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RESEARCH ARTICLE

A Generalized Area-Biased Power Ishita Distribution - Properties and Applications

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Abstract

Objectives: Generalized Area-Biased Power Ishita Distribution (GAPID) is a brand-new distribution that was suggested in this study. Its characterization is also mentioned in detail. **Methods:** The idea of weighted distributions is incorporated. This helps to derive new distributions that will be a best fit for many real-life data from many domains like agriculture, biomedical, financial, etc., where they may not match with the conventional distributions. Thus, it becomes necessary to create new distributions. The parameters have been calculated by maximum likelihood estimation. **Findings:** Its hazard rate function, survival function, and Moments, among other statistical features, were explored. **Novelty**: As results from classical distributions are insufficient for many Biomedical datasets, this novel distribution was fitted to a real data set of lung cancer patients' survival periods in months, allowing for a discussion of the data set's use. The superiority of the distribution is tested by comparing the same with known distributions and finding this one is better. Thereby the importance of the said distribution is established.

Keywords: Estimate; Weighted distributions; Parameters; Reliability; Ishita distribution

1 Introduction

Data analysis using probability distributions has enormous implications for the health sciences. Fisher's [1] study of weighted distributions provides a heuristic by applying many approaches to the unknown weight function. In order to classify the many sampling circumstances that might be described by weighted distributions, Rao [2] formally united the notion of weighted distributions into a coherent concept. Adding a weighting component to an existing probability distribution is a powerful method for expanding the applicability of such models. A weighted distribution is the result of recording observations from a random process with a non-uniform distribution of weights. The unit of interest length is the only criterion for the weight function in length-biased distributions. Length bias was first proposed by Cox [3]. The concept of weighted distributions allows us to more effectively deal with difficulties in model design and data interpretation. For precise statistical modeling, weighted distributions are used in many fields. Some examples include reliability, biology, ecology, meta-analysis family data analysis, and analysis of intervention data. In order to better manage data sets from a wide range of applications, several academics have created weighted probability models. Weighted generalised beta distributions (WGBDs) of the second sort were first described by Ye et al.[4]. Exponentiated weighted Weibull distribution (EWWD) was suggested by Idowu and Adebayo[5]. Weighted transmuted power distribution was first presented by Dar et al. [6] along with its characteristics and uses. Mir, Ahmad, and Reshi[7] addressed the structural features of the length beta distribution of first type, while Subramanian and Rather [8] investigated the length biased Shanker distribution with application to real-world data. Characterizing and estimating the length-biased Shukla distribution using real-world data was the topic of discussion between Rajagopalan and Ganaie[9]. New area biased distributions with applications in cancer data were investigated by Elangovan and Mohanasundari[10]. Oluwafemi and Olalekan [11] used forest inventory data to produce anarea and length biased exponentiated Weibull distribution. Agu Friday Ikechukwu et al. [12] detailed the Type II Topp-Leone Generalized Power Ishita Distribution with Properties and Applications. Kariema et al.[13]applied the Inverse Power Ishita Distribution under Progressive Type-II Censored Data to COVID-19 Data. Anwar Hassan Ishita at al.[14]explained Power Series Distribution with Properties and Applications. Kamlesh Kumar and Rama Shanker[15]studied Power Ishita distribution and its application to model lifetime data.The area biased Poison exponential distribution was the subject of Fazal's[16] research. Parameter estimation using an area-biase Rayleigh distribution is a topic that was recently explored by Rao and Pandey[17]. Praseeja at.al.[18]detailed the Biomedical Application and characteristics of Pras_Nikh Distribution which is also formed with weighted function method. Enogwe and Ibeh[19] analyzed Beta-Exponentiated Ishita Distribution and Its Applications. Applications in some Area-Biased Distribution discussed by Nuri celik[20]. A new Area-Biased Distribution with Applications in Cancer Data detailed by Elangovan and Mohanasundari[21]. The Power Ishita Distribution (PID) was given by Shanker and Shukla[22]as a two-parametric lifespan distribution that generalizes the one-parameter Ishita distribution.

2 Methodology

The pdf of PID is,

$$f(x;\,\theta,\alpha)=\frac{\alpha\theta^3}{\theta^3\ +2}(\theta+x^{\alpha-1}e^{-\theta x^{\alpha}};\,x>0,\,\theta>0,\,\alpha>0 \tag{1}$$

PID's cdf, or cumulative distribution function, is calculated as

$$F(x;\,\theta,\alpha)=1-(1+\ \frac{\theta x^{\alpha}\,(\theta x^{\alpha}+2)}{\theta^3+2})e^{-\theta x^{\alpha}};\,x>0,\,\theta>0,\,\alpha>0 \tag{2}$$

Assume X is positive random variable with pdf$f(x)$

$f_{w(x)=\frac{w(x)f(x)}{E(w(x))}},\ x>0$

Where w(x) be the non-negative waited function and$E(w(x))=\int w(x)f(x)dx<\infty$. We determine area biased power Ishita distribution. Thus, we shall derive the GAPID by arriving to the logical conclusion that w(x) = x^2. Then, we can find the pdf of GAPID from

$$f_{a(x)=}\ \frac{x^2\,f(x)}{E(x)} \tag{3}$$

From equation (4.1) of Shanker and Shukla[20]

r^{th} raw moment of PID, $\mu_r^1=E(x^1)=\{r\Gamma(\frac{r}{a})[\alpha^2\theta^3+(r+\alpha)(r+2\alpha)]\}/\alpha^2\theta^{r/\alpha}(\theta^3+2),\,r=1,2,3,...$

$$ie.,E(x^2)=\frac{\theta^3\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}}{\alpha\theta\frac{2}{\alpha}(\theta^3+2)} \tag{4}$$

When Equation (1) along with Equation (4) are substituted into Equation (3), pdf of GAPID is,

$$f_{a(x)=}\ \frac{\alpha\theta^{\overline{\frac{3\alpha+2}{\alpha}}}\quad x^{\alpha+1}(\theta+x^{2\alpha})\,e^{-\theta x^{\alpha}}}{\theta^3\ \Gamma\frac{\alpha+2}{\alpha}+\Gamma\frac{3\alpha+2}{\alpha}} \tag{5}$$

And the cumulative distribution function (cdf) of GAPID is

$$
\begin{aligned}
f_{\alpha(x)} &= \int_0^x \frac{\alpha\theta^{\frac{3\alpha+2}{\alpha}}}{\theta^3\,\Gamma\frac{\alpha+2}{\alpha}+\Gamma\frac{3\alpha+2}{\alpha}}\, x^{\alpha+1}\left(\theta+x^{2\alpha}\right) e^{-\theta x^{\alpha}}\,dx \\
&= \frac{1}{\theta^3\,\Gamma\frac{\alpha+2}{\alpha}+\Gamma\frac{3\alpha+2}{\alpha}}\left[\alpha\theta^{\frac{4\alpha+2}{\alpha}}\int_0^x x^{\alpha+1}e^{-\theta x^{\alpha}}\,\alpha\theta^{\frac{3\alpha+2}{\alpha}}\int_0^x x^{3\alpha+1}e^{-\theta x^{\alpha}}\,dx\right]
\end{aligned}
\qquad (6)
$$

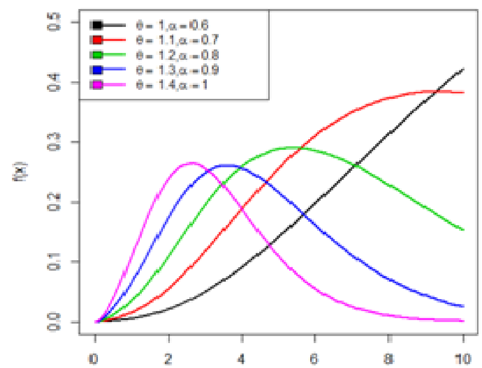

Fig 1. pdf of the GAPID

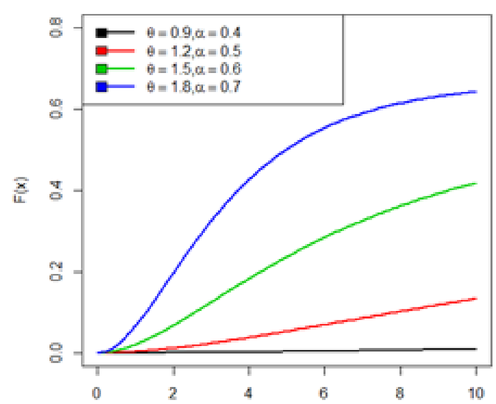

Fig 2. cdf of the GAPID

From Figure 1 and Figure 2 we can observe the nature of the pdf and cdf of the GAPID.

3 Results and discussion

Here we are discussing the main Characteristics & Properties of GAPID

3.1. Survival function

Survival function is a reliability function, and it is possible to calculate GAPID's survival function as

$$S(x) = 1 - F_a(x)$$
$$= 1 - \frac{1}{\theta^3 \Gamma \frac{(\alpha+2)}{\alpha} + \Gamma \frac{(3\alpha+2)}{\alpha}} \left(\theta^3 \gamma \left(\frac{(\alpha+2)}{\alpha}, \theta x^\alpha \right) + \gamma \left(\frac{(3\alpha+2)}{\alpha}, \theta x^\alpha \right) \right) \tag{7}$$

3.2. Hazard function

Other names for hazard function include instantaneous failure rate, hazard rate, and mortality force.

$$h(x) = \frac{f_{a(x)}}{1 - F_{a(x)}} = \frac{x^{\alpha+1} \quad \alpha\theta \frac{3\alpha+2}{\alpha} (\theta + x^{2\alpha} \ )^{e^{-\theta x^\alpha}}}{(\theta^3 \Gamma \frac{\alpha+2}{\alpha} + \Gamma \frac{3\alpha+2}{\alpha}) - (\theta^3 \gamma (\frac{\alpha+2}{\alpha}, \ \theta x^\alpha) + \gamma (\frac{3\alpha+2}{\alpha} \ , \theta x^\alpha))} \tag{8}$$

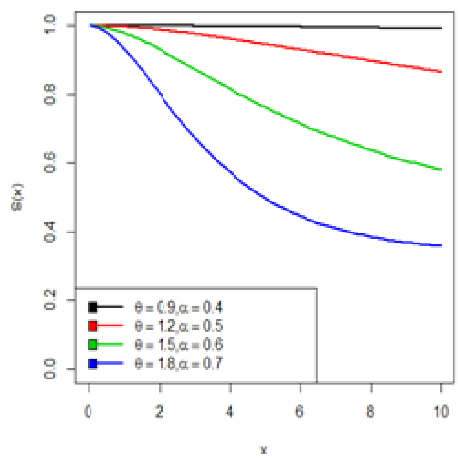

Fig 3. Reliability function

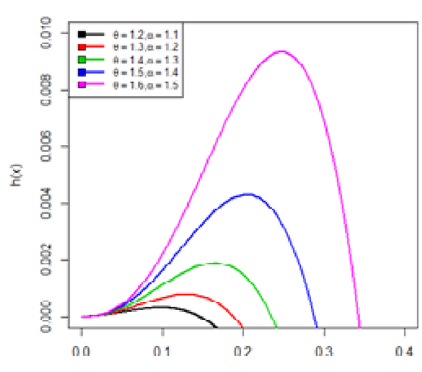

Fig 4. Hazard Function

From Figure 3 and Figure 4we can observe the nature of the Reliability function and Hazard Function of GAPID.

3.3. Moments

Let's assume that X is random variable that follows GAPID with parameters θ and α, we can calculate the rth order moment of this distribution, denoted by E(X^r), as

$$E(X^r) = \mu_r^{'} = \int_0^\infty x^r f_{a(x)} dx$$
$$= \frac{\alpha\theta^{\overline{\frac{3\alpha+2}{\alpha}}}}{\theta^3\Gamma\frac{\alpha+2}{\alpha}+\Gamma\frac{3\alpha+2}{\alpha}} (\theta\int_0^\infty x^{\alpha+r+1}e^{-\theta x^\alpha}dx + \int_0^\infty x^{3\alpha+r+1}e^{-\theta x^\alpha}\,dx) \tag{9}$$

$Put\ x^\alpha = t \Rightarrow x = (t)^{\frac{1}{\alpha}}\ Also\ \alpha x^{\alpha-1}dx = dt \Rightarrow dx = \frac{dt}{\alpha x^{\alpha-1}} = \frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$ Then the result of simplifying Equation (9) is,

$$E(X^r) = \mu^1{}_r = \frac{\theta^3\Gamma\frac{\alpha+r+2}{\alpha}+\Gamma\frac{3\alpha+r+2}{\alpha}}{\alpha\theta^{\frac{r}{\alpha}}(\theta^3\Gamma\frac{\alpha+2}{\alpha}+\Gamma\frac{3\alpha+2}{\alpha})} \tag{10}$$

The second central moment of the GAPID is obtained by substituting r = 1 and 2, into Equation (10) and simplifying, i.e.,

$$Variance = \frac{\theta^3\Gamma\frac{(\alpha+4)}{\alpha}+\Gamma\frac{(3\alpha+4)}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\left(\theta^3\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}\right)} - \left(\frac{\theta^3\Gamma\frac{(\alpha+3)}{\alpha}+\Gamma\frac{(3\alpha+3)}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^3\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}\right)}\right)^2 \tag{11}$$

3.4. Moment Generating Function (MGF)

MGF of a random variable, X, with parameters θ and α and, according to the area biased power Ishita distribution, found by using the formula

$$M_{X^{(t)}} = E(e^{tx}) = \int_0^\infty e^{tx} f_{a(x)} dx$$

Taylor's series provides us

$$\int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_{a(x)}\,dx = \int_0^\infty \left(1+tx+\frac{(tx)^2}{2!}+....\right) f_a(x)dx$$
$$M_X(t) = \frac{1}{\alpha\left(\theta^3\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}\right)}\sum_{j=0}^\infty \frac{t^j}{j!\,\theta^{\frac{j}{\alpha}}\left(\theta^3\Gamma\frac{(\alpha+j+2)}{\alpha}+\Gamma\frac{(3\alpha+j+2)}{\alpha}\right)} \tag{12}$$

3.5. Maximum Likelihood Estimation

GAPID parameters are estimated here with the help of the maximum likelihood estimation method. If we choose random size n sample from GAPID, we can write the likelihood function as where $X_1, X_{2,}\ldots,X_n$ are the observed values.

$L(x) = \prod_{i=1}^n f_a(x) = \frac{\alpha\theta^{n\left(\frac{3\alpha+2}{\alpha}\right)}}{\left(\theta^3\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}\right)^n}\prod_{i=1}^n (x_i{}^{\alpha+1}(\theta+x_i{}^{2\alpha})e^{-\theta x_i{}^\alpha})$ Then the partial differentiation with respect to α & θ of the logarithmic function of the likelihood function is:

$$\frac{\partial logL}{\partial\theta} = \frac{n\alpha}{\alpha\theta}\left(\frac{3\alpha+2}{\alpha}\right) - n\left(\frac{3\theta^2\Gamma\frac{(\alpha+2)}{\alpha}}{\theta\square^2\Gamma\frac{(\alpha+2)}{\alpha}+\Gamma\frac{(3\alpha+2)}{\alpha}}\right) + \sum_{i-1}^n \log x_i + \sum_{i-1}^n\left(\frac{x_i^{2\alpha}\log x_i}{(\theta+x_i^{2\alpha}}\right) \tag{13}$$

$$\frac{\partial logL}{\partial \alpha} = -\frac{2n\theta}{\alpha^3\theta} - n\varphi(\theta^3\Gamma\left(\frac{(\alpha+2))}{\alpha} + \Gamma\frac{(3\alpha+2))}{\alpha}\right) + \sum_{i=1}^{n} logx_i + \sum_{i=1}^{n}\left(\frac{x_i^{2\alpha} logx_i}{(\theta+x_i^{2\alpha})}\right) \\ -\theta\sum_{i=1}^{n} x_i^{\alpha} logx_i = 0 \quad (14)$$

Where $\varphi(.)$ is a digamma function defined as the logarithmic derivative of the Gamma function. Algebraic solutions to the aforementioned system of nonlinear equations are not possible. Since this distribution has unknown parameters, we estimate them using R.

With respect to real data analysis it can establish the importance of new distribution by showing that this one can be the best fit while comparing with the conventional distributions. It is narrated in the next session.

4 Application

Data analysis. Seventy patients were selected randomly from a medical college at Thrissur district of Kerala (India). We observe the Survival time in days Table 1 of lung cancer patients after their second cycle of chemotherapy.

(While analyzing the goodness of fit of the distribution, initially we converted the data set of Survival time in days to Survival time in months. Thereafter, we compare the fit of the Survival time (in months) of GAPID to that of PID and of the Ishita Distribution using a real-world data set.)

Table 1. Survival time (days) -after 2^{nd} cycle of chemotherapy

446	**484**	**494**	**546**	**551**	**459**	**528**	**451**	**523**	**419**	**38**	**65**	**328**	**349**
621	148	151	165	170	701	155	676	155	572	80	29	379	304
139	148	151	165	170	148	155	142	155	132	79	84	404	337
175	259	254	286	301	244	284	179	258	172	86	59	368	395
92	111	112	128	130	103	123	94	114	90	83	53	383	328

The unknown parameters are computed using R software, which is also used to calculate the model comparison criteria values. AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), AIC (Akaike Information Criterion), along with -2logL are used to evaluate GAPID's efficacy in comparison to ABR, ABA, PID and the Ishita Distribution. The distribution with smaller values of -2logL, AICC, BIC, and AIC, is preferred. The following formulae may be used to assess the criteria values of AIC, AICC, BIC, as well as -2logL.

$AIC = 2k - 2logL$, $BIC = klog\, n - 2logL$ and $AICC = AIC + \frac{2k(k+1)}{n-k-1}$

Table 2. Performance of Fitted Distributions

D	**S.E**	**MLE**	**AICC**	**BIC**	**AIC**	**-2logL**	**95% CI**	**K-S**	**p - value**
GAPI	$\widehat{\alpha} = 0.022, -\hat{\theta} = 0.080$	$\widehat{\alpha} = 0.630, -\hat{\theta} = 1.233$	198.1	202.22	197.91	193.91	(0.598, 0.661) (1.171,1.294)	0.031	0.8101
ABR	$\hat{\theta} = 0.052$	$\hat{\theta} = 1.145$	241.4	243.44	241.36	239.36	(1.087,1.202)	0.039	0.7810
ABA	$\hat{\theta}$=0.1013	$\hat{\theta}$=1.415	274.2	274.64	273.97	271.97	(1.087,1.202)	0.041	0.7670
Power Ishita	$\widehat{\alpha} = 0.037, -\hat{\theta} = 0.067$	$\widehat{\alpha} = 0.549, -\hat{\theta} = 0.798$	289.3	293.45	289.13	285.13	(0.521,0.576) (0.758,0.837)	0.151	0.0590
Ishita	$\hat{\theta} = 0.024$	$\hat{\theta} = 0.340$	396.3	398.42	396.26	394.25	(0.323,0.357)	0.1233	0.0660

Here, D Distributions, S.E Standard Error, KS Kolmogorov–Smirnov statistic, CI confidence interval ABR Area Biased Rayleigh Distributions ABA Area-biased Aradhana Distributions.

Under the considered model, the maximum value of the log-likelihood function is -2logL, and k, n represent the maximum number of parameters in the statistical model, maximum number of observations- sample size, respectively.

The GAPID has lower -2logL, AICC, BIC, and AIC, values and high p- value compared to the PID and Ishita Distributions, as seen in Table 2. It follows that the GAPID provides a more satisfactory match than the ABR, ABA, PID and the Ishita distribution.

5 Conclusion

Based on these findings, we suggest a novel distribution- Generalized Area-Biased Power Ishita Distribution (GAPID). The area biased method is used to the standard distribution to get the suggested distribution. Survival analysis and some of its statistical features were examined. The technique of estimating the greatest probability has also been investigated for potential use in establishing the values of its parameters. The innovative distribution was then tested on an actual medical data set to see how well it suited the data. The results indicate that the GAPID distribution fits the data more well when compared to the ABR, ABA, PID and the Ishita distributions.

References

1) Fisher RA. The effects of methods of ascertainment upon the estimation of frequencies. *Ann Eugenics*. 1934;6:13–25. Available from: https://doi.org/10.1111/j.1469-1809.1934.tb02105.x.
2) Rao C, Radhakrishna. On discrete distributions arising out of methods of ascertainment. *Sankhyā: The Indian Journal of Statistics, Series A*. 1965;p. 311–324. Available from: https://www.jstor.org/stable/25049375.
3) Cox DR. Some sampling problems in technology. In: Johnson, L N, Smith, H, J, editors. New Development in Survey Sampling. 1969;p. 506–527. Available from: https://www.nuff.ox.ac.uk/Users/Cox/Publications.html.
4) Ye Y, Oluyede BO, Pararai M. Weighted Generalized Beta Distribution of the Second Kind and Related Distributions. *Journal of Statistical and Econometric Methods*. 2012;1(1):13–31. Available from: https://ideas.repec.org/a/spt/stecon/v1y2012i1f1_1_2.html.
5) Idowu BN, Adebayo BT. Some Statistical Properties of Exponentiated Weighted Weibull Distribution. *Global Journal of Science Frontier Research*. 2014;14:2249–4626. Available from: https://globaljournals.org/GJSFR_Volume14/2-Some-Statistical-Properties.pdf.
6) Dar AA, Ahmed A, Reshi JA. An extension of power distribution. *Revista Investigation Operacional*. 2018;39(4):626–638. Available from: https://revistas.uh.cu/invoperacional/article/view/3869.
7) Mir KA, Ahmed A, Reshi JA. Structural properties of length biased beta distribution. *American Journal of engineering Research*. 2013;(02):1–06. Available from: https://www.ajer.org/papers/v2(2)/A0220106.pdf.
8) Subramanian C, Rather AA. A New extension of Shanker distribution with Real life data. *Journal of Xi'an University of Architecture & Technology*. 2020;12(3):2001–2009. Available from: https://www.researchgate.net/publication/339817308.
9) Rajagopalan V, Ganaie RA. Characterization and Estimation of length biased Shukla distribution with applications of real life time data. *Strad Research*. 2020;7(10):490–507. Available from: https://www.researchgate.net/publication/344725330.
10) Elangovan R, Mohanasundari R. A New Area-Biased Distribution with Applications. *Science, Technology and Development*. 2019;8(12):1–14. Available from: https://www.researchgate.net/publication/344377096.
11) Oluwafemi OS, Olalekan DM. Length and area biased exponentiated Weibull distribution based on forest inventories. *Biometrics & Biostatistics International Journal*. 2017;6(2). Available from: https://doi.org/10.15406/bbij.2017.06.00163.
12) Ikechukwu AF, Eghwerido JT, Francis R, Emmanuel. The Type II Topp-Leone Generalized Power Ishita Distribution with Properties and Applications. *Thailand Statistician*. 2021;19(3):472–483. Available from: https://www.researchgate.net/publication/352837114.
13) Elnagar KA, Ramadan DA, El-Desouky, Beih S. Statistical Inference to the Parameter of the Inverse Power Ishita Distribution under Progressive Type-II Censored Data with Application to COVID-19 Data. *Journal of Mathematics*. 2022;7704167. Available from: https://doi.org/10.1155/2022/7704167.
14) Hassan A, Akhtar N, Rashid A. A New Compound Lifetime Distribution: Ishita Power Series Distribution with Properties and Applications. *Journal of Statistics Applications & Probability*. 2021;10(3):883–896. Available from: https://doi.org/10.18576/jsap/100324.
15) Shukla RKK, Shanker. Power Ishita distribution and its application to model lifetime data Statistics in Transition new series. 2018. Available from: https://sit.stat.gov.pl/Article/354.
16) Fazal A. Area-biased poisson exponential distribution with applications. *Biometrics & Biostatistics International Journal*. 2018;7(3):256–261. Available from: https://doi.org/10.15406/bbij.2018.07.00216.
17) Rao AK, Pandey H. Parameter estimation of area biased Rayleigh distribution. *International Journal of Statistics and Applied Mathematics*. 2020;5(4):27–33. Available from: https://www.mathsjournal.com/archives/2020/vol5/issue4/PartA/5-3-24.
18) Praseeja CB, Prasanth CB, Subramanian C, Unnikrishnan T. Characterisation of PrasNikh-N Distribution and Its Biomedical Application. *Indian Journal of Science and Technology*;16(2023):4667–4675. Available from: https://doi.org/10.17485/IJST/v16i48.2522.
19) Enogwe S, Ibeh G. Beta-Exponentiated Ishita Distribution and Its Applications. *Open Journal of Statistics*. 2021;11:690–712. Available from: https://www.scirp.org/journal/paperinformation?paperid=112352.
20) Celik N. Applications in Some Area-Biased Distribution. *Asian Journal of Probability and Statistics*. 2022;19(3):45–50. Available from: https://doi.org/10.9734/ajpas/2022/v19i330472.
21) Elangovan R, Mohanasundari R. A New Area-Biased Distribution with Applications in Cancer Data. *Science, Technology and Development*. 2019;8(12):1–14. Available from: https://www.researchgate.net/publication/344377096_A_New_Area-Biased_Distribution_with_Applications_in_Cancer_Data.
22) Shukla KK, Shanker R. Power Ishita distribution and its application to model lifetime data. *Statistics in Transition New Series*. 2018;19(1):135–148. Available from: https://dx.doi.org/10.21307/stattrans-2018-008.