_{27}, P_{30} \ldots, P_{12m-3}, P_{12m}, P_{12m+3}, P_{12m+6}\}, m \geq 1$$

By using 1.5. Proposition, we have

$\gamma_0(H_1) = \gamma_0(K_{1,n}) = \min\{1, n\} = 1, n \geq 3$

The minimal isolate dominating set of H_2 is $\{(u,v_2),(u_1,v_2)\} = D_2$(say).
Thus $\gamma_0(H_2) = |D_2| = 2$.

$\gamma_0(G_{4m-3}) = \gamma_0(P_{12m-3}) = \left\lceil \frac{12m-3}{3} \right\rceil = 4m-1, m \geq 1.$

$\gamma_0(G_{4m-2}) = \gamma_0(P_{12m}) = \left\lceil \frac{12m}{3} \right\rceil = 4m, m \geq 1.$

$\gamma_0(G_{4m-1}) = \gamma_0(P_{12m+3}) = \left\lceil \frac{12m+3}{3} \right\rceil = 4m+1, m \geq 1.$

$\gamma_0(G_{4m}) = \gamma_0(P_{12m+6}) = \left\lceil \frac{12m+6}{3} \right\rceil = 4m+2, m \geq 1.$

It is clear that $\gamma_0(H_j) = j,\ 1 \leq j \leq 2$and $\gamma_0(G_i) = i+2,\ 1 \leq i \leq 4m$.
Hence, $W_{n+1} \wedge K_2$ admits an IDD as $4m+2$- parts.

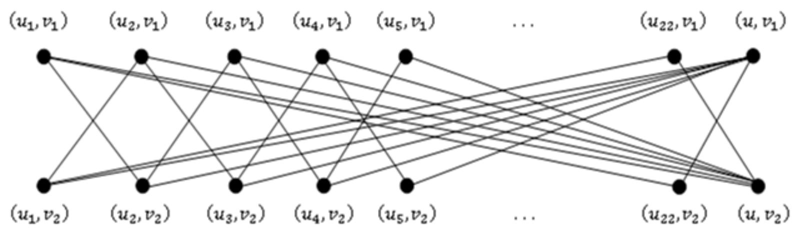

Fig 10. $W_{22+1} \wedge K_2$

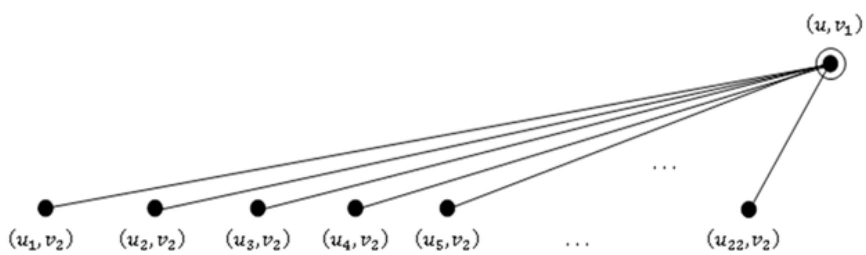

Fig 11. H_1

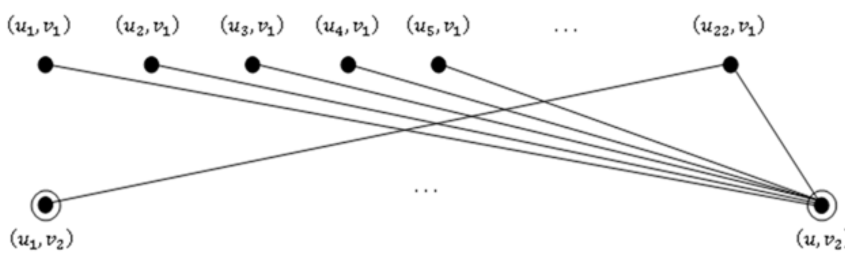

Fig 12. H_2

Illustration For $m=1$, $W_{22+1} \wedge K_2$ admits an IDD $\{H_1,H_2,G_1,G_2,G_3,G_4\}$ as 6- parts. The following Figures 11 and 12 represents an IDD of $W_{22+1} \wedge K_2 . Set\ H = (W_{22+1} \wedge K_2) \backslash (H_1 \bigcup H_2)$
Let $G_1 = P_7, G_2 = P_{10}, G_3 = P_{13}$ and $G_4 = P_{17}$in H.
Here $\gamma_0(H_j) = j,\ 1 \leq j \leq 2$and $\gamma_0(G_i) = i+2,\ 1 \leq i \leq 4$.

4 Conclusion

Thus we have found the range of vertices for the tensor product of cycle related graphs under which the conditions of IDD are satisfied along with the converse part.We will extend our future research for the graphs which not admits an IDD also to compare the theories with the existing graphs and examine their necessary and sufficient condition for which it admits an IDD.

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