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RESEARCH ARTICLE

On Topological Indices of Fuzzy Random Graph

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Abstract

Objectives: Fuzzy random graphs model uncertain relationships between entities, where edges possess degrees of membership rather than binary connections. This study aims to discover how operations such as union and intersection relate to each other on the topological indices of fuzzy random graphs. By analyzing the topological indices of fuzzy random graphs, our aim is to uncover meaningful insights into the connectivity, complexity and other topological features of fuzzy random graphs. **Method:** The topological indices of fuzzy random graphs were determined by merging the fuzzy and random graph indices and applying them to the corresponding edges. **Findings:** This study investigates various topological indices, such as the Connectivity index, the Wiener index, the Zagreb indices, the Harmonic index and the Randić index, adapted to account for the fuzzy nature of graph edges. **Novelty:** In this work, the topological indices of a particular fuzzy random graph are also examined, and the results are presented. Furthermore, an example-based description of the relationship between operations on the topological indices of fuzzy random graphs is provided. This study describes how topological indices of fuzzy random graphs can be used to locate electric charging stations, predict protein structure and more.

Keywords: Fuzzy graph; Random graph; Fuzzy random graph; Degree of Vertices; Topological indices

1 Introduction

Fuzzy graphs are a generalization of traditional graphs, where the edges are not simply present or absent but have degrees of membership. Random graphs are graphs in which the edges between vertices are assigned according to some probabilistic model. In a random graph, the presence or absence of each edge is typically determined by a random process. Fuzzy random graphs combine the concepts of randomness from random graphs with the flexibility of fuzzy graphs. In a fuzzy random graph, both the presence of edges and their degrees of membership are determined by random processes.

In graph theory, topological indices are numerical parameters associated with a graph that provide information about its structural properties. In 1947, Harry Wiener proposed the first topological index and it was used to correlate alkane boiling points[1]. Subsequently, in 1972, Gutman and Trinajstic presented the "Zagreb index," an additional topological indicator for crisp graphs. The Wiener index for fuzzy graphs was first presented by Binu et al. in 2020. Following the Wiener index, many topological indices were developed in 2021, including the Zagreb indices[2], Harmonic index and Randic index. Uzma Ahmad, Iqra Nawaz, and Said Broumi describe the neighborhood connectivity index (NCI) in directed rough fuzzy random graphs, which considers the strength of vertices to neighbor vertices[3].

Different topological indices are vital in a wide range of subjects, including the social sciences, medicine, electronics, chemistry, economics, and business studies. The Wiener indices are used to determine the characteristics of known paraffinic alkanes. Javeria Dinar, Zahid Hussain, Shahid Zaman and Shams Ur Rehman applied the Wiener and Average Wiener indices in water pipeline networks[4], which are crucial to the management of water supplies. The reader also refers to[5–10]. The topological indices of fuzzy graphs, intuitionistic fuzzy graphs, etc., are described in the earlier study. This study presents a novel concept: fuzzy random graphs, whose topological indices are explained. Overall, the topological indices of fuzzy random graphs involve understanding the concepts of graph theory, randomness, fuzziness and topology and how they intersect in the context of analyzing the structural properties of fuzzy random graphs.

Preliminaries

Definition 1.1

Let $V_{FR} = \{v_i, i = 1, 2, \ldots\ldots, n\}$ be a set of n vertices has $n(n-1)/2$ possible edges E_{FR}between them. Then there are two sets of edges,

$E_F = \{(v_i, v_j)/\, 1 \leq v_i < v_j \leq n\, ; i, j = 1, 2, \ldots\ldots, n,\ i \neq j\, and\ (v_i, v_j)\ are\ fuzzy\ edges\}$, $E_R = \{(v_i, v_j)/\, 1 \leq v_i < v_j \leq n;\ i, j = 1, 2, \ldots\ldots, n,\ i \neq j\, and\ (v_i, v_j)\ are\ random\ edges\}$that are disjoint. Consider the mapping

$$\varphi_G : V_{FR} \to [0,1]\, v_i \to \varphi_G(v_i)$$

$$\psi_G : V_{FR} \times V_{FR} \to [0,1]_P \times [0,1]_{f_A}$$

$$(v_i, v_j) \to \psi_G(v_i, v_j) = (P(v_i, v_j), f_A\,(v_i, v_j))$$

with $P(v_i, v_j) = 0$ if and only if $f_A\,(v_i, v_j) = 0$. Then $G_{FR} = (V_{FR}, E_{FR})$ is called a fuzzy random graph (FRG) if $\psi_G\,(v_i, v_j) \leq min\{\varphi_G\,(v_i)\,, \varphi_G\,(v_j)\}$. Where (v_i, v_j) corresponds to the edge between v_i and v_j, $P(v_i, v_j)$ and $f_A(v_i, v_j)$ represents the probability of the edge (v_i, v_j) and the membership function for the edge (v_i, v_j) within the fuzzy set A in X respectively.

Definition 1.2

The fuzzy random graph $H_{FR} = (V_{FR}, E_{FR}, \tau_G, \sigma_G)$ is called a Fuzzy random subgraph of $G_{FR} = (V_{FR}, E_{FR}, \varphi_G, \psi_G)$ induced by α if $\alpha \in V_{FR}$, $\tau_G\,(v_i) \leq \varphi_G(v_i)$ for all $v_i \in \alpha$ and $\sigma_G\,(v_i, v_j) \leq \psi_G(v_i, v_j)$ for all $v_i, v_j \in \alpha$.

Definition 1.3

In a fuzzy random graph $G_{FR} = (V_{FR}, E_{FR})$, the degree of a fuzzy random vertex v_i is defined as

$$d_{G_{FR}}(v_i) = \sum_{v_i \neq v_j \in E_F} \psi_G\,(v_i, v_j) + \sum_{v_i \neq v_j \in E_R} nP$$

for $(v_i, v_j) \in E_{FR}$ and $\psi_G\,(v_i, v_j) = 0$ for (v_i, v_j) not in E_{FR}. Where $\psi_G\,(v_i, v_j) = P$ and n- number of vertices incident at a random edge. i.e.,$n = 2$

$$d_{G_{FR}}(v_i) = \sum_{v_i, v_j \in E_F} \psi_G\,(v_i, v_j) + \sum_{v_i, v_j \in E_R} 2\,\psi_G\,(v_i, v_j)$$

Definition 1.4

The Minimal degree of vertex, $\delta\,(G_{FR}) = min\,\{d_{G_{FR}}\,(v_i, v_j)\,; (v_i, v_j) \in E_{FR}\}$

The Maximal degree of vertex, $\triangle\,(G_{FR}) = max\,\{d_{G_{FR}}\,(v_i, v_j)\,; (v_i, v_j) \in E_{FR}\}$

Definition 1.5

The order, $O(G_{FR})$ and size, $S(G_{FR})$ of fuzzy random graph $G_{FR}=(V_{FR},E_{FR})$ is defined as,$O(G_{FR})=\sum\varphi_G(v_i)$, $v_i\in V_{FR}$, $S(G_{FR})=\sum\psi_G(v_i,v_j)$, $(v_i,v_j)\in E_{FR}$

Definition 1.6

The fuzzy random graph $G_{FR}=(V_{FR},E_{FR})$is called a complete fuzzy random graph if $\psi_G(v_i,v_j)=min\{\varphi_G(v_i),\varphi_G(v_j)\}$ for all $(v_i,v_j)\in E_{FR}$.

Definition 1.7

In a fuzzy random graph $G_{FR}=(V_{FR},E_{FR})$, a path P is an array of different vertices, $v_0,v_1,\ldots\ldots v_m$ (potentially excluding u_0 and u_m) such that $\mu(v_{i-1},v_i)>0, i=1,2,\ldots\ldots,m$. The path's length is denoted by m in that case. The edges of the path are the consecutive pairings. If $v_0=v_m$ and $m\geq 3$ then P is a cycle and two vertices are said to be connected when a path connects them. The length of the shortest path connecting the vertices that are the furthest apart is known as the diameter of a fuzzy random graph.

Definition 1.8

The strength of the path P in a fuzzy random graph $G_{FR}=(V_{FR},E_{FR})$is defined to be $min\mu(v_{i-1},v_i) where\ i=1,2,\ldots\ldots,n$. i.e., The minimum membership value of an edge in P. (The edge may be fuzzy edge or random edge). It is denoted as $S_G(P)$.

Definition 1.9

The strength of connectivity of fuzzy random graph is defined as the maximum of all $S_G(P)$connecting vertices v_i and v_j.

$$CONN_G(v_i,v_j)=max\{\mu(v_i,u_1)\wedge\mu(u_1,u_2)\wedge\ldots\ldots\wedge\mu(u_{n-1},v_j)\}$$

Definition 1.10

An edge (v_i,v_j) is considered to be strong if and only if whenever the edge (v_i,v_j) is vanished, its membership value is at least as great as the connectedness of its end vertices.

ie., $\mu(v_i,v_j)\geq\mu^{\infty}(v_i,v_j)$ or $\mu(v_i,v_j)\geq CONN_{G-(v_i,v_j)}(v_i,v_j)$

Definition 1.11

A path $P:v_i=v_0,v_1,\ldots,v_n=v_j$ from v_i to v_j is called a strong path if (v_{i-1},v_i) is strong for all $1\leq i\leq n$

Definition 1.12

A fuzzy random graph $G_{FR}=(V_{FR},E_{FR})$ is connected if any two vertices are joined by a path.

2 Methodology

This section introduces the topological indices of fuzzy random graphs with an example and some results.

Definition 2.1

For a fuzzy random graph, the connectivity index $CI_{G_{FR}}$ is defined as $CI_{G_{FR}}=CI_{G_F}+CI_{G_R}$ with

$$CI_{G_F}=\sum_{(v_i,v_j)\in E_F}\varphi_G(v_i)\varphi_G(v_j)CONN_G(v_i,v_j)CI_{G_R}=\sum_{(v_i,v_j)\in E_R}\frac{1}{\sqrt{d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)}}$$

Where $CONN_G(v_i,v_j)$ is the strength of connectedness between v_i and v_j, $d_{G_{FR}}(v_i)$ is the degree of the vertex v_i.

Definition 2.2

The Wiener index of fuzzy random graph is defined as, $WJ_{G_{FR}}=WJ_{G_F}+WJ_{G_R}$

$$WJ_{G_{FR}}=\sum_{(v_i,v_j)\in E_F}\phi_G(v_i)\phi_G(v_j)d_s(v_i,v_j)+\frac{1}{2}\sum_{i=1}^{n}d(v_i/(v_i,v_j)\in E_R)$$

Where $d_s(v_i,v_j)$ denotes distance between the membership value of the edges and $d(v_i/(v_i,v_j)\in E_R)$ denotes the sum of distances between v_i and all other vertices ofG_{FR}, defined by

$$d(v_i/(v_i,v_j)\in E_R)=\sum_{j=1}^{n}d(v_i,v_j)$$

Definition 2.3

The Zagreb indices of fuzzy random graph are given by,

$$First\ Zagreb\ Index,\ \ M(G_{FR}) = \sum_{v_i \epsilon V_{FR}} v_i (d_{G_{FR}}(v_i))^2\ for\ all\ v_i \epsilon V_{FR}$$

Second Zagreb Index of fuzzy random graph is defined as,

$$M^*(G_{FR}) = \frac{1}{2} \sum_{(v_i,v_j)\in E_F} \phi_G(v_i) d_{G_{FR}}(v_i) \phi_G(v_j) d_{G_{FR}}(v_j) + \sum_{(v_i,v_j)\in E_R} d_{G_{FR}}(v_i) d_{G_{FR}}(v_j)$$

Definition 2.4

The Randić connectivity index of fuzzy random graph is given by

$$R(G_{FR}) = \frac{1}{2} \sum_{(v_i,v_j)\in E_F} \frac{1}{\sqrt{\phi_G(v_i)\, d_{G_{FR}}(v_i)\, \phi_G(v_j)\, d_{G_{FR}}(v_j)}} + \sum_{(v_i,v_j)\epsilon E_R} \frac{1}{\sqrt{d_{G_{FR}}(v_i) d_{G_{FR}}(v_j)}}$$

Definition 2.5

The Harmonic index of fuzzy random graph is defined as

$$H(G_{FR}) = \frac{1}{2} \sum_{(v_i,v_j)\epsilon E_F} \frac{1}{\phi_G(v_i) d_{G_{FR}}(v_i) + \phi_G(v_j) d_{G_{FR}}(v_j)} + \sum_{(v_i,v_j)\in E_R} \frac{2}{d_{G_{FR}}(v_i) + d_{G_{FR}}(v_j)}$$

Where $d_{G_{FR}}(v_i)$ is the degree of the vertex $v_i \epsilon V_{FR}$

3 Results and Discussion

Consider the fuzzy random graph,

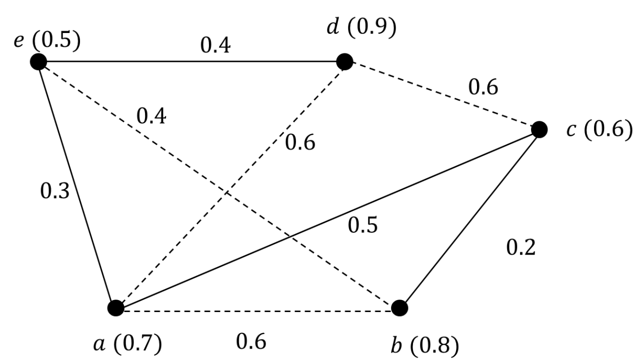

Fig 1.

In Figure 1, the edges $\{ab, ad, be, cd\}$ are random edges and $\{ac, ae, de, bc\}$ are fuzzy edges.

Degree of vertices:

$$d_{G_{FR}}(v_i) = \sum_{v_i,v_j\in E_F} \psi_G(v_i, v_j) + \sum_{v_i,v_j\in E_R} 2\, \psi_G(v_i, v_j)$$

$$d_{G_{FR}}(a) = 0.3 + 0.5 + 2(0.6) + 2(0.6) = 3.2$$

$$d_{G_{FR}}(b) = 0.2 + 2(0.4 + 0.6) = 2.2$$

$$d_{G_{FR}}(c) = 0.2 + 0.5 + 2(0.6) = 1.9$$

$$d_{G_{FR}}(d) = 0.4 + 2(0.6 + 0.6) = 2.8$$

$$d_{G_{FR}}(e) = 0.3 + 0.4 + 2(0.4) = 1.5$$

Strength of connectivity:

$$CONN_G(v_i, v_j) = max\{\mu(v_i, u_1) \wedge \mu(u_1, u_2) \wedge \ldots\ldots \wedge \mu(u_{n-1}, v_j)\}$$

$$CONN_G(a,b) = max\{0.3, 0.2, 0.2, 0.2, 0.6\} = 0.6$$

$$CONN_G(a,c) = max\{0.3, 0.5, 0.2, 0.6\} = 0.6$$

$$CONN_G(a,e) = max\{0.3, 0.4, 0.4, 0.2, 0.4\} = 0.4$$

$$CONN_G(b,c) = max\{0.2, 0.3, 0.4, 0.5, 0.6\} = 0.6$$

$$CONN_G(d,e) = max\{0.4, 0.3, 0.2\} = 0.4$$

The Connectivity Index of fuzzy random graph:

$$CI_{G_{FR}} = CI_{G_F} + CI_{G_R}$$

$$CI_{G_F} = \sum_{(v_i, v_j) \in E_F} \varphi_G(v_i)\varphi_G(v_j) CONN_G(v_i, v_j) CI_{G_R} = \sum_{(v_i, v_j) \in E_R} \frac{1}{\sqrt{d_{G_{FR}}(v_i) d_{G_{FR}}(v_j)}}$$

$$CI_{G_F} = (0.7)(0.6)(0.6) + (0.7)(0.5)(0.4) + (0.8)(0.6)(0.6) + (0.9)(0.5)(0.4) = 0.86$$

$$CI_{G_R} = \frac{1}{\sqrt{(3.2)(2.2)}} + \frac{1}{\sqrt{(3.2)(2.8)}} + \frac{1}{\sqrt{(2.2)(1.5)}} + \frac{1}{\sqrt{(1.9)(2.8)}} = 1.695$$

$$CI_{G_{FR}} = 0.86 + 1.695 = 2.56$$

$$CI_{G_{FR}} = 2.56$$

The Wiener Index of fuzzy random graph:

$$WJ_{G_{FR}} = WJ_{G_F} + WJ_{G_R}$$

$$WJ_{G_{FR}} = \sum_{(v_i,v_j)\in E_F} \phi_G(v_i)\phi_G(v_j)d_s(v_i,v_j) + \frac{1}{2}\sum_{i=1}^{n} d(v_i/(v_i,v_j)\in E_R)$$

$$WJ_{G_F} = \sum_{v_i,v_j\in V_{FR}} \phi_G(v_i)\phi_G(v_j)d_s(v_i,v_j) = (0.7)(0.6)(0.5) + (0.7)(0.5)(0.3) + (0.8)(0.6)(0.2) + (0.9)(0.5)(0.4) = 0.59$$

$$WJ_{G_R} = \frac{1}{2}\sum_{i=1}^{n} d(v_i/(v_i,v_j)\in E_R) = (1/2)(4.4) = 2.2$$

$d(a) = 0.6 + 0.6 = 1.2$, $d(b) = 0.6 + 0.4 = 1.0$, $d(c) = 0.6$, $d(d) = 0.6 + 0.6 = 1.2$, $d(e) = 0.4$

$$WJ_{G_{FR}} = 2.79$$

The first Zagreb index of fuzzy random graph:

$$M(G_{FR}) = \sum_{v_i \epsilon V_{FR}} v_i (d_{G_{FR}}(v_i))^2 \, for\, all\, v_i \epsilon V_{FR}$$

$$M(G_{FR}) = (0.7)(3.2)^2 + (0.8)(2.2)^2 + (0.6)(1.9)^2 + (0.9)(2.8)^2 + (0.5)(1.5)^2$$

$$M(G_{FR}) = 21.39$$

The second Zagreb index of fuzzy random graph:

$$M^*(G_{FR}) = \frac{1}{2}\sum_{(v_i,v_j)\in E_F} \phi_G(v_i)d_{G_{FR}}(v_i)\phi_G(v_j)d_{G_{FR}}(v_j) + \sum_{(v_i,v_j)\epsilon E_R} d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)$$

$$M^*(G_{FR}) = \tfrac{1}{2}\{(0.7\times 3.2\times 0.6\times 1.9) + (0.7\times 3.2\times 0.5\times 1.5) + (0.8\times 2.2\times 0.6\times 1.9) + (0.9\times 2.8\times 0.5\times 1.5)\}$$
$$+\{(3.2\times 2.2) + (2.2\times 1.5) + (3.2\times 2.8) + (1.9\times 2.8)\}$$

$$M^*(G_{FR}) = 4.07 + 24.62 = 28.69$$

$$M^*(G_{FR}) = 28.69$$

The Randić index of fuzzy random graph:

$$R(G_{FR}) = \frac{1}{2}\sum_{(v_i,v_j)\in E_F} \frac{1}{\sqrt{\phi_G(v_i)\, d_{G_{FR}}(v_i)\, \phi_G(v_j)\, d_{G_{FR}}(v_j)}} + \sum_{(v_i,v_j)\epsilon E_R} \frac{1}{\sqrt{d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)}}$$

$$= \left(\frac{1}{2}\right)(2.831) + 1.695 = 3.11$$

$$R(G_{FR}) = 3.11$$

The Harmonic index of fuzzy random graph:

$$H(G_{FR}) = \frac{1}{2}\sum_{(v_i,v_j)\epsilon E_F}\frac{1}{\phi_G(v_i)d_{G_{FR}}(v_i)+\phi_G(v_j)d_{G_{FR}}(v_j)} + \sum_{(v_i,v_j)\in E_R}\frac{2}{d_{G_{FR}}(v_i)+d_{G_{FR}}(v_j)}$$

$$H(G_{FR}) = \tfrac{1}{2}\left\{\tfrac{1}{(0.7\times3.2)+(0.6\times1.9)}+\tfrac{1}{(0.7\times3.2)+(0.5\times1.5)}+\tfrac{1}{(0.8\times2.2)+(0.6\times1.9)}+\tfrac{1}{(0.9\times2.8)+(0.5\times1.5)}\right\} + \left\{\tfrac{2}{(3.2\times2.2)}+\tfrac{2}{(2.2\times1.5)}+\tfrac{2}{(3.2\times2.8)}+\tfrac{2}{(1.9\times2.8)}\right\}$$

$$H(G_{FR}) = 1.28 + 1.49 = 2.77$$

$$H(G_{FR}) = 2.77$$

Results

$i.\ CI_{G_{FR}} \le H(G_{FR}) \le R(G_{FR})$
$ii.\ CI_{G_{FR}} \le WJ_{G_{FR}}$
$iii.\ M(G_{FR}) \le M^*(G_{FR})$

Theorem 3.1

Let G_{FR} be a fuzzy random graph. Then the inequality holds $CI_{G_{FR}} \le WJ_{G_{FR}}$

Proof:

To show that the connectivity index of a fuzzy random graph is always less than or equal to the wiener index of the graph, we need to consider the relationship between the probabilities of edges and their corresponding shortest path distances.

Let us denote $d_s(v_i,v_j)$ as the shortest path distance between vertices v_i and v_j, and $P(v_i,v_j)$ as the probability of the edge between vertices v_i and v_j. Since $P(v_i,v_j)$ is a probability, it lies between 0 and 1.

Then we have,

$$CI_{G_{FR}} = \sum_{i<j} P(v_i,v_j)\ and\ WJ_{G_{FR}} = \sum_{i<j} d_s(v_i,v_j)$$

For any pair of vertices v_i and v_j, the shortest path distance $d_s(v_i,v_j)$ between them is always non-negative. Therefore, for any pair of vertices, $P(v_i,v_j) \le d_s(v_i,v_j)$since $P(v_i,v_j)$ is a probability and probabilities are always less than or equal to 1.

Hence, for each term in the summation, we have $P(v_i,v_j) \le d_s(v_i,v_j)$. Therefore,

$$CI_{G_{FR}} = \sum_{i<j} P(v_i,v_j) \le WJ_{G_{FR}} = \sum_{i<j} d_s(v_i,v_j)$$

Thus, the Connectivity index of a fuzzy random graph is always less than or equal to the Wiener index of the graph.

Theorem 3.2

Let H_{FR} be a fuzzy random subgraph of a fuzzy random graph G_{FR}. Then $R(H_{FR}) \le R(G_{FR})$.

Proof:

Since $H_{FR} = (V_{FR}, E_{FR}, \tau_G, \sigma_G)$ is an fuzzy random subgraph of $G_{FR} = (V_{FR}, E_{FR}, \varphi_G, \psi_G)$, therefore $\tau_G(u) \le \varphi_G(u)$ and $\sigma_G(uv) \le \psi_G(uv)$ for all $u, v \in V_{FR}$. This implies that $d_{H_{FR}}(v_i) \le d_{G_{FR}}(v_i)$ for all vertices in G_{FR}.

Now,

$$R(G_{FR}) = \frac{1}{2}\sum_{(v_i,v_j)\in E_F}\frac{1}{\sqrt{\phi_G(v_i)d_{G_{FR}}(v_i)\phi_G(v_j)d_{G_{FR}}(v_j)}} + \sum_{(v_i,v_j)\epsilon E_R}\frac{1}{\sqrt{d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)}}$$

$$\geq \frac{1}{2}\sum_{(v_i,v_j)\in E_F}\frac{1}{\sqrt{\tau_G(v_i)\,d_{H_{FR}}(v_i)\,\tau_G(v_j)\,d_{H_{FR}}(v_j)}}+\sum_{(v_i,v_j)\epsilon E_R}\frac{1}{\sqrt{d_{H_{FR}}(v_i)d_{H_{FR}}(v_j)}}=R(H_{FR})$$

Hence, $R(H_{FR})\leq R(G_{FR})$. This inequality is also applicable to the Harmonic index of fuzzy random graph. i.e., $H(H_{FR})\leq H(G_{FR})$.

Relation between the operations such as union and intersection of topological indices of fuzzy random graph:

Theorem 3.3

Let G_{FR} and H_{FR} be two fuzzy random graphs. The union of two fuzzy random graphs $G_{FR}\cup H_{FR}$ is

i. $M(G_{FR}\cup H_{FR})\geq max\{M(G_{FR}),M(H_{FR})\}$
ii. $M^{*}(G_{FR}\cup H_{FR})\geq max\{M^{*}(G_{FR}),M^{*}(H_{FR})\}$
iii. $R(G_{FR}\cup H_{FR})\leq max\{R(G_{FR}),R(H_{FR})\}$
iv. $H(G_{FR}\cup H_{FR})\leq max\{H(G_{FR}),H(H_{FR})\}$

Proof:

There are 4 cases arise.

Case (i):

Let $G_{FR}=(V_{FR},\,E_{FR})$ and $H_{FR}=(V^{*}_{FR},\,E^{*}_{FR})$ be two fuzzy random graphs. Consider$V_{FR}\neq {V_{FR}}^{*}\neq\phi$, then $V_{FR}\cap {V_{FR}}^{*}=\phi$ which implies that $\varphi_G(v_i)\cap\varphi_G({v_i}^{*})=\phi$and$\psi_G(v_i,v_j)\cap\psi_G({v_i}^{*},{v_j}^{*})=\phi$. Consequently, the first kind of Zagreb index determined by union operations is given by

$$M(G_{FR}\cup H_{FR})\geq max\{M(G_{FR}),M(H_{FR})\}$$

which indicates that the fuzzy random graphs are distinct.

Case (ii):

Suppose $G_{FR}=(V_{FR},E_{FR})$ and $H_{FR}=(V^{*}_{FR},E^{*}_{FR})$ be two fuzzy random graphs. Consider some of the vertices in V_{FR} and${V_{FR}}^{*}$ are common, i.e., $V_{FR}\cap {V_{FR}}^{*}\neq\phi$ but there exists as$\psi_G(v_i,v_j)\cap\psi_G({v_i}^{*},{v_j}^{*})=\phi$. Then the two fuzzy random graphs are not equal and satisfies the condition as

$$M(G_{FR}\cup H_{FR})\geq max\{M(G_{FR}),M(H_{FR})\}$$

Case (iii):

There exist $\varphi_G(v_i)=\varphi_G({v_i}^{*})$and $\varphi_G(v_j)=\varphi_G({v_j}^{*})$with$\psi_G(v_i,v_j)=\psi_G({v_i}^{*},{v_j}^{*})$. Suppose, adjacent to the vertices of G_{FR} and H_{FR} are different. Then the two fuzzy random graphs have different $M(G_{FR})$ and $M(H_{FR})$ values which shows that

$$M(G_{FR}\cup H_{FR})\geq max\{M(G_{FR}),M(H_{FR})\}$$

Case (iv):

Consider one of the fuzzy random graph structures coincides with other but not equal i e.,$H_{FR}-\{u\}$ where $u\in H_{FR}$. Then

$$M(G_{FR}\cup H_{FR})\geq max\{M(G_{FR}),M(H_{FR})\}$$

For the Zagreb index of first kind, the theorem is quite proved. The same process is used to establish the relationship between the union and intersection of topological indices, including the Harmonic index, the Randić connectivity index and the second Zagreb index of fuzzy random graph.

Theorem 3.4

Let G_{FR} and H_{FR} be two fuzzy random graphs. The intersection of two fuzzy random graphs $G_{FR}\cap H_{FR}$ is

i. $M(G_{FR}\cap H_{FR})\leq min\{M(G_{FR}),M(H_{FR})\}$
ii. $M*(G_{FR}\cap H_{FR})\leq min\{M*(G_{FR}),M*(H_{FR})\}$
iii. $R(G_{FR}\cap H_{FR})\geq min\{R(G_{FR}),R(H_{FR})\}$
iv. $H(G_{FR}\cap H_{FR})\geq min\{H(G_{FR}),H(H_{FR})\}$

Proof:

The proof is similar to the previous theorem (3.3).

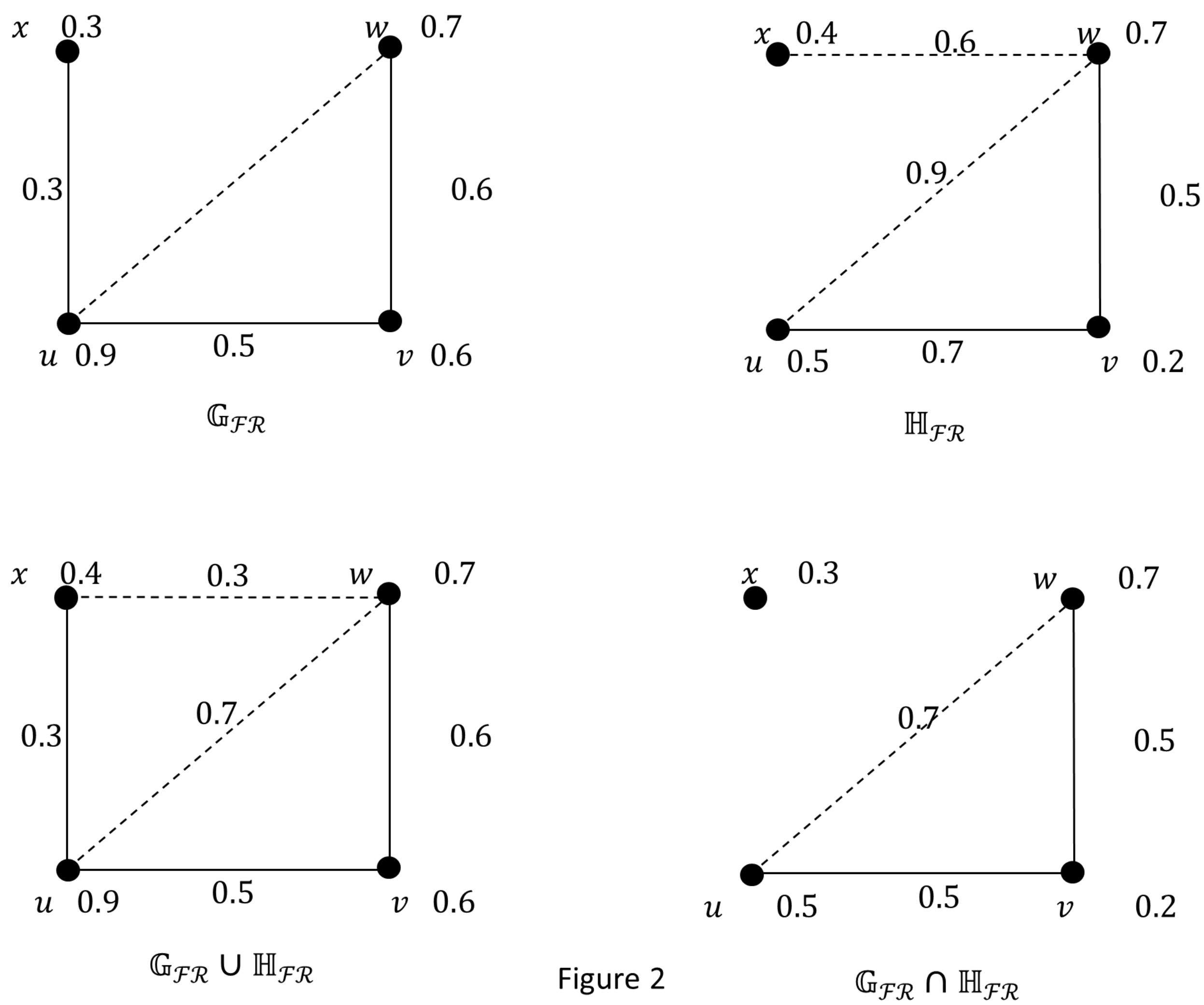

Fig 2.

Example 3.1

Two fuzzy random graphs G_{FR} and H_{FR} and the union $G_{FR} \cup H_{FR}$ and the intersection $G_{FR} \cap H_{FR}$ of them are given in Figure 2.

Degree of vertices:

For fuzzy random graphs in Figure 2,

$$d_{G_{FR}}(u) = 2.2 \qquad d_{G_{FR}}(v) = 1.1 \qquad d_{G_{FR}}(w) = 2.0 \qquad d_{G_{FR}}(x) = 0.3$$

$$d_{H_{FR}}(u) = 1.2 \qquad d_{H_{FR}}(v) = 0.4 \qquad d_{H_{FR}}(w) = 2.0 \qquad d_{H_{FR}}(x) = 0.8$$

$$d_{G_{FR} \cup H_{FR}}(u) = 2.2 \qquad d_{G_{FR} \cup H_{FR}}(v) = 1.1 \qquad d_{G_{FR} \cup H_{FR}}(w) = 2.8 \qquad d_{G_{FR} \cup H_{FR}}(x) = 1.1$$

$$d_{G_{FR} \cap H_{FR}}(u) = 1.2 \qquad d_{G_{FR} \cap H_{FR}}(v) = 0.4 \qquad d_{G_{FR} \cap H_{FR}}(w) = 1.2 \qquad d_{G_{FR} \cap H_{FR}}(x) = 0$$

The first Zagreb index of fuzzy random graph:

$$M\left(G_{FR}\right)=\sum_{v_i\epsilon V_{FR}} v_i(d_{G_{FR}}(v_i))^2 for\ all\ v_i\epsilon V_{FR}$$

$$M\left(G_{FR}\right)=\left(0.9\right)\left(2.2\right)^2+\left(0.6\right)\left(1.1\right)^2+\left(0.7\right)\left(2\right)^2+\left(0.3\right)\left(0.3\right)^2$$

$$M\left(G_{FR}\right)=7.91$$

$$M\left(H_{FR}\right)=\sum_{v_i\epsilon V_{FR}} v_i(d_{H_{FR}}(v_i))^2\ for\ all\ v_i\epsilon V_{FR}$$

$$M\left(H_{FR}\right)=\left(0.5\right)\left(1.2\right)^2+\left(0.2\right)\left(0.4\right)^2+\left(0.7\right)\left(2\right)^2+\left(0.4\right)\left(0.8\right)^2$$

$$M\left(H_{FR}\right)=3.81$$

$$M\left(G_{FR}\cup H_{FR}\right)=\sum_{v_i\epsilon V_{FR}} v_i(d_{G_{FR}\cup H_{FR}}(v_i))^2\ for\ all\ v_i\epsilon V_{FR}$$

$$M\left(G_{FR}\cup H_{FR}\right)=\left(0.9\right)\left(2.2\right)^2+\left(0.6\right)\left(1.1\right)^2+\left(0.7\right)\left(2.8\right)^2+\left(0.4\right)\left(1.1\right)^2$$

$$M\left(G_{FR}\cup H_{FR}\right)=11.10$$

$$M\left(G_{FR}\cap H_{FR}\right)=\sum_{v_i\epsilon V_{FR}} v_i(d_{G_{FR}\cap H_{FR}}(v_i))^2\ for\ all\ v_i\epsilon V_{FR}$$

$$M\left(G_{FR}\cap H_{FR}\right)=\left(0.5\right)\left(1.2\right)^2+\left(0.2\right)\left(0.4\right)^2+\left(0.7\right)\left(1.2\right)^2+\left(0.3\right)\left(0\right)$$

$$M\left(G_{FR}\cap H_{FR}\right)=1.76$$

The second Zagreb index of fuzzy random graph:

$$M^*(G_{FR})=\frac{1}{2}\sum_{(v_i,v_j)\in E_F}\phi_G(v_i)d_{G_{FR}}(v_i)\phi_G(v_j)d_{G_{FR}}(v_j)+\sum_{(v_i,v_j)\in E_R}d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)$$

$$M^*\left(G_{FR}\right)=\frac{1}{2}\left\{(0.9\times2.2\times0.6\times1.1)+(0.6\times1.1\times0.7\times2)+(0.9\times2.2\times0.3\times0.3)\right\}+(2.2\times2)$$

$$M^*(G_{FR})=6.41$$

$$M^{*}\left(H_{FR}\right)=\frac{1}{2}\left\{(0.5\times1.2\times0.2\times0.4)+(0.2\times0.4\times0.7\times2)\right\}+(1.2\times2)+(2\times0.8)$$

$$M^{*}(H_{FR})=4.08$$

$$M^{*}\left(G_{FR}\cup H_{FR}\right)=\frac{1}{2}\left\{(0.9\times2.2\times0.6\times1.1)+(0.6\times1.1\times0.7\times2.8)+(0.9\times2.2\times0.4\times1.1)\right\}+(2.2\times2.8)+(2.8\times1.1)$$

$$M^{*}(G_{FR}\cup H_{FR})=10.98$$

$$M^{*}\left(G_{FR}\cap H_{FR}\right)=\frac{1}{2}\left\{(0.5\times1.2\times0.2\times0.4)+(0.2\times0.4\times0.7\times1.2)\right\}+(1.2\times1.2)$$

$$M^{*}(G_{FR}\cap H_{FR})=1.50$$

The Randić index of fuzzy random graph:

$$R\left(G_{FR}\right)=\frac{1}{2}\sum_{(v_i,v_j)\in E_F}\frac{1}{\sqrt{\phi_G\left(v_i\right)d_{G_{FR}}\left(v_i\right)\phi_G\left(v_j\right)d_{G_{FR}}\left(v_j\right)}}+\sum_{(v_i,v_j)\epsilon E_R}\frac{1}{\sqrt{d_{G_{FR}}(v_i)d_{G_{FR}}(v_j)}}$$

$$R\left(G_{FR}\right)=2.62$$

$$R\left(H_{FR}\right)=5.21$$

$$R\left(G_{FR}\cup H_{FR}\right)=2.39$$

$$R\left(G_{FR}\cap H_{FR}\right)=5.04$$

The Harmonic index of fuzzy random graph:

$$H\left(G_{FR}\right)=\frac{1}{2}\sum_{(v_i,v_j)\epsilon E_{FR}}\frac{1}{\phi_G\left(v_i\right)d_{G_{FR}}\left(v_i\right)+\phi_G\left(v_j\right)d_{G_{FR}}\left(v_j\right)}+\sum\frac{2}{d_{G_{FR}}\left(v_i\right)+d_{G_{FR}}\left(v_j\right)}$$

$$H\left(G_{FR}\right)=0.90$$

$$H\left(H_{FR}\right)=2.12$$

$$H\left(G_{FR}\cup H_{FR}\right)=1.07$$

$$H\left(G_{FR}\cap H_{FR}\right)=1.97$$

Applications

Topological indices of fuzzy random graphs find applications across several fields due to their ability to capture structural characteristics of complex networks with uncertain or varying connectivity. These are just a few applications.

1. To identify the location for charging stations for electric vehicles: Electric charging stations can be seen as vertices in a fuzzy random graph, and the roads or paths connecting them can be represented as edges. By analyzing the network topology using topological indices of fuzzy random graph, one can identify optimal locations for charging stations based on factors such as connectivity, centrality, and efficiency of the network.

2. Drug Discovery and chemo informatics: Topological indices are used to model molecular structures in drug discovery. Fuzzy random graph indices help predict molecular properties, such as bioactivity or toxicity, by capturing the structural features of molecules with uncertain or variable bonds. These predictions aid in virtual screening and lead optimization processes.

3. Protein structure prediction: In bioinformatics, topological indices of fuzzy random graphs are applied to model protein structures and interactions. By representing proteins as graphs with fuzzy edges, researchers can analyze the structural properties of protein networks, predict protein functions and identify potential drug targets.

4. Social network analysis: Social networks are inherently dynamic and uncertain, making traditional graph analysis challenging. Topological indices of fuzzy random graphs enable the study of social networks with uncertain or probabilistic connections. These indices provide insights into network dynamics, information diffusion and community detection in social media or collaboration networks.

5. **Transportation planning and Infrastructure Design:** Fuzzy random graph indices are applied in transportation planning to analyze road networks, public transportation systems, and logistics networks. By quantifying network connectivity and robustness, these indices aid in optimizing transportation routes, improving traffic flow and enhancing the resilience of transportation infrastructures.

6. Power Grid analysis: Power grids are complex networks with uncertain connections and dynamic load patterns. Topological indices of fuzzy random graphs help assess the reliability and optimization, demand forecasting and energy management strategies.

7. Ecological network modeling: Ecological networks, such as food webs, mutualistic networks and habitat networks, exhibit complex interactions and uncertain species relationships. Fuzzy random graph indices are used to model ecological networks and assess their stability, resilience and biodiversity. These indices aid in ecosystem management, conservation planning and species coexistence analysis.

8. Machine learning and pattern recognition: Fuzzy random graph indices are utilized in machine learning and pattern recognition tasks, such as image analysis, pattern matching and object recognition. By representing data as fuzzy graphs, these indices capture spatial relationships, connectivity patterns and structural features, improving the performance of classification, clustering and anomaly detection algorithms.

The applications of topological indices of fuzzy random graphs span diverse domains, including drug discovery, bioinformatics, social network analysis, transportation planning, ecology, communication networks and machine learning. These indices provide valuable insights into the structural characteristics and dynamics of complex networks with uncertain or variable connectivity, informing decision making, optimization and prediction tasks in various fields.

4 Conclusion

Specific topological indices, such as the connectedness index, Wiener index, Zagreb index, Randić connectivity index, and Harmonic index of fuzzy random graphs, are introduced in this study. Additionally, a relationship between the operations on topological indices of fuzzy random graphs and their applications discussed in this study can be stated. Applications of topological indices, like the fuzzy random graph's connectedness index, in the discipline of medicine will be covered in afterwards.

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