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Modified Slope Rotatability for Balanced Ternary Designs of Second Order Response Surfaces

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Abstract

Objective: To evaluate the best optimal solutions of modified Second Order Slope Rotatable Designs (SOSRD) using Balanced Ternary Designs (BTD) and to compare with different methods of Modified SOSRD for $3 \leq v \leq 8$. **Methods:** To estimate the parameters of the response surface design of second order the principle of least squares was applied for design of experiments, we also evaluated modified slope rotatable value and variance of estimated responses. **Findings:** The outcome of this research reveals the influence of various design parameters on key aspects such as the variance of the estimated response, slope rotatable value, axial points, central points and also total number of design points of modified SOSRD framework. **Novelty:** The variance of the estimated responses in the new method of Modified SOSRD using BTD provides the best optimal solutions with minimum variances while compared to different methods of Modified SOSRD.

Keywords: Response Surface Experimental Designs; Second Order Rotatable Designs; Second Order Slope Rotatability; Second Order Modified Slope Rotatability; Balanced Ternary Designs

1 Introduction

The response surface methodology, has been widely used in experiment design and analysis. Response surface methodology is mainly used to obtain optimal solutions. Such as; To study the optimal response for second order designs using BTD [1].While assessing the degree of rotatability for second order response surface designs using a class of BTD [2].To assess the second order response surface designs' slope rotatable value using the newly developed supplementary difference sets method [3]. To obtain the optimal design points for the construction of SOSRD using Supplementary Difference Sets (SDS) [4]. To evaluate the level of slope rotatability in the modified slope for designs of second order response surfaces [5]. To estimate the level of modified slope rotatability for design of second order response surfaces [6]. To evaluate the slope rotatability value of SOSRD under tri-diagonal correlation structure [7]. To assess the level of slope rotatability for design of second order response surface under tri – diagonal correlation structure [8]. To assess the directions of the design on the precision of the determination of an isotropy of magnetic susceptibility [9].

This work presents a new approach for modified slope - rotatability for balanced ternary designs of second - order response surfaces, building on the construction methods of[1] &[2]. In this paper we studied the variances of the estimated responses, which give the best optimal solutions with minimum variances compared to different methods of modified SOSRD".

2 Methodology

For instance, in order to fit the surface, we wish to use the response surface design of second-order.

$$\mathrm{Y_u} = \omega_0 + \sum_{\mathrm{i}=1} \omega_{\mathrm{i}} \mathrm{x_{iu}} + \sum_{\mathrm{i}=1} \omega_{\mathrm{ii}} \mathrm{x_{iu}^2} + \sum_{\mathrm{i}<\mathrm{j}} \sum_{\mathrm{ij}} \mathrm{x_{iu}} \mathrm{x_{ju}} + \varepsilon_{\mathrm{u}} \tag{1}$$

where $\mathbf{X}_{\mathrm{iu}}$ is the i$^{\mathrm{th}}$ factor's level $(i = 1, 2, \ldots, v)$ in the experiment's u$^{\mathrm{th}}$ run. $\mathcal{E}_u{}'S$ are random errors that lack correlation and have a mean of zero and variance ψ^2 is referred to as SOSRD if, for each independent variable (x_i), The distance to the point is the only factor influencing the variance of the initial-order partial derivatives estimate of ($\mathrm{d}^2 = \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{x_i^2}$) from the design's center.

It is possible to get the general criteria "for second order slope - rotatability as follows, which were provided by[1–3]. We enforce the following fundamental symmetry restrictions on D to facilitate simple solutions of the normal equations and reduce the second order polynomial's fit dimension from design points"D" by the process of least squares".

(A) $\sum \pi_{\mathrm{i}=1}^{\mathrm{v}} \mathrm{x_{iu}} = 0$ if any $\mathrm{x_i}$ is odd

(B) (i) $\sum \mathrm{x_{iu}^2}$ = Consistent = $\mathrm{N}\theta_2$;

(ii) $\sum \mathrm{x_{iu}^4}$ = Consistent = $\mathrm{cN}\theta_4$; for all i

(C) $\sum \mathrm{x_{iu}^2 x_{ju}^2}$ = Consistent = $\mathrm{N}\theta_4$; for $\mathrm{i} \neq \mathrm{j}$ (2)

Where c, θ_2 and θ_4 are constants.

The assessed parameters, variances and covariance's are

$$\mathrm{V}(\hat{\omega}_0) = \frac{\theta_4(\mathrm{c}+\mathrm{v}-1)\psi^2}{\mathrm{N}\left[\theta_4(\mathrm{c}+\mathrm{v}-1) - \mathrm{v}\theta_2^2\right]},$$

$$\mathrm{V}(\hat{\omega}_{\mathrm{i}}) = \frac{\psi^2}{\mathrm{N}\theta_2},$$

$$\mathrm{V}(\hat{\omega}_{\mathrm{ij}}) = \frac{\psi^2}{\mathrm{N}\theta_4},$$

$$\mathrm{V}(\hat{\omega}_{\mathrm{ii}}) = \frac{\psi^2}{(\mathrm{c}-1)\mathrm{N}\theta_4}\left[\frac{\theta_4(\mathrm{c}+\mathrm{v}-2) - (\mathrm{v}-1)\theta_2^2}{\theta_4(\mathrm{c}+\mathrm{v}-1) - \mathrm{v}\theta_2^2}\right],$$

$$\mathrm{Cov}(\hat{\omega}_0, \hat{\omega}_{\mathrm{ii}}) = \frac{-\theta_2\psi^2}{\mathrm{N}\left[\theta_4(\mathrm{c}+\mathrm{v}-1) - \mathrm{v}\theta_2^2\right]},$$

$$\mathrm{Cov}(\hat{\omega}_{\mathrm{ii}}, \hat{\omega}_{\mathrm{ij}}) = \frac{(\theta_2^2 - \theta_4)\psi^2}{(\mathrm{c}-1)\mathrm{N}\theta_4\left[\theta_4(\mathrm{c}+\mathrm{v}-1) - \mathrm{v}\theta_2^2\right]} \tag{3}$$

and the remaining covariance's disappear.

Examining the variance of $\hat{\omega}_0$ exposes that in order for a non-singular second - order experimental design to survive, it must (D) $\frac{\theta_4}{\theta_2^2} > \frac{v}{(c+v-1)}$.

As for the model of second order

$$\frac{\partial \hat{Y}}{\partial x_i} = \hat{\omega}_{\mathrm{i}} + 2\hat{\omega}_{\mathrm{ii}} \mathrm{x_{iu}} + \sum_{\mathrm{j}\neq\mathrm{i}} \hat{\omega}_{\mathrm{ij}} \mathrm{x_{ju}} \tag{5}$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right) = \mathrm{V}\left(\hat{\omega}_{\mathrm{i}}\right) + 4\mathrm{x}_{\mathrm{iu}}^2\, \mathrm{V}\left(\hat{\omega}_{\mathrm{ij}}\right) + \sum_{\mathrm{j}\neq 1} \mathrm{x}_{\mathrm{ju}}^2\, \mathrm{V}\left(\hat{\omega}_{\mathrm{ij}}\right) \tag{6}$$

To convert the portion of the equation on the right into a function of slope rotatability $d^2 = \sum_{i=1}^m x_i^2$ alone, the following condition must be met

$$4\,\mathrm{V}\left(\hat{\omega}_{\mathrm{ii}}\right) = \mathrm{V}\left(\hat{\omega}_{\mathrm{ij}}\right) \tag{7}$$

Here, conditions (2), (3), and (7) result in

$$\text{(E)}\ \left[\mathrm{v}(5-\mathrm{c}) - (\mathrm{c}-3)^2\right]\theta_4 + [\mathrm{v}(\mathrm{c}-5)+4]\theta_2^2 = 0 \tag{8}$$

Therefore, any general design of a second-order response surface, the A, B, C, D, and E from equation (8) provide the necessary conditions for slope rotatability.

Conditions required for modified second - order slope rotatable designs

The A, B, C equations of (2), D equation of (4), and E equation of (8) provide the required and adequate constraints for modified second - order slope - rotatable designs, in accordance with [3].

The regulations employed in SOSRD are changed, $\left(\sum x_{iu}^2\right)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e., $(N\theta_2)^2 = N\left(N\theta_4\right)$ i.e., $\theta_2^2 = \theta_4$ thus to obtain a modified SOSRD. We obtain c=1 or c=5 by using the new rule in equation E of (8). c=5 follows from the non-singularity criterion D of (4). Note that $\theta_2^2 = \theta_4$ and c=5 are comparable circumstances. The assessed parameters variances and covariance's are,

$$\mathrm{V}\left(\hat{\omega}_0\right) = \frac{(\mathrm{v}+4)\psi^2}{4\,\mathrm{N}}$$

$$\mathrm{V}\left(\hat{\omega}_{\mathrm{i}}\right) = \frac{\psi^2}{\mathrm{N}\sqrt{\theta_4}}$$

$$\mathrm{V}\left(\hat{\omega}_{\mathrm{ij}}\right) = \frac{\psi^2}{\mathrm{N}\theta_4}$$

$$\mathrm{V}\left(\hat{\omega}_{\mathrm{ii}}\right) = \frac{\psi^2}{4\,\mathrm{N}\theta_4}$$

$\mathrm{Cov}\left(\hat{\omega}_0, \hat{\omega}_{\mathrm{ii}}\right) = \frac{-\psi^2}{4\,\mathrm{N}\sqrt{\theta_4}}$ and there are zero other covariance's.

$$\mathrm{V}\left[\frac{\partial \hat{\mathrm{Y}}}{\partial \mathrm{X}_{\mathrm{i}}}\right] = \left[\frac{\sqrt{\theta_4} + \mathrm{d}^2}{\mathrm{N}\theta_4}\right]\psi^2. \tag{9}$$

3 Results and Discussion

Balanced Ternary designs: BTD is represented by (v, b, ρ_1, ρ_2, r, k, λ) for rows and columns that start with "v." The number of times the element i appears in block j is shown by the (i, j)$^{\text{th}}$ entry in this case. The incidence matrix form of a balanced ternary design, denoted by n= (n_{ij}), should meet the following requirements.

(i) A treatment may show up or not; if it does, it shows up 0–1 or 2 times in a block.

(ii) The number of replications, or r, is the same for every treatment.

(iii) Every element appears in BTD twice in ρ_2 blocks and once in ρ_1 blocks.

"Assuming a BTD of (v, b, ρ_1, ρ_2, r, k, λ) $2^{t(k)}$ signifies a fractional replication of 2^k with $+1$ or -1 in which no muddle interaction with fewer than five components exists. $[1-(\mathrm{v},\mathrm{b},\rho_1,\rho_2,\mathrm{r},\mathrm{k},\lambda)]$represents the design points obtained by transposing the BTD incidence matrix. $[1-(\mathrm{v},\mathrm{b},\rho_1,\rho_2,\mathrm{r},\mathrm{k},\lambda)]\,2^{t(\mathrm{k})}$ are the $\mathrm{b}2^{\mathrm{t}(\mathrm{k})}$design points obtained by "multiplication" of BTD. U represents the combination of the design points created from several sets of points, and let n_0 be the number of central points in the modified SOSRD. The design points $(a,0,0,\ldots,0)2^1$ derived from the point set are indicated by $(a,0,0,\ldots,0)$. Say n_a times over this series of extra design points"[3]

Theorem 3.1:

The design points, " $[1-(\mathrm{v},\mathrm{b},\rho_1,\rho_2,\mathrm{r},\mathrm{k},\lambda)]\,2^{\mathrm{t(k)}}\mathrm{Un}_a(\pm \mathrm{a},0,0,\ldots.0)2^1\mathrm{U}(\mathrm{n}_0)$ will provide a v-dimensional modified SOSRD with BTD in $N=\frac{\left(\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2\mathrm{n_a a^2}\right)^2}{\lambda 2^{\mathrm{tk(k)}}}$ design points in the event that $a^4=\frac{(c\lambda-\rho_1-2\rho_2)2^{t(k)-1}}{n_a}, n_0=\frac{\left(\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2n_a a^2\right)^2}{\lambda 2^{t(k)}}-\left[b2^{t(k)}+2n_a v\right]$ and n_0 must be an integer".

The design points, "

Proof:

For the simple symmetric conditions to produce from the BTD we have

$$\sum x_{iu}^2=\rho_1 2^{\mathrm{tk(k)}}+\rho_2 2^{\mathrm{t(k)}}+2\mathrm{n_a a^2}=\mathrm{N}\theta_2 \tag{10}$$

$$\sum x_{iu}^4=\rho_1 2^{\mathrm{t(k)}}+2\rho_2 2^{\mathrm{tk(k)}}+2\mathrm{n_a a^4}=\mathrm{cN}\theta_4 \tag{11}$$

$$\sum x_{iu}^2 x_{ju}^2=\lambda 2^{\mathrm{tk)}}=N\theta_4 \tag{12}$$

The modified condition $\left(\sum x_{iu}^2\right)^2=\mathrm{N}\sum \mathrm{x_{iu}^2 x_{ju}^2}$ results in N, , which is provided by $\mathrm{N}=\frac{\left(\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2\mathrm{n_a a^2}\right)^2}{\lambda 2^{\mathrm{tk(k)}}}$ alternatively N, may be obtained directly as $\mathrm{b}2^{\mathrm{t(k)}}+2\mathrm{n_a v}+\mathrm{n_0}$, where $\mathrm{n_0}$ is given by $\mathrm{n_0}=\frac{\left(\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2\mathrm{n_a a^2}\right)^2}{\lambda 2^{(k)}}-\left[\mathrm{b}2^{t(k)}+2\mathrm{n_a v}\right]$ and $\mathrm{n_0}$ must be an integer. From equations (10) and (12) and on shortening we get $\theta_2=\frac{\rho_1 2^{t(k)}+2\rho_2 2^{(k)}+2\mathrm{n_a a^2}}{\mathrm{N}}$ and $\theta_4=\frac{\lambda 2^{\mathrm{t(k)}}}{\mathrm{N}}$.

Example 3 .1: With the use of a BTD, we demonstrate how to construct modified SOSRD for v=3 factors. The outline elements,

$[1-(\mathrm{v}=3,\ \mathrm{b}=3,\rho_1=1,\rho_2=1,\mathrm{r}=3,\mathrm{k}=3,\lambda=2)]\,2^3\mathrm{Un_a}(\pm\mathrm{a},0,0,\ldots.0)2^1\mathrm{U}(\mathrm{n_0})$ will provide the second order response surface design's modified slope rotatability in N = 169 design points. Equations Equations (10), (11) and (12) give us

$$\sum \mathrm{x_{iu}^2}=24+2\mathrm{n_a a^2}=\mathrm{N}\theta_2 \tag{13}$$

$$\sum \mathrm{x_{iu}^4}=24+2\mathrm{n_a a^4}=\mathrm{cN}\theta_4 \tag{14}$$

$$\sum x_{iu}^2 x_{ju}^2=16=N\theta_4 \tag{15}$$

Equations Equations (14) and (15) lead to $\mathrm{n_a a^4}=28$, which suggests that $\mathrm{a^2}=2 \Rightarrow \mathrm{a}=1.414214$,

by applying the modified condition $\left(\theta_2^2=\theta_4\right)$ with $\mathrm{a^2}=2, \mathrm{n_a}=7$, we obtain $\mathrm{N}=169, \mathrm{n_0}=103$from equations Equations (13) and (15). The table below provides examples of modified SOSRDs that use BTD for illustration.

Table 1. Results of Modified - SOSRD using BTD

$(\mathrm{v},\mathrm{b},\rho_1,\rho_2,\mathrm{r},\mathrm{k},\lambda)$	C	n_a	a^2	n_o	N	$\mathrm{V}\left[\frac{\partial\hat{\mathrm{Y}}}{\partial \mathrm{X_i}}\right]=\left[\frac{\sqrt{\theta_4}+\mathrm{d}^2}{\mathrm{N}\theta_4}\right]\psi^2$
(3,3,1,1,3,3,2)	5	7	2	103	169	0.0817
(5,5,1,2,5,5,4)	5	30	2	245	625	0.0206
(6,6,2,1,4,4,2)	5	12	2	152	392	0.0402
(7,7,3,1,5,5,3)	5	5	4	118	300	0.0291
(8,8,4,1,6,6,4)	6	18	4	338	882	0.0107

Note: (Here $\mathrm{d}=\psi=1$)

Table 2. Comparison of variance of estimated responses for different methods Modified SOSRD Note: (**Here** $\mathrm{d}=\psi=1$)

Factors: v	CCD	BIBD	PBD	SUBA with two unequal block sizes	BTD	Minimum variance
3	0.1875	-	-	-	0.0817	BTD
4	0.0938	0.3125	-	-	-	CCD
5	0.0938	0.0583	-	-	0.0206	BTD
6	0.0521	0.3	0.1563	0.1563	0.0402	BTD
7	0.026	0.1563	-	-	0.0291	CCD
8	0.026	0.0278	0.0402	0.1528	0.0107	BTD

"From the Table 2, we compare the variance of estimated responses among the methods of modified SOSRD using Central Composite Design (CCD), Balanced Incomplete Block Design (BIBD), Pairwise Balanced Design (PBD), Symmetrical Unequal Block Arrangements (SUBA) with two unequal block sizes and BTD. For v = 3, 5, 6 and 8 factors Modified SOSRD using BTD gives the minimum variance compared with Modified SOSRD using CCD, BIBD, PBD and SUBA with two unequal block sizes. For v = 4 factors modified SOSRD using CCD gives the minimum variance compared with Modified SOSRD using BIBD. Similarly it observe that for v =7 factors modified SOSRD using CCD gives the minimum variance compared with Modified SOSRD using BIBD, PBD, SUBA with two unequal block sizes and BTD".

4 Conclusion

The variance of the estimated responses, in the new proposed method modified SOSRD using BTD, provides the best optimal solutions in some factors(i.e. v = 3, 5, 6 and 8) with minimum variances while compared to different methods of modified SOSRD.

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