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RESEARCH ARTICLE

A Portrayal of Integer Solutions to Transcendental Equation with Six Unknowns $\sqrt[3]{\mathbf{X}+\mathbf{Y}}+\sqrt{\mathbf{xy}}=z^{2}+w^{2}$

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Abstract

Objectives: This research article focuses on finding non-zero integer solutions to the Transcendental equation with six unknowns represented by $\sqrt[3]{X+Y}+\sqrt{xy}=z^{2}+w^{2}$. **Methods:** There is no universal method for finding integer solutions to Diophantine equations involving surds. Various substitutions strategies are employed to obtain non-zero distinct integer solutions to the surd equation under consideration. **Findings:** Five different choices of substitutions are utilized to determine many non-zero distinct integer solutions to the transcendental equation with six unknowns in title**. Novelty:** In this analysis, the transcendental equation with six unknowns given by $\sqrt[3]{X+Y}+\sqrt{xy}=z^{2}+w^{2}$ has been reduced to either Space Pythagorean equation or non-homogeneous ternary quadratic equation through suitable transformations and for which integer solutions can be found elegantly.

Keywords: Transcendental Equation With Six Unknowns; Surd Equation; Integer Solutions; Space Pythagorean Equation; Ternary Quadratic Equation

1 Introduction

The subject of Diophantine equation, one of the interesting areas of Number Theory, plays a significant role in higher arithmetic and has a marvelous effect on credulous people and always occupies a remarkable position due to unquestioned historical importance. The Diophantine equations may be either polynomial equation with at least two unknowns for which integer solution, are required or transcendental equation involving trigonometric, logarithmic, exponential and surd function such that one may be interested in getting integer solution. Most of the Diophantine problems solved by the researchers are polynomial equations[1–5]. Note that, the transcendental equation can be solved by transforming it into an equivalent polynomial equation. Adhoc methods exists for some classes of transcendental equations in one variable to transform them into polynomial equations which then might be solved. Some transcendental equation in more than one unknown can be solved by separation of the unknowns reducing them to polynomial equations. In this context, one may refer[6–15].

In this paper, we are interested in obtaining integer solutions to transcendental equation involving surds. In particular, we obtain five different sets of integer solutions to the transcendental equation with six unknowns given by $\sqrt[3]{X+Y}+\sqrt{xy}=z^2+w^2$.

2 Methodology

The transcendental equation with six unknowns to be solved is

$$\sqrt[3]{X+Y}+\sqrt{xy}=z^2+w^2 \tag{1}$$

The above equation (1) is reduced to polynomial equations of degree two and three through suitable transformations . The resulting quadratic equations are analyzed using elementary mathematical procedures.

3 Results and Discussion

3.1 Technical procedure-I

Launching the transformations

$$X=p(p+q)^2, Y=q(p+q)^2, x=p^2+q^2+2pq, y=p^2+q^2-2pq \tag{2}$$

in (1) ,it simplifies to the polynomial equation

$$(2p+1)^2=(2q-1)^2+(2z)^2+(2w)^2 \tag{3}$$

The above equation (3) is similar to the Space Pythagorean equation
$w^2=x^2+y^2+z^2$
and is satisfied by

$$z=ab, w=ac \tag{4}$$

$$p=\frac{a^2+b^2+c^2-1}{2}, q=\frac{a^2-b^2-c^2+1}{2} \tag{5}$$

As we require the solutions of (1) to be integers ,observe that it is possible to choose a ,b ,c in such a way that p ,q are integers. By inspection , the choices ofa ,b ,c and the respective integer solutions to (1) are presented below :

Choice 3.1:

$$a=2A+1, b=2B, c=2C$$

From (4) ,we have

$$z=2B(2A+1), w=2C(2A+1) \tag{6}$$

From (5) ,we get

$$p=2A^2+2A+2B^2+2C^2, q=2A^2+2A+1-2B^2-2C^2$$

In view of (2) , it is seen that

$$X=(2A^2+2A+2B^2+2C^2)(4A^2+4A+1)^2,$$

$$Y=(2A^2+2A+1-2B^2-2C^2)(4A^2+4A+1)^2,$$

$$x=\left(2A^2+2A+2B^2+2C^2\right)^2+\left(2A^2+2A+1-2B^2-2C^2\right)^2+$$
$$2\left(2A^2+2A+2B^2+2C^2\right)\left(2A^2+2A+1-2B^2-2C^2\right)$$

Table 1. Numerical examples

A	B	C	x	y	X	Y	z	w
1	2	3	9^2	51^2	$30*9^2$	$-21*9^2$	12	18
2	3	5	25^2	135^2	$80*25^2$	$-55*25^2$	30	50

$$y=\left(2A^2+2A+2B^2+2C^2\right)^2+\left(2A^2+2A+1-2B^2-2C^2\right)^2-2\left(2A^2+2A+2B^2+2C^2\right)\left(2A^2+2A+1-2B^2-2C^2\right) \tag{7}$$

Thus, (7) & (6) represent the integer solutions to (1).

Few numerical examples are given in Table 1 below:

Choice 3.2 :

$$a=2A, b=2B+1, c=2C$$

From (4) ,we have

$$z=2A(2B+1), w=4AC \tag{8}$$

From (5) ,we get

$$p=2A^2+2B+2B^2+2C^2, q=2A^2-2B-2B^2-2C^2$$

In view of (2) , it is seen that

$$X=(2A^2+2B+2B^2+2C^2)(4A^2)^2,$$

$$Y=(2A^2-2B-2B^2-2C^2)(4A^2)^2,$$

$$\begin{aligned} x&=\left(2A^2+2B+2B^2+2C^2\right)^2+\left(2A^2-2B-2B^2-2C^2\right)^2+2\left(2A^2+2B+2B^2+2C^2\right)\left(2A^2-2B-2B^2-2C^2\right),\\ y&=\left(2A^2+2B+2B^2+2C^2\right)^2+\left(2A^2-2B-2B^2-2C^2\right)^2-2\left(2A^2+2B+2B^2+2C^2\right)\left(2A^2-2B-2B^2-2C^2\right)\end{aligned} \tag{9}$$

Thus, (9) & (8) represent the integer solutions to (1).

Few numerical examples are given in Table 2 below:

Table 2. Numerical examples

A	B	C	x	y	X	Y	z	w
1	2	3	4^2	60^2	$2*16^2$	$-7*8^2$	10	12
2	3	5	16^2	148^2	$82*16^2$	$-66*16^2$	28	40

Choice 3.3:

$$a=2A+1, b=2B+1, c=2C+1$$

From (4) ,we have

$$z=(2A+1)(2B+1), w=(2A+1)(2C+1) \tag{10}$$

From (5) ,we get

$$p=2A^2+2A+2B+2B^2+2C^2+2C+1, q=2A^2+2A-2B-2B^2-2C^2-2C$$

In view of (2) , it is seen that

$$X=(2A^2+2A+2B+2B^2+2C^2+2C+1)(4A^2+4A+1)^2,$$

$$Y=(2A^2+2A-2B-2B^2-2C^2-2C)(4A^2+4A+1)^2,$$

$$\begin{aligned} x &= \left(2A^2+2A+2B+2B^2+2C^2+2C+1\right)^2+\left(2A^2+2A-2B-2B^2-2C^2-2C\right)^2+ \\ &\quad 2\left(2A^2+2A+2B+2B^2+2C^2+2C+1\right)\left(2A^2+2A-2B-2B^2-2C^2-2C\right) \\ y &= \left(2A^2+2A+2B+2B^2+2C^2+2C+1\right)^2+\left(2A^2+2A-2B-2B^2-2C^2-2C\right)^2- \\ &\quad 2\left(2A^2+2A+2B+2B^2+2C^2+2C+1\right)\left(2A^2+2A-2B-2B^2-2C^2-2C\right) \end{aligned} \tag{11}$$

Thus, (11) & (10) represent the integer solutions to (1).

Few numerical examples are given in Table 3 below:

Table 3. Numerical examples

A	B	C	x	y	X	Y	z	w
1	2	3	9^2	73^2	$41*9^2$	$-32*9^2$	15	21
2	3	5	25^2	169^2	$97*25^2$	$-72*25^2$	35	55

Note 3.1

It is to be noted that ,apart from (2) ,the following transformations may be considered for finding other patterns of integer solutions to (1)

$(i)X=p^3+3pq^2, Y=3p^2q+q^3, x=p^2+q^2+2pq, y=p^2+q^2-2pq, (ii)X=p^3+3pq^2, Y=-3p^2q-q^3, x=p^2+q^2+2pq, y=p^2+q^2-2pq,$

3.2 Technical procedure-II

Taking

$$z=kw, k>1 \tag{12}$$

in (3) ,it reduces to the quadratic equation

$$(2p+1)^2=4(k^2+1)w^2+(2q-1)^2 \tag{13}$$

Considering

$$2p+1=u+v, 2q-1=u-v, u\neq v\neq 0 \tag{14}$$

in (13) ,we get

$$u*v=(k^2+1)w^2 \tag{15}$$

Choosing

$$u=(k^2+1)w, v=w$$

in (15) and using (14) ,it is seen that

$$p=\frac{(k^2+2)w-1}{2}, q=\frac{k^2w+1}{2} \tag{16}$$

It is observed that the values of p, q are integers provided both k & w are odd.

Taking

$$k = 2s+1, w = 2t+1$$

in (16), one obtains

$$p = \frac{(4s^2+4s+3)(2t+1)-1}{2} = (4s^2+4s+3)t+(2s^2+2s+1),$$

$$q = \frac{(4s^2+4s+1)(2t+1)+1}{2} = (4s^2+4s+1)t+(2s^2+2s+1)$$

In view of (2) and (12) ,the corresponding integer solutions to (1) are given by

$$X = [(4s^2+4s+3)t+(2s^2+2s+1)][(8s^2+8s+4)t+2(2s^2+2s+1)]^2,$$

$$Y = [(4s^2+4s+1)t+(2s^2+2s+1)][(8s^2+8s+4)t+2(2s^2+2s+1)]^2,$$

$$x = \left[(4s^2+4s+3)t+(2s^2+2s+1)\right]^2 + \left[(4s^2+4s+1)t+(2s^2+2s+1)\right]^2 + 2\left[(4s^2+4s+3)t+(2s^2+2s+1)\right]*\left[(4s^2+4s+1)t+(2s^2+2s+1)\right],$$

$$y = \left[(4s^2+4s+3)t+(2s^2+2s+1)\right]^2 + \left[(4s^2+4s+1)t+(2s^2+2s+1)\right]^2 - 2\left[(4s^2+4s+3)t+(2s^2+2s+1)\right]*\left[(4s^2+4s+1)t+(2s^2+2s+1)\right],$$

$$z = (2s+1)*(2t+1),$$

$$w = 2t+1$$

Few numerical examples are given in Table 4 below:

Table 4. Numerical examples

s	t	x	y	X	Y	z	w
1	1	30^2	2^2	120^2	$14*30^2$	9	3
1	2	50^2	4^2	$27*50^2$	$23*50^2$	15	5

Note 3.2

In (15), the values of u, v may be chosen as shown below:

Representation I: $u = w^2, v = k^2+1$

Representation II: $u = k^2+1, v = w^2$

Considering Representation I and following the above procedure, thecorresponding integer solutions to (1) are given by

$$X = \frac{(w^2+k^2)w^4}{2}, Y = \frac{(w^2-k^2)w^4}{2},$$

$$x = w^4, y = k^4, z = k*w$$

where k, w are of the same parity.

Few numerical examples are given in Table 5 below:

Considering Representation II and following the above procedure ,the corresponding integer solutions to (1) are given by

$$X = \frac{(w^2+k^2)(k^2+1)^2}{2}, Y = \frac{(k^2-w^2+2)(k^2+1)^2}{2},$$

$$x = (k^2+1)^2, y = (w^2-1)^2, z = k*w$$

where k, w are of the same parity.

Few numerical examples are given in Table 6 below:

Table 5. Numerical examples

k	x	y	X	Y	z	w
4	4^2	16^2	$10*4^2$	$-6*4^2$	8	2
3	49^2	9^2	$29*49^2$	$20*40^2$	21	7

Table 6. Numerical examples

k	x	y	X	Y	z	w
4	17^2	3^2	$10*17^2$	$7*17^2$	8	2
3	10^2	48^2	$29*10^2$	$-19*10^2$	21	7

3.3 Technical procedure-III

Launching the transformations

$$X = (1+p)(1+p+q)^2, Y = q(1+p+q)^2, x = p, y = pq^2 \tag{17}$$

in (1), it simplifies to the second degree polynomial equation

$$1+p+q+p*q = z^2+w^2 \tag{18}$$

Again, taking

$$p = r+s, q = r-s, r \neq s \neq 0 \tag{19}$$

in (18), it leads to the space Pythagorean equation

$$(r+1)^2 = s^2+z^2+w^2 \tag{20}$$

which is satisfied by

$$s = a^2-b^2-c^2, r = a^2+b^2+c^2-1$$

and

$$z = 2ab, w = 2ac \tag{21}$$

From (19), we get

$$p = 2a^2-1, q = 2(b^2+c^2)-1$$

In view of (17) ,one obtains

$$X = 2a^2(2a^2+2b^2+2c^2-1)^2, Y = (2b^2+2c^2-1)(2a^2+2b^2+2c^2-1)^2,$$

$$x = 2a^2-1, y = (2a^2-1)(2b^2+2c^2-1)^2 \tag{22}$$

Thus, (21) &(22) represent the integer solutions to (1).

Few numerical examples are given in Table 7 below:

Note 3.3

The integer solutions to (20) may be taken as

$$s = 2ab, r = a^2+b^2+c^2-1$$

and

$$z = 2ac, w = a^2-b^2-c^2 \tag{23}$$

Table 7. Numerical examples

a	b	c	x	y	X	Y	z	w
2	1	3	7	$7*19^2$	$8*27^2$	$19*27^2$	4	12
1	2	1	1	9^2	$2*11^2$	$9*11^2$	4	2
1	2	3	1	25^2	$2*27^2$	$25*27^2$	4	6

Correspondingly , we have

$$p=(a+b)^2+c^2-1, q=(a-b)^2+c^2-1$$

In view of (17) ,one obtains

$X=[(a+b)^2+c^2](2a^2+2b^2+2c^2-1)^2,$

$Y=[(a-b)^2+c^2-1](2a^2+2b^2+2c^2-1)^2,$

$$x=(a+b)^2+c^2-1, y=\left[(a+b)^2+c^2-1\right]\left[(a-b)^2+c^2-1\right]^2 \tag{24}$$

Thus, (24) & (23) represent the integer solutions to (1).

Few numerical examples are given in Table 8 below:

Table 8. Numerical examples

a	b	c	x	y	X	Y	z	w
2	1	3	17	$17*9^2$	$18*27^2$	$9*27^2$	12	-6
1	2	1	3	$9*9^2$	$10*11^2$	11^2	2	-4

3.4 Technical procedure-IV

Launching the transformations

$$X=(1-p)(1-p-q)^2, Y=-q(1-p-q)^2, x=p, y=pq^2 \tag{25}$$

in (1), it simplifies to the second degree polynomial equation

$$1-p-q+p*q=z^2+w^2 \tag{26}$$

Again, taking

$$p=r+s, q=r-s, r\neq s\neq 0 \tag{27}$$

in (26), it leads to the space Pythagorean equation

$$(r-1)^2=s^2+z^2+w^2 \tag{28}$$

which is satisfied by

$$s=a^2-b^2-c^2, r=a^2+b^2+c^2+1$$

and

$$z=2ab, w=2ac \tag{29}$$

From (27), we get

$$p=2a^2+1, q=2(b^2+c^2)+1$$

In view of (25), one obtains

$$X=-2a^2(2a^2+2b^2+2c^2+1)^2, Y=-(2b^2+2c^2+1)(2a^2+2b^2+2c^2+1)^2,$$

Table 9. Numerical examples

a	b	c	x	y	X	Y	z	w
2	1	3	9	$9*21^2$	$-8*29^2$	$-21*29^2$	4	12
1	1	2	3	$3*11^2$	$-2*13^2$	$-11*13^2$	2	4
3	5	1	19	$19*53^2$	$-18*71^2$	$-53*71^2$	30	6

$$x=2a^2+1, y=(2a^2+1)(2b^2+2c^2+1)^2 \tag{30}$$

Thus , (29) &(30) represent the integer solutions to (1).

Few numerical examples are given in Table 9 below:

Note 3.4

The integer solutions to (28) may be taken as

$$s=2ab, r=a^2+b^2+c^2+1$$

jointly with (23).

Correspondingly, we have

$$p=(a+b)^2+c^2+1, q=(a-b)^2+c^2+1$$

In view of (25), one obtains

$$X=-[(a+b)^2+c^2](2a^2+2b^2+2c^2+1)^2,$$

$$Y=-\left[(a-b)^2+c^2+1\right]\left(2a^2+2b^2+2c^2+1\right)^2,$$

$$x=(a+b)^2+c^2+1, y=\left[(a+b)^2+c^2+1\right]\left[(a-b)^2+c^2+1\right]^2 \tag{31}$$

Thus, (31) & (23) represent the integer solutions to (1).

3.5 Technical procedure-V

Taking

$$P=2p+1, Q=2q-1 \tag{32}$$

in (13), it is written as

$$P^2-Q^2=4(k^2+1)w^2 \tag{33}$$

which is expressed in the form of ratio as

$$\frac{P+Q}{(k^2+1)w}=\frac{4w}{P-Q}=\frac{\alpha}{\beta}, \beta\neq 0$$

Solving the above system of double equations through employing the method of cross-multiplication and using (32), one obtains

$$w=2\alpha\beta,$$

$$p=2\beta^2+\frac{(k^2+1)\,\alpha^2-1}{2},$$

$$q=-2\beta^2+\frac{\left(k^2+1\right)\alpha^2+1}{2}. \tag{34}$$

As our thrust is on finding integer solutions, taking

$$k=2t,\alpha=2s+1$$

in the above equations ,we get

$$w=2\beta(2s+1) \tag{35}$$

And

$$p=2\beta^2+2t^2(2s+1)^2+2s(s+1),$$

$$q=-2\beta^2+2t^2(2s+1)^2+2s(s+1)+1.$$

In view of (2) & (12) ,we have

$$z=4t\beta(2s+1),$$

$$x=[(2s+1)^2(4t^2+1)]^2,$$

$$y=(4\beta^2-1)^2,$$

$$X=[2\beta^2+2t^2(2s+1)^2+2s(s+1)][(2s+1)^2(4t^2+1)]^2, \tag{36}$$

Thus, (35) & (36) represent the integer solutions to (1).

Note 3.5

One can also write (33) as the system of double equations as

$$\frac{P+Q}{4(k^2+1)w}=\frac{w}{P-Q}=\frac{\alpha}{\beta},\beta\neq 0$$

Following the procedure as above ,the corresponding integer solutions to (1) are given by

$$x=16(k^2+1)^2\alpha^4,y=(4s^2+4s)^2,$$

$$w=2\alpha(2s+1),z=2\alpha(2s+1)k,$$

$$X=[2(k^2+1)\alpha^2+2s(s+1)][16(k^2+1)^2\alpha^4],$$

$$Y=[2(k^2+1)\alpha^2-2s(s+1)][16(k^2+1)^2\alpha^4].$$

4 Conclusion

In this paper, the transcendental equation involving surds with six unknowns given by $\sqrt[3]{X+Y}+\sqrt{xy}=z^2+w^2$ has been reduced to polygonal Diophantine equations of degree two with three and four unknowns for which integer solutions are obtained in an elegant way through suitable transformations.

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