INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Tree Domination Number of Semi Total Point Graphs

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OPEN ACCESS

Received: 16-05-2024

Accepted: 10-07-2024

Published: 01-08-2024

Citation: Revathi R, Vidhya P (2024) Tree Domination Number of Semi Total Point Graphs. Indian Journal of Science and Technology 17(30): 3109-3115. https://doi.org/10.17485/IJST/v17i30.1612

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: The main aim of the research work is to study the concept of a tree domination number of semi-total point graphs. **Method:** A set D $\subseteq V$ of a graph G = (V, E) is a dominating set, if every vertex in V- D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set D. A dominating set D is called a tree dominating set, if the induced subgraph $\langle D \rangle$is a tree. The minimum cardinality of a tree dominating set is called the tree-domination number of G and is denoted by $\gamma_{td}(G)$. For any graph G = (V, E), the semi-total point graph T_2(G) = H is the graph whose vertex set is the union of vertices and edges in which two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex and other is an edge of G incident with it. **Findings:** The characterizations of tree dominating set in semi total point graph sets to be minimal are found. To obtain the exact values, lower and upper bounds of tree dominating set in semi total point graph in some standard graphs. **Novelty:** This study provides bounds and results concerning the tree domination number of the semi-total point graph. This implies that the study likely presents new findings or insights into the behavior of these graphs.

Keywords: Dominating set; Domination number; Tree dominating set; Tree domination number; Semi total point graph

1 Introduction

Graphs that are discussed in this paper are undirected and simple graphs. For a graph G, let V (G) and E (G) denote its vertex set and edge set respectively. For v ϵ V (G), the **neighborhood N (v)** of v is a set of all vertices adjacent to v in G,

N[v] =N (v) {v} is called the **closed neighborhoodof v**. The **degree of a vertex v** in a graph G is the number of edges of G incident with v. A vertex vεV (G) is called a **support vertex** if it is adjacent to a pendant vertex (that is, a vertex of degree one). The maximum (minimum) degree of a vertex v is denoted by Δ(G)(δ(G)). The maximum d(u,v) for all u in G is **eccentricity of v** and the maximum eccentricity is the **diameter of G** and is denoted by diam(G). A **path** on p vertices P_p, is the simple graph consisting of a path, the end-vertices have degree 1 and all vertices in between have degree 2. A **cycle** on p vertices C_p, is the simple graph consisting of a cycle where each vertex has degree two. A graph is **connected** if every pair of points are joined by a path. A connected acyclic graph is called a tree. A graph H = (V_1,X_1) is called a **subgraph** of G (V, X) if $V_1 \subseteq V$ and $X_1 \subseteq X$. H is a subgraph of G then we say that G is a **super graph** of H. H is called a **spanning graph** of G if V_1 = V. If H is an **induced subgraph** of G then two point are adjacent in H if and only if they are adjacent in G. A graph in which any two distinct points are adjacent is called a **complete graph**. The complete graph with p points is denoted by K_p. K($_{m,n}$) is the **complete bipartite graph** with two partite sets containing m and n as vertices. The **Corona** $G_1{}^\circ G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 of order p_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Graphs that are considered here are finite and connected with p vertices and q edges. Any undefined terms in this paper may be found in Harary[1]. The concept of domination in graphs was introduced by Ore[2]. A set D⊆V(G) is said to be a dominating set of G, if every vertex in V(G) –D is adjacent to some vertex in D. D is said to be a **minimal dominating set** if D-{u} is not a dominating set, for any u ∈D. The **domination number** $\gamma(G)$of G is the minimum cardinality of a dominating set. Sampathkumar and Walikar[3] introduced the concept of connected domination number of a graph. A dominating set D of G is called a **connected dominating set**, if the induced subgraph $\langle D\rangle$ is connected. The minimum cardinality of a connected dominating set of G is called the **connected domination number** of G and is denoted by $\gamma_c(G)$.

Xue-Gang Chen, Liang Sun and Alice A McRae[4] introduced the concept of tree domination in graphs. A dominating set D of G is called a **tree dominating set**, if the induced sub graph $\langle D\rangle$ **is a tree**. The minimum cardinality of a tree dominating set of G is called the tree domination number of G and is denoted by $\gamma_{td}(G)$. Sampathkumar and Chikkodimath[5] introduced the concept of semi- total graphs of a graph-I. Basavanagoud, Hosamani and Malghan[6] introduced the concept of domination in semi total-point graph. The semi total point graph T_2(G) of a graph G is the graph whose vertex set is $V(G) \cup E(G)$, where two vertices are adjacent if and only if (i) they are adjacent vertices of G or (ii) one is a vertex of G and the other is an edge of G, incident with it. In this paper we determine tree domination number of semi total point graph of some standard graphs and its bounds are obtained.

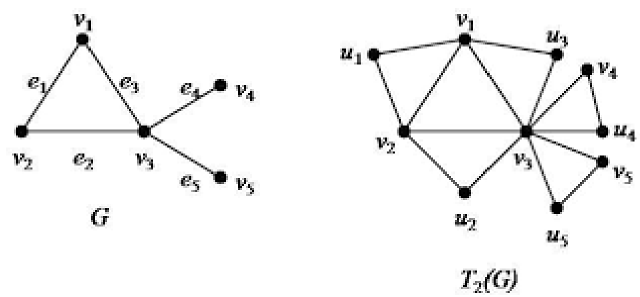

Fig 1. Graph G and its semi total point graph T_2 (G)

Here, $D = \{v_1, v_3\}$ is a tree-dominating set of T_2(G). Hence, $|D| = \gamma_{td}(T2(G)) = 2.$

2 Methodology

A dominating set D of G is called a tree-dominating set, imposing the condition the induced subgraph ⟨D⟩ is a connected and acyclic.

3 Tree Domination Number of Some Standard Graphs

Observation 3.1

1. For any connected graph G, $\gamma(G) \leq \gamma_{td}(G)$
2. For any spanning sub graph H of G, $\gamma_{td}(G) \leq \gamma_{td}(H)$

Proposition 3.2[7]

Every support vertices of G are members of tree dominating set.

Proof:

Let v be a support vertex in G, such that deg(v)≥2 and let D be a td-set of G.

If vε V-D, then a vertex adjacent to v must be in D. Since deg(v) ≥2 then D contains at least two vertices but it is not connected. So <D> is not a tree. Which is a contradiction. Hence the proposition follows.

Proposition 3.3

$\gamma_{td}(G)$ = p-m, where m is the number of pendent vertices in G,
Since every support vertex is a member of tree dominating set. Here, the proposition follows.

Observation 3.4

1. For any path P_p with p $\geq$ 3 vertices, $\gamma_{td}(P_p) = p-2$

Proof:

Let P_p(p≥3) be a path with vertex set $\{v_1, v_2, v_3, ..., v_p\}$and $D = \{v_2, v_3, ..., v_{p-1}\}$be the minimum tree-dominating set. Hence, |D| = $\gamma_{td}(Pp) = p-2$.

2. For any cycle C_p with p $\geq$ 3 vertices, $\gamma_{td}(Cp) = p-2$.

Proof:

Let $V(Cp) = \{ v_1, v_2, v_3, ..., v_p\}$Let $D = V(G) - \{v_{p-1}, v_p\}$ be the minimum td-set.
Hence, $|D| = \gamma_{td}(Cp) = p-2$.

3. For any complete bipartite graph $K_{m,n}$ with m,n $\geq$ 2, $\gamma_{td}(km, n) = 2$.

Proof:

Let $V\left(K_{m,n}\right) = \left\{u_i, v_j \backslash i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ m, n \geq 2\right\}$. Take any one vertex from u_i say u_1 and any one vertex from $v_j$say v_2 which dominates all the vertices of $K_{m,n}$. Hence, $\gamma_{td}(K_{m,n}) = 2$.

4. For star graph $K_{1,p-1}$ with p $\geq$ 2 vertices, $\gamma_{td}(K1, p-1) = 1$.

Proof:

Let v be the vertex of degree p – 1 in $K_{1,p-1}$. Hence, D = {v} is a tree dominating set.
Therefore, |D| = $\gamma_{td}(K1, p-1) = 1$.

5. For a wheel graph W_p with p $\geq$ 4 vertices, $\gamma_{td}(Wp) = 1$.

Proof:

Let G = $W_p \cong K_1 + C_{p-1}$. Let v be the vertex of K_1 and $v_1, v_2, ..., v_{p-1}$ be the vertices of cycle C_{p-1}. Hence, D = {v} be a minimum tree dominating set.
Therefore, |D| = $\gamma_{td}(Wp) = 1$.

6. For complete graph K_p with p≥3 vertices, $\gamma_{td}(K_p) = 1$.

Proof:

In a complete graph K_p , each vertex is adjacent to each other vertex. Hence, take any one vertex of K_p which dominates all other vertices.
Hence ,$\gamma_{td}(Kp) = 1$.

7. For any fan graph Fp with p≥3 vertices, $\gamma_{td}(F_p) = 1$.

Proof:

Let $V(Fp) = \{v_1, v_2, ..., v_{p-1}\}$and hence D = {v} is a minimum tree dominating set of F_p. Therefore, |D| = $\gamma_{td}(Fp) = 1$.

4 Main Results

Tree domination number of semi total point graphs

In the following, exact values of tree domination number of semi total point graph of graphs are given.

Proposition 4.1

For any path P_p with p $\geq$ 2 vertices, $\gamma_{td}(T2(Pp)) = p-2$.

Proof:

Let P_p(p≥2) be a path with vertex set $\{v_1, v_2, v_3, ..., v_p\}$and edge set $\{e_1, e_2, e_3, ..., e_{p-1}\}$. For semi total point graph $T_2(P_p)$whose vertex set is V(G) $\bigcup$ E(G). Then the edge vertices are labelled as $v_1', v_2', v_3', ..., v_{p-1}'$.
Then $V(T2(Pp)) = \{v_1, v_2, v_3, ..., v_p, v_1', v_2', v_3', ..., v_{p-1}'\}$
Let $D = \{v_2, v_3, ..., v_{p-1}\}$be the minimum tree dominating set of T_2 (P_p).
Hence, $|D| = \gamma_{td}(T2(Pp)) = p-2$.

Proposition 4.2

For any cycle C_p with p $\geq$ 3 vertices, $\gamma_{td}(T2(Cp)) = p-1$.

Proof:

Let C_p(p≥3) be a cycle with vertex set $\{v_1, v_2, v_3, ..., v_p\}$and edge set $\{e_1, e_2, e_3, ..., e_p\}$. Let $v_1', v_2', v_3', ..., v_p'$ be the corresponding edge vertices of G in $T_2(C_p)$. Then V($T_2(C_p)$ = $(\bigcup_{i=1}^{p} v_i) \cup (\bigcup_{i=1}^{p} v_i')$. Let $D = \{v_1, v_2, v_3, ..., v_{p-1}\}$be the minimum td-set of $T_2(C_p)$.
Hence, |D| = $\gamma_{td}(T2(Cp)) = p-1$.

Proposition 4.3

For any star graph $K_{1,p-1}$ with p $\geq$ 2 vertices, $\gamma_{td}(T_2(K_{1.p-1})) = 1$

Proof.

Let $K_{1,p-1}$ be a star graph with p vertices say $v, v_1, v_2, v_3, ..., v_{p-1}$ and corresponding edge vertices say $v_1', v_2', v_3', ..., v_{p-1}'$.

Then $V(T_2(K_{1,p-1})) = \{v, v_1, v_2, v_3, ..., v_{p-1}, v_1', v_2', v_3', ..., v_{p-1}'\}$

Let D = {v} is a tree dominating set of $T_2(K_{1,p-1})$.

Hence, |D|= $\gamma_{td}(T_2(K_{1.p-1})) = 1$

Proposition 4.4

For any complete bipartite graph $K_{m,n}$, $\gamma_{td}(T_2(K_{m,n}))$= min (m,n)+1, where m, n $\geq$2.

Proof.

Let V ($K_{m,n}$) =$\{v_1, v_2, v_3, ..., v_m, u_1, u_2, u_3, ..., u_n\}$, m, n $\geq$2 be the vertex set, where $v_i \epsilon V_1(G)\, for\ i = 1, 2, \ldots, m$ and $u_i \epsilon V_2(G)\ for\ i = 1, 2, \ldots, n.$

$Let\ w_{11}, w_{12}, w_{13}, \ldots, w_{ij}\ for\ i = 1, 2, \ldots, m\ and\ j = 1, 2, \ldots, n$ be the corresponding edge vertices of $T_2(K_{m,n})$.

Then $V(T2(Km,n)) = (\bigcup_{i=1}^{m} v_i) \cup (\bigcup_{i=1}^{n} u_i) \cup (\bigcup_{i=1}^{m} \bigcup_{j=1}^{n} w_{ij})$.

Case (i) if m < n

Let $D = \bigcup_{i=1}^{m} v_i \cup (u_1\}$ is a minimum tree dominating set of $T_2(K_{m,n})$.

Hence, |D|= m +1.

Case (ii) if n $< m$

Let $D = \bigcup_{i=1}^{n} u_i \cup \{v_1\}$ is a minimum tree dominating set of $T_2(K_{m,n})$

Hence, |D|= n +1.

From cases (i) and (ii), we conclude |D|= min (m, n) + 1.

Therefore, $\gamma_{td}(T2(Km,n)) = min\,(m, n) + 1.$

Proposition 4.5

For any wheel W_p (p$\geq$ 5) vertices, $\gamma_{td}(T2(Wp)) = \left\{ \begin{matrix} [\frac{p}{2}], if\, p\, is\, odd \\ does\ not\ exists,\ if\ p\ is\ even \end{matrix} \right\}$.

Proof.

Let $v, v_1, v_2, ..., v_{p-1}$ be the vertices of wheel W_p, where d(v) = p – 1 and corresponding edge vertices are denoted by $v_1', v_2', v_3', ..., v_{2p-2}'$, in $T_2(W_p)$.

Case (i) p is odd

Let $D = \{v, v_1, v_2, ..., v_{\frac{p-1}{2}}\}$ be a minimum tree dominating set of $T_2(W_p)$.

Hence,

$$|D| = \left\lceil \frac{p}{2} \right\rceil \tag{1}$$

Case (ii) p is even

$$Tree\ dominating\ set\ does\ not\ exist \tag{2}$$

From Equation (1) and **(2)** we conclude the result.

Proposition 4.6[8]

For a corona graph $P_p \circ K_1$, p $\geq$ 2 ,$\gamma_{td}(T_2(P_p \circ K_1))$ = p.

Proof:

Let G= $P_p \circ K_1$, then $V(G) = \{v_1, v_2, ..., v_p, u_1, u_2, ..., u_p\}$where u_i be the pendant vertex adjacent to v_i for i= 1,2,..,p.

Let $v_1, v_2, ..., v_p, u_1, u_2, ..., u_p, v_1', v_2', ..., v_{p-1}', u_1', u_2', ..., u_p'$ be the vertices of T_2 (G) respectively. Let $D = \{v_1, v_2, ..., v_p\}$is a minimum tree dominating set of $T_2(P_p^\circ K_1)$. Hence,$|D| = \gamma_{td}(T_2(P_p^\circ K_1)) = p.$

5 Bounds for Tree Domination Number of Semi Total Point Graph of Graphs

Observation 5.1

For any connected graph G, then $\gamma_{td}(G) \leq \gamma_{td}(T_2(G))$.

Theorem 5.2

Some vertices of G are only the members of tree dominating set of semi total point graph.

Proof:

Let G be a connected graph with vertices $v_1, v_2, v_3, ..., v_p$, p≥ 2. Then $V(T_2(G)) = \{v_1, v_2, v_3, ..., v_p, v_1^{'}, v_2^{'}, v_3^{'}, ..., v_q^{'}\}$ each $v_i^{'}(1 \leq i \leq q)$is adjacent with two vertices of $v_i(1 \leq i \leq p)$which forms a cycle C_3. Suppose $v_i^{'} \in D$ which dominates all the vertices of T_2(G). But $\langle D \rangle$ is not a tree which is a contradiction. Hence, some vertices of G which dominates the vertices of G and $V(T_2(G)) - V(G)$

Theorem 5.3

For any connected graph G with p≥2 then $1 \leq \ \gamma_{td}(T_2(G)) \leq p + q - 2$.

Proof:

T_2(G) has p + q vertices and radius of T_2(G) has one. Hence, T_2(G) has no vertex of degree p+ q - 1. Therefore, $1 \leq \ \gamma_{td}(T_2(G)) \leq p + q - 2$.

Theorem 5.4

For any connected graph G with $p \geq 2$, $\left\lceil \frac{p}{1+\triangle(G)} \right\rceil \leq \gamma_{td}(T_2(G)) \leq p - 1$.

Proof:

First, we consider the lower bound, a tree dominating set which dominates each vertex at most itself and vertices adjacent to maximum degree vertex of G. Hence $\left\lceil \frac{p}{1+\triangle(G)} \right\rceil \leq \gamma_{td}(G) \leq \gamma_{td}(T_2(G))$ .The lower bound is attained if $G \ \cong K_{1,p-1}$.

By theorem 5.3, for any graph $T_2(G)$there is no vertex of degree p+q-1, therefore $\gamma_{td}(T_2(G)) \leq p + q - 2$.

Since every vertex of $T_2(G)$−V(G) is not a member of $T_2(G)$. Hence, $|V(T_2(G)) - D| \geq q$.

Therefore, $\gamma_{td}(T_2(G)) \leq p + q - 2 - (q - 1)$.

$\gamma_{td}(T_2(G)) \leq p - 1$.

Theorem 5.5

$\gamma_{td}(T_2(G)) = 1$ if and only if $G \cong K_{1,p-1}$, $p \geq 2$.

Proof:

Let D = {v} be a γ_{td} set of $T_2(G)$ such that $|D| \ = \ 1$.

Since each vertex of $T_2(G) - \{v\}$ is adjacent to the vertex in D. Hence $G \cong \ K_{1,p-1}$

Conversely, If $G \cong \ K_{1,p-1}$ then $\gamma_{td}(T_2(G)) \ = 1$.

Theorem 5.6

$\gamma_{td}(T_2(G)) = 2$if and only if diam $T_2(G) = \ 2 \, or \, 3$.

Proof:

Assume $\gamma_{td}(T_2(G)) = \ 2$. Let D = {u,v} be a tree dominating set of $T_2(G)$. Since every member of td set of $T_2(G)$ are vertices of G in $T_2(G)$

Hence, u, v $\in G$ in $T_2(G)$.

Case (i)

Suppose u and v are adjacent to one vertex say w in $V(T_2(G)) - D$,, be the corresponding edge vertices are $e_1^{'}, e_2^{'}, e_3^{'}$ where $e_1^{'} = (u, v), e_2^{'} = (v, w) \, and \, e_3^{'} = (w, u)$ then $d\ (u,\ e_2^{'}) = \ d(v,\ e_3^{'}) = 2$.

Hence, diam (G) = 2.

Case (ii)

Suppose u and v are adjacent to two different vertices say x and y in $V(T_2(G)) - D$) and corresponding edge vertices $e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, e_5^{'}$Where $e_1^{'} = \ (u,\ v)$,$e_2^{'} = \ (u,\ x)$,$e_3^{'} = \ (x,\ v)$$e_4^{'} = \ (v,\ y)$ and $e_5^{'} = \ (y, u)$. Then $d\ (u,\ e_4^{'}) = \ d\ (u,\ e_5^{'}) = d\ (v,\ e_2^{'}) \ = \ d\ (v,\ e_5^{'}) = 2$.

Hence, diam (G) = 2.

Case (iii)

Suppose u and v are adjacent to one vertex say w and pendant vertex x is adjacent to u and pendant vertex y is adjacent to v and corresponding edge vertices $e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, e_5^{'}$ Where $e_1^{'} = \ (u,\ v)$,$e_2^{'} = \ (u,\ x)$,$e_3^{'} \ = (v,\ y)$,$e_4^{'} = (u,\ w)$ and $e_5^{'} \ = (v,\ w)$.

Then d (w, x) = d (w, y) = 2.

Hence, diam (G) =2.

Case (iv)

Suppose u is adjacent to the vertex x and v is adjacent to the vertex y in $V(T_2(G)) - D$ and the corresponding edge vertices $e_1^{'}, e_2^{'}, e_3^{'}$. Where $e_1^{'} = (u,\ v)$,$e_2^{'} \ = (u,\ x)$and $e_3^{'} \ = (\ v,\ y)$. Then $d\ (e_2^{'},\ y) = d\ (e_3^{'},\ x) = 3$.

Hence, diam (G) =3.

Case (v)

Suppose u and v are adjacent to one vertex say x and pendant vertex say y is adjacent to any one of the vertex u or v say u, then the corresponding edge vertices $e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}$. Where $e_1^{'}$ = (u, v), $e_2^{'}$ = (u, x), $e_3^{'}$ = (x, v) and $e_4^{'}$ = (v, y).

Then d (y, $e_3^{'}$) = 3

Hence, diam (G) = 3.

Case (vi)

Suppose u and v are adjacent to one vertex say w and two pendant vertices say x and y are adjacent to any one of the vertex u or v, say u then the corresponding edges $e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}\ and\ e_5^{'}$ Where $e_1^{'} = (u, v)$,$e_2^{'} = (u, x)$,$e_3^{'} = (u, y)$,$e_4^{'} = (u, w)$ and $e_5^{'} = (v, w)$.

Then $d\,(x,\ e_5^{'}) = d\,(y,\ e_5^{'}) = 3$.

Hence, diam (G) = 3.

From case (i), (ii), (iii), (iv), (v) and (vi)

$\gamma_{td}\,(T_2(G)) = 2$ then diam $(T_2(G)) = 2\ or\ 3$.

Conversely

If diam $T_2(G)\ =\ 2$, since D is a tree$|D| \leq\ 2$.

Case (i)

$|D| = 1$

Let v $\in D$ then v is adjacent to the vertices $v_1, v_2, ..., v_{p-1}$ of G in $V(T_2(G)) - D$ and vertices $v_1^{'}, v_2^{'}, ..., v_{p-1}^{'}.d(v_i^{'}, v_j^{'}) = 2$ then $T_2(G) \cong K_{1,p-1}$. But $\gamma_{td}\,(T_2(G)) = 1$.

Which is a contradiction.

Case (ii) $|D| = 2$ then $G \cong G_1, G_2, G_3, G_4, G_5, G_6$ then $\gamma_{td}\,(T_2(G)) = 2$.

Figure 2, semi-total point graphs with diameter 2 or 3.

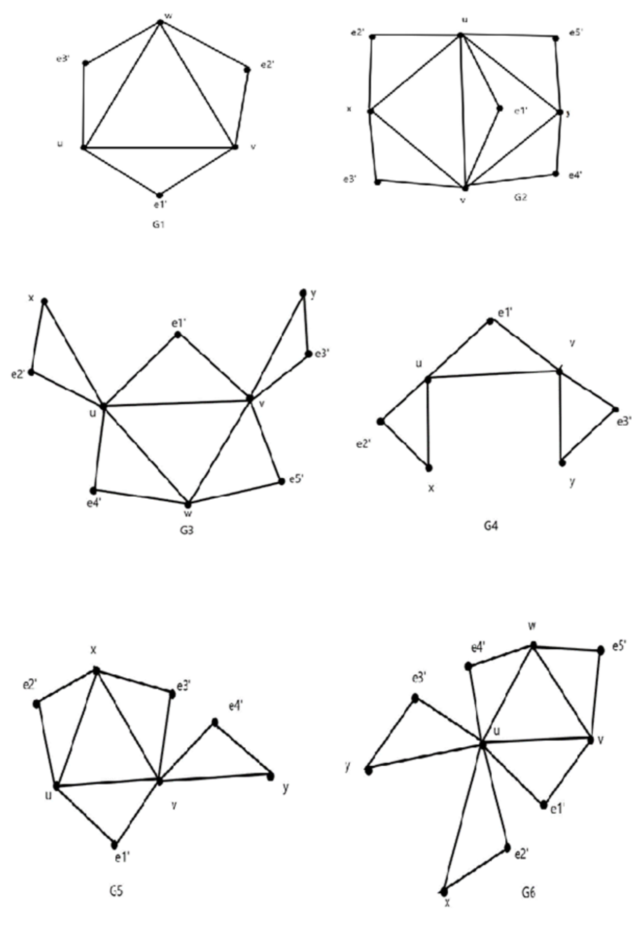

Fig 2. Semi total point graphs with diameter 2 or 3

Theorem 5.7

For any tree T with p $\geq 2\gamma_{td}(T) = \gamma_{td}(T_2(T))$

Proof:

Let T be any nontrivial tree, Let $D_1 = \{v_1, v_2, \ldots, v_s\}$ be a subset of vertex set such that deg $(v_i) \geq 2$for $1 \leq i \leq s$ and $D_2 = \{u_1, u_2, \ldots, u_r\}$ where deg $(u_i) = 11 \leq i \leq r$ be the subset of V(T) where $V(T) = D_1 \cup D_2$, clearly D_1 is a minimum td set of T. Therefore $\gamma_{td}(T) = |D|$ Let $v_1^{'}, v_2^{'}, \ldots, v_s^{'}$ and $u_1^{'}, u_2^{'}, \ldots, u_r^{'}$ be the corresponding vertices of D_1 and D_2in $(T_2(T))$ respectively. Then each $v_i\,(1 \leq i \leq s)$ dominates all the vertices of $(T_2(T))$.

Hence, D_1 is also a tree dominating set of $(T_2(T))$.

$\gamma_{td}(T) = \gamma_{td}(T_2(T))$.

6 Applications of Tree Dominating Set

Network Design and Optimization:

Sensor Networks: In a sensor network, sensors need to communicate with each other and ensure coverage of an area. A tree dominating set can help in identifying a subset of sensors that can communicate directly (dominating set) while the rest of the sensors form a structure (induced subgraph) that is also efficient (tree). This can optimize the deployment and energy usage in the network.

Broadcasting in Networks:

Telecommunication: For efficient broadcasting, ensuring that a small set of nodes can dominate the network and that the remaining nodes still form a connected structure (tree) can improve broadcasting efficiency and reduce redundancy.

Hierarchical Clustering:

In hierarchical clustering algorithms, a tree dominating set can represent cluster heads or leaders that ensure connectivity and coverage within clusters, while the subclusters themselves maintain a tree structure for efficient traversal and communication.

Biological Networks:

Phylogenetic Trees: In evolutionary biology, a tree dominating set can be used to identify key species (or genes) that need to be studied or conserved to ensure the overall integrity of the phylogenetic tree structure.

Social Networks:

In social network analysis, influential nodes (influencers) can be seen as a dominating set. Ensuring that the network without these influencers remains connected (tree structure) can provide insights into the robustness of the network and how information or influence propagates.

Graph Theory and Algorithm Design:

Algorithm Optimization: In designing algorithms for graph problems, a tree dominating set can help in simplifying the problem by focusing on a smaller subset of vertices while ensuring the problem structure (tree) is maintained in the rest of the graph

7 Conclusion

Understanding the tree domination number in semi total point graphs enables comparative analysis with other domination numbers and contributes to various practical applications, including network design, social network analysis, and algorithmic efficiency.

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