INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Application of Improved Class of Weighted Averaging Operators in Intuitionistic Fuzzy Ann Guided MAGDM E-Commerce Problems

P John Robinson[1]*, **P Wilson Arul Prakash**[2]

1 Assistant Professor of Mathematics, Bishop Heber College, Affiliated to Bharathidasan University, Tiruchirappalli, Tamil Nadu, India
2 Assistant Professor of Mathematics, Loyola College, Affiliated to Thiruvalluvar University, Vettavalam, Tamil Nadu, India

OPEN ACCESS

Received: 03-06-2024
Accepted: 13-07-2024
Published: 30-07-2024

Citation: Robinson PJ, Prakash PWA (2024) Application of Improved Class of Weighted Averaging Operators in Intuitionistic Fuzzy Ann Guided MAGDM E-Commerce Problems. Indian Journal of Science and Technology 17(29): 3026-3036. https://doi.org/10.17485/IJST/v17i29.1872

* **Corresponding author**.

johnrobinson.ma@bhc.edu.in

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: To propose a novel and improvised Artificial Neural Network (ANN) technique to solve Multiple Attribute Group Decision Making (MAGDM) problems and an improved class of Aggregation operators for combining Intuitionistic Fuzzy Set (IFS) matrices for the ANN algorithm. **Methods:** A novel class of improved aggregation operators namely the Improved Intuitionistic Fuzzy Weighted Arithmetic Averaging (IM-IFWAA) operator and the Improved Intuitionistic Fuzzy Ordered Weighted Averaging (IM-IFOWA) operator are proposed in this work. The proposed improved class of aggregation operators will aggregate the IFS matrix data sets appearing in the form of matrices and then the revised input vectors which is then fed into the ANN algorithm following Delta, Perceptron, and Hebb Learning Rule for the next phase. **Findings:** Aggregating the Intuitionistic Fuzzy set information or data in today's digital world is a tedious task in the field of MAGDM problem-solving. In this work, two new classes of operators for aggregating the data sets are proposed namely the IM-IFWAA operator and the IM-IFOWA operator which will improve the performance of the ANN. Necessary theorems for the proposed operators are proved to provide consistency of the same. The input created using the aforementioned operators is then fed into the novel ANN algorithm for further computations. Varieties of learning rules are then engaged for ranking of the best alternative for the decision problem. The developed theory is supported by a numerical example which is computed using all the proposed techniques. **Novelty:** Most of the research done on Intuitionistic Fuzzy Artificial Neural Network models are based on learning rules or using some other calculations. The proposed methods of using novel and improvised aggregation operators for ANN are used to find the inputs for ANN where varieties of learning rules for ANN are employed for effective decision analysis.

Keywords: ANN; Learning Rules; MAGDM; Intuitionistic Fuzzy Sets; Weighted Aggregation Operator

1 Introduction

In the present-day scenario, addressing the question of 'why investments in e-commerce should be a good option?' should be dealt with to a very large extent, since most of the shopping and commerce is done through online platforms. Online shopping through e-commerce provides a more convenient choice for many services. It saves money and time for people to buy or sell something online instead of going in person to stores of different types for varied purposes. The power of Digital Marketing has shifted the dimension of the globe to a large extent. Since, nowadays e-commerce is destroying the Brick-and-Mortar stores, proper investments based on the digital market trends are also very crucial. Many online platform companies have grown in their finance also due to the post-pandemic growth of M-commerce which permits people to buy and sell from a mobile phone. In various ways, smartphones play a more crucial role in shopping tasks than ever in history. In this view, most of the real-life data are ambiguous in nature. A single real number, known as the grade of membership, between zero and one is assigned to each item in fuzzy set (FS) theory. A generalization of the idea of a fuzzy set whose fundamental component is merely a membership function, Atanassov[(1)] developed the notion of an Intuitionistic Fuzzy Set (IFS), which is characterized by a membership function and a non-membership function. IFS theory essentially refutes the argument that an element x should "not belong" to a fuzzy set A to the extent that it "belongs" to a specific degree (let's say μ) in A. In contrast, IFSs relax the mandated duality from fuzzy set theory by assigning to each element in the universe a degree of both membership and non-membership. Obviously, the standard fuzzy set idea is recovered when applied to all elements of the universe. The concept of varieties of domains of Fuzzy sets and IFSs are already available to a large extent in the literature.

Decision-making problems in general have been dealt with by various authors and researchers in the past and several solution methods have been proposed with a humongous list of aggregation operators. Problems involving Multi-Attribute Group Decision Making (MAGDM) are common in everyday life. The goal of a MAGDM problem is to select a desired solution from a limited number of workable options that have been evaluated on a variety of quantitative and qualitative criteria. The decision-maker frequently presents preferred information in the form of numerical values, such as exact values, interval numbers, and fuzzy numbers, in order to select a desired outcome. Numerical numbers, however, are frequently insufficient or insufficient to represent real-world decision difficulties. In fact, intuitionistic fuzzy information[(2)] can be used to express human judgments, including preference information. Therefore, researchers[(3–6)] have lately found that MAGDM problems under intuitionistic fuzzy environments are an important field of study and can also be approached through neural networks. It is presumed that there are a fixed number of possible solutions for MAGDM problems. Sorting and ranking are required while solving a MAGDM problem, which may be seen as an alternate technique for fusing information from the decision maker's additional data with the information in the problem's decision matrix to arrive at a final ranking or choice among the options[(7,8)] even in neural networks. To arrive at a final ranking or selection, all but the most basic MAGDM procedures require additional information from the decision matrix in addition to the information already included in the matrix. For several higher-order intuitionistic fuzzy sets as in[(7–10)] there are suggested correlation coefficients that have been used extensively to rank the options in MAGDM issues. Different types of fuzzy sets and new techniques of decision-making and Deep Learning are nowadays utilized for several decision problems involving a lot of uncertainty[(11–16)].

MAGDM problems with digitally enhanced AI techniques are an open area of research and there are a lot of openings in combining ANN techniques with Decision Support Systems (DSS). In this research work, the intuitionistic fuzzy set data are utilized to create the input for ANN especially when the decision-making situation is vague in nature. Recently, the work proposed in[(10)] concentrated on creating the data set for ANN using the Gram-Schmidt orthogonalization process which was novel to the field of MAGDM. Following the work done by the authors in[(10)], here the data set for ANN is composed from the process of using some novel and improvised aggregation operators which is fed into the algorithm of ANN for a rigorous learning rule process where the decision alternatives receive the threshold for the next level by the activation phase. Finally, when the activated dataset from the ANN passes through the threshold, they are assumed to be ready for the choice making of which should be the best alternative from the available ones which is vague in nature from the beginning of the definition of the problem. The final decision results are compared with other methods and the proposed method is an effective one in line with the earlier methods[(5,6,10)] where different techniques were employed for the creation of input data sets for ANN. Comparison of computations with Delta, Perceptron, and Hebb Learning Rules are performed in addition to the existing proposed computations. The comparative analysis helps the decision maker to choose an appropriate tool for decision-making depending on his/her preferences on the nature of the problem/situation.

2 Preliminaries

Here, we will require to present some basic definitions and theory for novel modified aggregation operators needed for proposing the novel algorithm for Decision Support System (DSS).

Definition 1: Intuitionistic fuzzy set [1]

Provided A in X, where $\mu_A : X \to [0,1]$, $\gamma_A : X \to [0,1]$, with the rule $0 \le \mu_A(x) + \gamma_A(x) \le 1$," "$\forall x \in X$, gives an IFS A in X i.e., $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ where the elements x's membership is $\mu_A(x)$ and non-membership degrees to the set A are denoted by $\gamma_A(x)$.

Definition 2: [1]

For an IFS A in X, the hesitancy degree is defined as $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$," "$\forall x \in X$

In the following, some new aggregation operators are proposed for the fact of utilizing them in the process of group decision-making.

Definition 3: The Improved IFWAA operator (IM-IFWAA)

Let $\tilde{a}_j = (\mu_j, \gamma_j)$, for all j = 1,2,…,n be a clump of intuitionistic fuzzy values. The Improved Intuitionistic Fuzzy Weighted Arithmetic Averaging (IM-IFWAA) operator, $IM-IFWAA : Q^n \to Q$ is defined as:

$$IM-IFWAA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \omega_j \tilde{a}_j = \left(1 - \prod_{j=1}^{n} (1-\mu_j)^{\omega_j}, \prod_{j=1}^{n} (1-\gamma_j)^{\omega_j}\right),$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ will project as the weighting vector of $\tilde{a}_j$, for each j = 1, 2, …,n and furthermore $\omega_j > 0$ with $\sum_{j=1}^{n} \omega_j = 1$.

Definition 4: The Improved IFOWA operator (IM-IFOWA)

Let $\tilde{a}_j = (\mu_j, \gamma_j)$, for all j = 1,2,…,n be a clump of intuitionistic fuzzy values. The Improved Intuitionistic Fuzzy Ordered Weighted Averaging (IM-IFOWA) operator

$IM-IFOWA : Q^n \to Q$ is defined as:

$$IM-IFOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_{\sigma(j)}$$

$$= \left(1 - \prod_{j=1}^{n} \left(1 - \mu_{\sigma(j)}\right)^{w_j}, \prod_{j=1}^{n} \left(1 - \gamma_{\sigma(j)}\right)^{w_j}\right),$$

where $w = (w_1, w_2, ..., w_n)^T$ is the affiliated lading vector, $w_j > 0$ with $\sum_{j=1}^{n} w_j = 1$. Over and above that, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1, 2, …,n), and perhaps $\tilde{a}_{\sigma(j-1)} \ge \tilde{a}_{\sigma(j)}$ for all j = 2, …, n.

Especially, if $\omega = (\omega_1, \omega_2, ..., \omega_n)^T = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then the IM-IFOWA operator is reduced to an IM-IF Averaging operator of dimension n.

Theorem: 1

Let $\tilde{\alpha}_j\,(j = 1, 2, ..., n)$ be a clump of intuitionistic fuzzy numbers; then it can be proved that the aggregated value is also an intuitionistic fuzzy number.

Proof: Let $\tilde{\alpha}_1 = \left(\mu_{\tilde{\alpha}_1}, \gamma_{\tilde{\alpha}_1}\right)$ *and* $\tilde{\alpha}_2 = \left(\mu_{\tilde{\alpha}_2}, \gamma_{\tilde{\alpha}_2}\right)$ be two intuitionistic fuzzy numbers, and $\lambda \le 0$. Then we have the following operational rules:

$$(i)\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \left(\mu_{\tilde{\alpha}_1} + \mu_{\tilde{\alpha}_2} - \mu_{\tilde{\alpha}_1}\mu_{\tilde{\alpha}_2}, \gamma_{\tilde{\alpha}_1}\gamma_{\tilde{\alpha}_2}\right)$$

$$(ii)\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \left(\mu_{\tilde{\alpha}_1}\mu_{\tilde{\alpha}_2}, \gamma_{\tilde{\alpha}_1} + \gamma_{\tilde{\alpha}_2} - \gamma_{\tilde{\alpha}_1}\gamma_{\tilde{\alpha}_2}\right)$$

$$(iii)\ \lambda\tilde{\alpha} = \left(1 - (1-\mu_\alpha)^\lambda, (\gamma_\alpha)^\lambda\right)$$

$$(iv)\ \tilde{\alpha}^\lambda = \left[(\mu_\alpha)^\lambda, 1 - (1-\gamma_\alpha)^\lambda\right],\ for\ \lambda > 0.$$

we also have the following rules or properties:

$$(v)\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \tilde{\alpha}_2 \otimes \tilde{\alpha}_1$$

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \left(\mu_{\tilde{\alpha}_1}\mu_{\tilde{\alpha}_2}, \gamma_{\tilde{\alpha}_1} + \gamma_{\tilde{\alpha}_2} - \gamma_{\tilde{\alpha}_1}\gamma_{\tilde{\alpha}_2}\right)$$

$$= \left(\mu_{\tilde{\alpha}_2}\mu_{\tilde{\alpha}_1}, \gamma_{\tilde{\alpha}_2} + \gamma_{\tilde{\alpha}_1} - \gamma_{\tilde{\alpha}_2}\gamma_{\tilde{\alpha}_1}\right)$$

$$= \tilde{\alpha}_2 \otimes \tilde{\alpha}_1$$

$$(vi)\ (\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^\lambda = (\tilde{\alpha}_2 \otimes \tilde{\alpha}_1)^\lambda$$

$$(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2) = \left(\mu_{\widetilde{\alpha}_1}\mu_{\widetilde{\alpha}_2}, \gamma_{\widetilde{\alpha}_1} + \gamma_{\widetilde{\alpha}_2} - \gamma^+_{\widetilde{\alpha}_1}\gamma^+_{\widetilde{\alpha}_2}\right)$$

$$(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^\lambda = \left(\mu_{\widetilde{\alpha}_1}\mu_{\widetilde{\alpha}_2}, \gamma_{\widetilde{\alpha}_1} + \gamma_{\widetilde{\alpha}_2} - \gamma_{\widetilde{\alpha}_1}\gamma_{\widetilde{\alpha}_2}\right)^\lambda.$$

$$= \left((\mu_{\widetilde{\alpha}_1}\mu_{\widetilde{\alpha}_2})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda(1-\gamma_{\widetilde{\alpha}_2})^\lambda\right)$$

Now if $\tilde{\alpha}_1^\lambda = \left((\mu_{\widetilde{\alpha}_1})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda\right)$ *and* $\tilde{\alpha}_2^\lambda = \left((\mu_{\widetilde{\alpha}_2})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda\right),$

Then we have, $\tilde{\alpha}_2^\lambda \otimes \tilde{\alpha}_1^\lambda = \left((\mu_{\widetilde{\alpha}_1})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda\right) \otimes \left((\mu_{\widetilde{\alpha}_2})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda\right)$

$= ((\mu_{\widetilde{\alpha}_1}\mu_{\widetilde{\alpha}_2})^\lambda, [1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda + 1-(1-\gamma_{\widetilde{\alpha}_2})^\lambda - (1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda)(1-(1-\gamma_{\widetilde{\alpha}_2})^\lambda]).$

$= \left((\mu_{\widetilde{\alpha}_1}\mu_{\widetilde{\alpha}_2})^\lambda, 1-(1-\gamma_{\widetilde{\alpha}_1})^\lambda(1-\gamma_{\widetilde{\alpha}_2})^\lambda\right).$

Hence, $(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^\lambda = \tilde{\alpha}_2^\lambda \otimes \tilde{\alpha}_1^\lambda$.

$$(vii)(\tilde{\alpha}^{\lambda_1} \otimes \tilde{\alpha}^{\lambda_2}) = \tilde{\alpha}^{(\lambda_1+\lambda_2)}$$

$$\tilde{\alpha} = (\mu_{\widetilde{\alpha}}, \gamma_{\widetilde{\alpha}})$$

$$(\tilde{\alpha})^{\lambda_1} = ((\mu_{\widetilde{\alpha}})^{\lambda_1}, 1-(1-\gamma_{\widetilde{\alpha}})^{\lambda_1}),$$

$$(\tilde{\alpha})^{\lambda_2} = ((\mu_{\widetilde{\alpha}})^{\lambda_2}, 1-(1-\gamma_{\widetilde{\alpha}})^{\lambda_2})$$

$$(\tilde{\alpha})^{\lambda_1} \otimes (\tilde{\alpha})^{\lambda_2} = \left((\mu_{\widetilde{\alpha})^{\lambda_1}} \quad (\mu_{\widetilde{\alpha})^{\lambda_2}}, \quad 1-(1-\gamma_{\widetilde{\alpha})^{\lambda_1}}, \quad (1-\gamma_{\widetilde{\alpha})^{\lambda_2}}\right)$$

$$= \left((\mu_{\widetilde{\alpha})^{\lambda_1+\lambda_2}}, \quad 1-(1-\gamma_{\widetilde{\alpha})^{\lambda_1+\lambda_2}}\right)$$

$$= (\tilde{\alpha})^{\lambda_1+\lambda_2}.$$

Hence, the aggregated value of intuitionistic fuzzy number is also an intuitionistic fuzzy number.

Theorem: 2

Let $\tilde{\alpha}_j, (j = 1, 2, \ldots.., n)$ be a clump of intuitionistic fuzzy numbers; then the aggregated value by using the IM-IFWAA operator is not necessarily an intuitionistic fuzzy number and

$$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots.., \tilde{\alpha}_n) = \sum_{j=1}^n \tilde{a}\theta_j = \left(1-\prod_{j=1}^n \left(1-\mu_{\widetilde{\alpha_j}}\right)^{\omega_j}, \prod_{j=1}^n \left(1-\gamma_{\widetilde{\alpha}_j}\right)^{\omega_j}\right)$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weight vector of IM-IFWAA operator with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Proof: Let us prove this by the method of induction. Let us consider $\widetilde{\alpha_1}\omega_1 \; and \; \widetilde{\alpha_2}\omega_2$.

$$\tilde{\alpha}_1\omega_1 = \left(1-(1-\mu_{\widetilde{\alpha}_1})^{\omega_1}, (1-\gamma_{\widetilde{\alpha}_1})^{\omega_1}\right).$$

$$\tilde{\alpha}_2\omega_2 = \left(1-(1-\mu_{\widetilde{\alpha}_2})^{\omega_2}, (1-\gamma_{\widetilde{\alpha}_2})^{\omega_2}\right).$$

Then

$$IM-IFWAA_\omega(\tilde{\alpha}_{\sigma(1)}, \tilde{\alpha}_{\sigma(2)}) = \tilde{\alpha}_{\sigma(1)}\omega_1 \oplus \tilde{\alpha}_{\sigma(2)}\omega_2$$

$$= (1-(1-\mu_{\widetilde{\alpha}_{\sigma(1)}})^{\omega_1}(1-\mu_{\widetilde{\alpha}_{\sigma(2)}})^{\omega_2}, (1-\gamma_{\widetilde{\alpha}_{\sigma(1)}})^{\omega_1}(1-\gamma_{\widetilde{\alpha}_{\sigma(2)}})^{\omega_2})$$

Continuing the process with $\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_k$, then we have

$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_k) = \left(1-\prod_{j=1}^{k}\left(1-\mu_{\widetilde{\alpha}_j}\right)^{\omega_j}, \prod_{j=1}^{k}\left(1-\gamma_{\widetilde{\alpha}_j}\right)^{\omega_j}\right)$.

Then when n = k+1, we have by theorem-1;

$$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_k, \tilde{\alpha}_{k+1}) = (1-\prod_{(j=1)}^{(k+1)}(1-\mu_{\alpha\sim j})^{\omega_j}, \prod_{j=1}^{k+1}(1-\gamma_{\alpha\sim_j})^{\omega_j})$$

, is also true.

Hence the operator holds for n=k+1. Then by the principle of induction, the operator is true for all n, which completes the proof.

$$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_k, \tilde{\alpha}_{k+1}, ..., \tilde{\alpha}_n) = \left(1-\prod_{j=1}^{n}\left(1-\mu_{\widetilde{\alpha}_j}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}_j}\right)^{\omega_j}\right).$$

Moreover, if $1-\prod_{j=1}^{n}\left(1-\mu_{\widetilde{\alpha}_j}\right)^{\omega_j} > 0.5$ and $\prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}_j}\right)^{\omega_j} > 0.5$,

then the resultant $IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_k, \tilde{\alpha}_{k+1}, ..., \tilde{\alpha}_n) \in (0,2)$.

Hence, the aggregated value by using the $IM-IFWAA$ operators is not necessarily an intuitionistic fuzzy number.

Theorem: 3

Let $\tilde{\alpha}_j, (j = 1, 2, ..., n)$ be a clump of intuitionistic fuzzy numbers and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of $\tilde{\alpha}_j, (j = 1, 2, ..., n)$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$; Then we prove that the IM-IFWAA operator is (i) Idempotent, (ii) Bounded, and (iii) Monotonic.

Proof: (i) Idempotent: If all $\tilde{\alpha}_j \, (j = 1, 2, ..., n)$ are equal,

i.e, $\widetilde{\alpha_j} = \tilde{\alpha}$ for all j, then we have to prove $IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \tilde{\alpha}$.

Since $\tilde{\alpha}_j = \tilde{\alpha}$, for all j, then we should have,

$$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \sum_{j=1}^{n} \tilde{a}\omega_j$$

$$= \left(1-\prod_{j=1}^{n}(1-\mu_{\widetilde{\alpha}})^{\omega_j}, \prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}}\right)^{\omega_j}\right)$$

$$= (\mu_{\widetilde{\alpha}}, \gamma_{\widetilde{\alpha}})$$

$$= \tilde{\alpha}.$$

(ii) Boundedness: Let, $\widetilde{\alpha}^{-} \leq IM-IFWAA_{\omega}\left(\widetilde{\alpha_1},\widetilde{\alpha_2},....,\widetilde{\alpha_n}\right) \leq \widetilde{\alpha}^{+}$.

This is evident from: $\tilde{\alpha}^{-} = \left(\left[\underset{i}{min}\mu_{\widetilde{\alpha_i}^{-}}, \underset{i}{min}\mu_{\widetilde{\alpha_i}^{+}}\right], \left[\underset{i}{max}\gamma_{\widetilde{\alpha_i}^{-}}, \underset{i}{max}\gamma_{\widetilde{\alpha_i}^{+}}\right]\right)$.

(iii) Monotonicity: Let $\tilde{\alpha}_j^*, (j = 1,2,....,n)$, be a collection of intuitionistic fuzzy numbers. If $\tilde{\alpha}_j \leq \tilde{\alpha}_j^*$ for all j, then we prove that

$IM-IFWAA_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n) \leq IM-IFWAA_{\omega}(\tilde{\alpha}_1^*,\tilde{\alpha}_2^*,....,\tilde{\alpha}_n^*)$ for all ω.

Let, $IM-IFWAA_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n) = \sum_{j=1}^{n}\tilde{\alpha}_i\omega_i$,

$IM-IFWAA_{\omega}(\tilde{\alpha}_1^*,\tilde{\alpha}_2^*,....,\tilde{\alpha}_n^*) = \sum_{j=1}^{n}\tilde{\alpha}_i^*\omega_i$.

Since $\tilde{\alpha}_j \leq \tilde{\alpha}_j^*$ for all j, then we have

$$IM-IFWAA_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n) \leq IM-IFWAA_{\omega}(\tilde{\alpha}_1^*,\tilde{\alpha}_2^*,....,\tilde{\alpha}_n^*).$$

This is evident from the fact when $\tilde{\alpha}_j = (\mu_{\tilde{\alpha}_j},\gamma_{\tilde{\alpha}_j}) and\ \tilde{\alpha}_j^* = (\mu_{\tilde{\alpha}_j^*},\gamma_{\tilde{\alpha}_j^*})$.

We should have, $a_j \leq a_j^*; a_j \leq a_j^*; a_j \leq a_j^*; a_j \leq a_j^* and\ \mu_{\tilde{\alpha}_j} \leq \mu_{\tilde{\alpha}_j^*}; \gamma_{\tilde{\alpha}_j} \leq \gamma_{\tilde{\alpha}_j^*},\ for\ all\ j.$

Theorem: 4

Let $\tilde{\alpha}_j, (j = 1,2,....,n)$, be a clump of intuitionistic fuzzy numbers; then the aggregated value by using the $IM-IFOWA$ operators is not necessarily an intuitionistic fuzzy number and

$$IM-IFOWA_{\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n\right) = \left(1-\prod_{j=1}^{n}\left(1-\mu_{\widetilde{\alpha}_{\sigma(1)}}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(1)}}\right)^{\omega_j}\right),$$

where $\omega = (\omega_1,\omega_2,....,\omega_n)^T$ is the weight vector of the $IM-IFOWA$ operator with $\omega_j \in (0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

Proof:

Let us prove this also by the method of induction. Let us consider $\tilde{\alpha}_{\sigma(1)}\omega_1\ and\ \tilde{\alpha}_{\sigma(2)}\omega_2$.

$$\tilde{\alpha}_{\sigma(1)}\omega_1 = (1-(1-\mu_{\widetilde{\alpha}_{\sigma(1)}})^{\omega_1},(1-\gamma_{\widetilde{\alpha}_{\sigma(1)}})^{\omega_1})$$

$$and\ \tilde{\alpha}_{\sigma(2)}\omega_2 = (1-(1-\mu_{\widetilde{\alpha}_{\sigma(2)}})^{\omega_2},(1-\gamma_{\widetilde{\alpha}_{\sigma(2)}})^{\omega_2}).$$

Then, $IM-IFOWA_{\omega}\left(\tilde{\alpha}_{\sigma(1)},\tilde{\alpha}_{\sigma(2)}\right) = \tilde{\alpha}_{\sigma(1)}\omega_1 \oplus \tilde{\alpha}_{\sigma(2)}\omega_2 .= (1-(1-\mu_{\alpha\ \sigma(1)})^{\omega_1}\left(1-\mu_{\alpha\ \sigma(2)}\right)^{\omega_2},\left(1-\gamma_{\alpha\ \sigma(1)}\right)^{\omega^1}(1-\gamma_{\alpha\ \sigma(2)})^{\omega^2})$

Continuing the process with $\tilde{\alpha}_{\sigma(1)},\tilde{\alpha}_{\sigma(2)},....,\tilde{\alpha}_{\sigma(k)}$ we have:

$$IM-IFOWA_{\omega}\left(\tilde{\alpha}_{\sigma(1)},\tilde{\alpha}_{\sigma(2)},....,\tilde{\alpha}_{\sigma(n)}\right) = \left(1-\prod_{j=1}^{k}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{k}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right).$$

Then when n=k+1 we have by Theorem (2):

$$IM-IFOWA_{\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_k,\tilde{\alpha}_{k+1}\right)$$

$= \left(1-\prod_{j=1}^{k+1}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j+1)}}\right)^{\omega_{j+1}}, \prod_{j=1}^{k+1}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right)$

$= \left(1-\prod_{j=1}^{k+1}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{k+1}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right)$.

Hence, we see that the operator holds for n=k+1. Then by the principle of induction, the operator is true for all n. Hence,

$$IM-IFOWA_{\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_k,\tilde{\alpha}_{k+1},...,\tilde{\alpha}_n\right) = \left(1-\prod_{j=1}^{n}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right).$$

Moreover, if $1-\prod_{j=1}^{n}\left(1-\mu_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j} > 0.5$ and $\prod_{j=1}^{n}\left(1-\gamma_{\widetilde{\alpha}_{\sigma(j)}}\right)^{\omega_j} > 0.5$, then the resultant $IM-IFOWA_{\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_k,\tilde{\alpha}_{k+1},...,\tilde{\alpha}_n\right) \in (0,2)$.

Hence, the aggregated value by using the $IM-IFOWA$ operators is not necessarily an intuitionistic fuzzy number.

Theorem: 5

Let $\tilde{\alpha}_j, (j = 1, 2, ..., n)$, be a clump of intuitionistic fuzzy numbers and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of $\widetilde{\alpha_j}$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$; Then we prove that the $IM-IFOWA$ operator is (i) Idempotent, (ii) Bounded, (iii) Monotonic and (iv) Commutative.

Proof: The proof for (i), (ii) and (iii) is the same as the proof of Theorem (3). Now we proceed to prove (iv).

(iv) Commutative:

Let $\tilde{\alpha}_j$ be a collection of intuitionistic trapezoidal fuzzy numbers; then we have to prove, $IM-IFOWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = IM-IFOWA_\omega\left(\overline{\tilde{\alpha}}_1, \overline{\tilde{\alpha}}_2, ..., \overline{\tilde{\alpha}}_n\right), j = 1, 2, ..., n$ and for all ω where $\left(\tilde{\alpha}\right.$

$_1, \tilde{\alpha}$

$_2, ..., \tilde{\alpha}$

$_n$ is any permutation of$(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n)$. Let

$$IM-IFOWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1 - \gamma_{\tilde{\alpha}_{\sigma()}}\right)^{\omega_j}\right).$$

Now,

$$IM-IFOWA_\omega\left(\overline{\tilde{\alpha}}_1, \overline{\tilde{\alpha}}_2, ..., \overline{\tilde{\alpha}}_n\right) = \left(1 - \prod_{j=1}^{n}\left(1 - \bar{\mu}_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1 - \bar{\gamma}_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right).$$

Since $\overline{\tilde{\alpha}}_1, \overline{\tilde{\alpha}}_2, ..., \overline{\tilde{\alpha}}_n$ is any permutation of $\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n$, we can have $\overline{\tilde{\alpha}}_{\sigma(j)} = \tilde{\alpha}_{\alpha(j)}$, for $j = 1, 2, ..., n$ and hence $IM-IFOW\ A_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = IM-IFOW\ A_\omega\left(\overline{\tilde{\alpha}}_1, \overline{\tilde{\alpha}}_2, ..., \overline{\tilde{\alpha}}_n\right)$.

Theorem: 6 Let $\tilde{\alpha}_j, (j = 1, 2, ..., n)$, be a collection of intuitionistic trapezoidal fuzzy numbers and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of the IM-IFOWA operator, with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Then we have the following:

1. If $\omega = (1, 0, 0, ..., 0),^T$then $IM-IFOWA_\omega(\alpha\, 1, \alpha\, 2, ..., \alpha\, n) = (max)j\,(\alpha\, j\,)$
2. If $\omega = (0, 0, 0, ..., 1)^T$then$IM-IFOWA_\omega(\alpha\, 1, \alpha\, 2, ..., \alpha\, n) = (min)j\,(\alpha\, j\,)$
3. If $\omega_j = 1, \omega_i = 0$ and $i \neq j$, then $IM-IFOWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left(\tilde{\alpha}_{\sigma(j)}\right)$ where $\tilde{\alpha}_{\sigma(j)}$ is the jth largest of $\tilde{\alpha}_i, (i = 1, 2, ..., n)$.

3 An approach to group decision-making with intuitionistic fuzzy information

Pseudo-code for Improvised Aggregated ANN:

C$_n$: n *Matrix item set of size* k x m

Input *{Intuitionistic Fuzzy Decision Matrices}*

A$_n$ = *{Collection of n Matrices of size k};*

//* Aggregation Phase*//

Compute *{IM-IFWAA aggregator & the Initial Weight Vector}*

For (n=1; A$_n$∅; n++) **do design**

Generate *{Individual Preference Intuitionistic Fuzzy Decision Matrices, X$_n$}*

//* X$_N$ *is the collection of Individual Preference IF-Decision Matrices* *//

Generate *{Intuitionistic Fuzzy Attribute Weight Vector}*

While $i \leq m$**do**

Compute *{IM-IFOWA aggregator & the Initial Weight Vector}*

//{Defuzzify the IF column matrix into the Fuzzy Column matrix}//

Generate *{Collective Overall Preference Linguistic Intuitionistic Fuzzy Decision Matrices using Fully Linguistic Intuitionistic Fuzzy Weight Vector, W^T}*

//*Improvise the input vector by Improvised Aggregation Operator*//

$IM-IFWAA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left(1 - \prod_{j=1}^{n}(1 - \mu_{\tilde{\alpha}})^{\omega_j}, \prod_{j=1}^{n}\left(1 - \gamma_{\tilde{\alpha}}\right)^{\omega_j}\right),$

$IM-IFOWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}, \prod_{j=1}^{n}\left(1 - \gamma_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right)$

//* Learning Phase*//

Generate *{Weight Matrix by IF-Delta Rule/Perceptron Rule/Hebb Rule}*
Update weights for the next step with different Learning Rules
Continue the weight updation until the error is minimized to the desired level
//*Activation function*//
Fix *{The Threshold Value-Binary Step Function}*
While Activated values $\geq$ *Threshold* **do**
Generate *{Binary Matrix for final decision with values exceeding the Threshold}*
Output *{Best Alternative(s) to be chosen}*
{the final decision variable can be converted into a crisp variable and computations can be performed}. End

4 Numerical illustration

An investment enterprise, which includes five members, is to plan for a successful investment plan to be done on five different choices of investments. Suppose there are three possible information given by the decision makers as three matrices to be evaluated, it is necessary to compare or aggregate these information matrices to select the most important of them as well as order them from the point of view of their importance, taking into account four attributes suggested given below:

G1- Shopping Market Analysis; G2- Stock Market Analysis; G3- Fraud Detection; G4 -Digital Surveillance. The five possible alternatives $A_i\,(i=1,2,3,4,5)$ are to be evaluated using intuitionistic fuzzy numbers and the three decision makers' information given as $\gamma=(0.45,0.20,0.35)^T$ and the attributes' weighting vector is given as $\omega=(\ 0.2\ ,\ 0.1\ ,\ 0.3\ ,\ 0.4\ )^T$. The computations of the ANN-guided MAGDM problem are as follows:

$$\widetilde{R}_1=\begin{pmatrix}(0.4,0.3)(0.5,0.2)(0.2,0.5)(0.1,0.6)\\(0.6,0.2)(0.6,0.1)(0.6,0.1)(0.3,0.4)\\(0.5,0.3)(0.4,0.3)(0.4,0.2)(0.5,0.2)\\(0.7,0.1)(0.5,0.2)(0.2,0.3)(0.1,0.5)\\(0.5,0.1)(0.3,0.2)(0.6,0.2)(0.4,0.2)\end{pmatrix};$$

$$\widetilde{R}_2=\begin{pmatrix}(0.5,0.4)(0.6,0.3)(0.3,0.6)(0.2,0.7)\\(0.8,0.1)(0.6,0.3)(0.3,0.4)(0.2,0.6)\\(0.8,0.1)(0.6,0.3)(0.3,0.4)(0.2,0.6)\\(0.8,0.1)(0.6,0.3)(0.3,0.4)(0.2,0.6)\\(0.6,0.2)(0.4,0.3)(0.7,0.1)(0.5,0.3)\end{pmatrix};$$

$$\widetilde{R}_3=\begin{pmatrix}(0.4,0.5)(0.5,0.4)(0.2,0.7)(0.1,0.8)\\(0.6,0.4)(0.6,0.3)(0.6,0.3)(0.3,0.6)\\(0.5,0.5)(0.4,0.5)(0.4,0.4)(0.5,0.4)\\(0.7,0.2)(0.5,0.4)(0.2,0.5)(0.1,0.7)\\(0.5,0.3)(0.3,0.4)(0.6,0.2)(0.4,0.4)\end{pmatrix}.$$

Running the computations by employing the IM-IFWAA operator for the above matrices, we can observe the following results:

$$r_{11}=(1-[(1-0.4)^{0.45}.\,(1-0.5)^{0.2}.\,(1-0.4)^{0.35}],[(1-0.3)^{0.45}.\,(1-0.4)^{0.2}.\,(1-0.5)^{0.35}])$$

$r_{11}=(0.421,0.603)$;$r_{12}=(0.521,0.704)$;$r_{13}=(0.221,0.399)$;
$r_{14}=(0.121,0.296)$;$r_{21}=(0.622,0.704)$;$r_{22}=(0.622,0.805)$;
$r_{23}=(0.622,0.805)$;$r_{24}=(0.321,0.502)$;$r_{31}=(0.522,0.603)$;
$r_{32}=(0.421,0.603)$;$r_{33}=(0.421,0.704)$;$r_{34}=(0.522,0.704)$;
$r_{41}=(0.723,0.864)$;$r_{42}=(0.522,0.704)$;$r_{43}=(0.221,0.603)$;
$r_{44}=(0.121,0.399)$;$r_{51}=(0.522,0.805)$;$r_{52}=(0.321,0.704)$;
$r_{53}=(0.622,0.819)$;$r_{54}=(0.421,0.704)$.

$$\widetilde{R}=\begin{pmatrix}(0.421,0.603)(0.521,0.704)(0.221,0.399)(0.121,0.296)\\(0.622,0.704)(0.622,0.805)(0.622,0.805)(0.321,0.502)\\(0.522,0.603)(0.421,0.603)(0.421,0.704)(0.522,0.704)\\(0.723,0.864)(0.522,0.704)(0.221,0.603)(0.121,0.399)\\(0.522,0.805)(0.321,0.704)(0.622,0.819)(0.421,0.704)\end{pmatrix}.$$

Running the IM-IFOWA operator, we obtain the concerted comprehensive preference values (i=1, 2, …,5).

$$\tilde{r}_1=(1-[(1-0.421)^{0.2}.\,(1-0.521)^{0.1}.\,(1-0.221)^{0.3}.\,(1-0.121)^{0.4}],)$$

$([(1-0.380)^{0.2}.(1-0.276)^{0.1}.(1-0.583)^{0.3}.(1-0.684)^{0.4}])$

$\tilde{r}_1 = (0.266,\ 0.427)$,

$\tilde{r}_2 = (0.522,\ 0.669)$,

$\tilde{r}_3 = (0.484,\ 0.691)$,

$\tilde{r}_4 = (0.367,\ 0.575)$,

$\tilde{r}_5 = (0.502,\ 0.774)$.

If these concerted comprehensive preference values are compared with Hamming distance measures or any correlation measures [5,6,9] the following ranking can be observed:

$A_1 > A_4 > A_3 > A_2 > A_5$.

Hence, the best alternative is

Table 1. Comparison of computations with Delta, Perceptron, and Hebb Learning Rules

ANN WITH DELTA LEARNING RULE & IFS INPUT VALUES			
Sl. No.	**Target Values**	**Threshold Value**	**Selected Alternatives**
1	d1= -1; d2= 1; d3= -1	0.20395	A1, A4
2	d1= 1; d2= -1; d3= -1	0.20266	A1, A4
3	d1= -1; d2= -1; d3= -1	0.1915	A1, A4
4	d1= 1; d2= -1; d3= 1	0.19432	A2, A3, A5
ANN WITH DELTA LEARNING RULE & DE-FUZZIFIED INPUT VALUES			
Sl. No.	**Target Values**	**Threshold Value**	**Selected Alternatives**
1	d1= -1; d2= 1; d3= -1	0.11884	A1, A2
2	d1= 1; d2= -1; d3= -1	0.11309	A1, A2
3	d1= -1; d2= -1; d3= -1	0.03679	A1, A2
4	d1= 1; d2= -1; d3= 1	0.19329	A1, A2
ANN WITH PERCEPTRON LEARNING RULE & IFS INPUT VALUES			
Sl. No.	**Target Values**	**Threshold Value**	**Selected Alternatives**
1	d1= -1; d2= 1; d3= -1	0.17755	A1, A4
2	d1= 1; d2= -1; d3= -1	0.15263	A1, A4
3	d1= -1; d2= -1; d3= -1	0.13017	A1, A4
4	d1= 1; d2= -1; d3= 1	0.11099	A1, A4
ANN WITH PERCEPTRON LEARNING RULE & DE-FUZZIFIED INPUT VALUES			
Sl. No.	**Target Values**	**Threshold Value**	**Selected Alternatives**
1	d1= -1; d2= 1; d3= -1	-0.1726	A1, A3, A5
2	d1= 1; d2= -1; d3= -1	-0.17255	A1, A3, A5
3	d1= -1; d2= -1; d3= -1	-0.17255	A1, A3, A5
4	d1= 1; d2= -1; d3= 1	0.01377	A1, A5
ANN WITH HEBB LEARNING RULE			
Sl. No.	**Data set**	**Threshold Value**	**Selected Alternatives**
1	IFS	0.35331	A2, A3, A5
2	De-fuzzified Values	2.99435	A2, A5

The computations are programmed through Python and results are tabulated as above.

5 Discussion

This paper deals with improvising a class of aggregation operators and utilizing them in a novel ANN algorithm to solve MAGDM problems. The Intuitionistic Fuzzy Artificial Neural Network model here is based on aggregating the intuitionistic fuzzy information by the new and improvised operators namely the IM-IFWAA and IM-IFOWA operators. For the Neural Network computations, different learning rules namely Delta, Perceptron, and Hebb learning rules are employed where the ranking of the alternatives is provided at the end of the activation phase. The option of updating the weights through the learning phase of the ANN provided more room for error minimization which may be considered as an option when only MAGDM problems are solved using ANN.

A numerical example is provided based on the published results in [10] and comparisons made reveal the authenticity of the proposed technique in this research work and also presents a novel way of approaching to produce a valid input vector for the ANN to solve and DSS related problem. The authors in [10] proposed the Gram-Scmidt process of orthogonalization for providing an input vector for the ANN and now following the work by the same authors, in this work, the model is improvised by providing input for ANN through Aggregation operators. Table 1 reveals the different choices of the available alternatives when different aggregation operators are employed, which depends on the choice of the decision maker of the problem. The comparisons between the Target values, Threshold values, and the changes in the outputs accordingly are recorded in the table for the perusal of the observer. Out of the comparative study made here, A1 is found to be the profound alternative for the best choice provided in the problem.

6 Conclusion

Arriving at a consensus on any business or e-commerce problem is a herculean task. The role of artificial neural networks is crucial in solving such complex problems. Perhaps in this research work, artificial neural networks and multiple attribute group decision-making problems are coupled to solve real-world business problems very sensibly and effectively which arise in an intuitionistic fuzzy environment. Here, unlike the earlier models suggested in [5,9,10] are replaced with the approach of determining the inputs for ANN through a novel class of improvised weighted aggregation operators namely, the IM-IFWAA and the IM-IFOWA operators. These operators are extremely resourceful in producing the appropriate input entries for the process of ANN whereafter the vectors are allowed to learn using the Delta, Perceptron, and Hebb learning rules. Finally, after the activation phase is completed, the data variables going through the activation function will be ready for the final ranking for the best selection of the alternatives. The advantage of the proposed model is that it relies completely on the aggregated nature of the input which is Mathematically sound as well as hypothetically working for the vague nature of the data set. Further, ranking of the order of the variables will be reserved for future work which will be coupled with some data mining techniques.

References

1) Atanassov K, Sotirov S, Pencheva T. Intuitionistic Fuzzy Deep Neural Network. *Mathematics*. 2023;11:1–14. Available from: https://doi.org/10.3390/math11030716.
2) Ding J, Zhang C, Li D, Zhan LWJ, Yao Y. Three-way decisions in generalized intuitionistic fuzzy environments: survey and challenges. *Artificial Intelligence Review*. 2024;57:38–38. Available from: https://doi.org/10.1007/s10462-023-10647-5.
3) Firouzkouhi N, Amini A, Nazari M, Fadi A, Bordbar C, Cheng B, et al. Advanced artificial intelligence system by intuitionistic fuzzy Γ-subring for automotive robotic manufacturing. *Artificial Intelligence Review*. 2023;56:9639–9664. Available from: https://doi.org/10.1007/s10462-023-10396-5.
4) Hajek P, Olej V, Froelich W, Novotny J. Intuitionistic Fuzzy Neural Network for Time Series Forecasting - The Case of Metal Prices. In: Maglogiannis, I, Macintyre, J, Iliadis, L, editors. Artificial Intelligence Applications and Innovations. AIAI 2021. IFIP Advances in Information and Communication Technology;vol. 627. Springer. 2021. Available from: https://doi.org/10.1007/978-3-030-79150-6_33.
5) Leonishiya A, Robinson JP. Varieties of Linguistic Intuitionistic Fuzzy Distance Measures for Linguistic Intuitionistic Fuzzy TOPSIS Method. *Indian Journal of Science and Technology*. 2023;16(33):2653–2662. Available from: https://doi.org/10.17485/IJST/v16i33.640-icrsms.
6) Leonishiya A, Robinson JP. A Fully Linguistic Intuitionistic Fuzzy Artificial Neural Network Model for Decision Support Systems. . *Indian Journal of Science and Technology*. 2023;16:29–36. Available from: https://doi.org/10.17485/IJST/v16iSP4.ICAMS136.
7) Nirmalsingh SS, Robinson PJ. Triangular Intuitionistic Fuzzy Linear Programming Problem with a New Ranking Method based on Correlation Coefficient. *Indian Journal of Science and Technology*. 2023;16(3):75–83. Available from: https://doi.org/10.17485/IJST/v16iSP3.icrtam297.
8) Petkov T, Bureva V, Popov S. Intuitionistic fuzzy evaluation of artificial neural network model. *Notes on Intuitionistic Fuzzy Sets*. 2021;27(4):71–77. Available from: https://doi.org/10.7546/nifs.2021.27.4.71-77.
9) Robinson JP, Leonishiya M. Application of Varieties of Learning Rules in Intuitionistic Fuzzy Artificial Neural Network. *Machine Intelligence for Research & Innovations*. 2024;832:35–45. Available from: https://doi.org/10.1007/978-981-99-8129-8_4.
10) Robinson JP, Saranraj A. Intuitionistic Fuzzy Gram-Schmidt Orthogonalized Artificial Neural Network for solving MAGDM Problems. *Indian Journal of Science and Technology*. 2024;17(24):2529–2537. Available from: https://doi.org/10.17485/IJST/v17i24.1386.
11) Banu S, Prasanna S, A. To Solve Pythagorean Fuzzy Decagonal Transportation Problem Using Bipartite Graph with Optimal Solution. *Indian Journal of Science and Technology*. 2024;17(1):103–108. Available from: https://doi.org/10.17485/IJST/v17sp1.193.
12) Taherdoost H. Deep Learning and Neural Networks: Decision-Making Implications. *Symmetry*. 2023;15(9):17–23. Available from: https://doi.org/10.3390/sym15091723.
13) Thakur P, Kizielewicz B, Gandotra N, Shekhovtsov A, Saini N, Saeid AB, et al. A New Entropy Measurement for the Analysis of Uncertain Data in MCDA Problems Using Intuitionistic Fuzzy Sets and COPRAS Method. *Axioms*. 2021;10:335–335. Available from: https://doi.org/10.3390/axioms10040335.
14) Touqeer M, Sulaie SA, Lone SA, Shaheen K, Gunaime NM, Elkotb MA. A fuzzy parametric model for decision making involving F-OWA operator with unknown weights environment. *Heliyon*. 2023;9(9):19969–19969. Available from: https://doi.org/10.1016/j.heliyon.2023.e19969.
15) Vincy CG, Merlin, M M. An integrated approach of Multi-Criteria Decision-Making method with intuitionistic fuzzy entropy for an exploration of food grain storage structures. *Nonlinear Studies*. 2023;30:43–48. Available from: http://www.nonlinearstudies.com/index.php/nonlinear/article/view/3166.

16) Zhang Q, Li H, Lu L, Wu C. A Neural Network-Based Approach to Multi-Attribute Group Decision-Making with Heterogeneous Preference Information. *Scientific Programming*. 2022;30(1):43–48. Available from: http://www.nonlinearstudies.com/index.php/nonlinear/article/view/3166.