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RESEARCH ARTICLE

Finite M_X/M/C Queue with State Dependent Service, Two-Class Arrivals and Impatient Customers

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Abstract

Objectives: 1. To compute the transition probabilities for the queueing system to be in different states. 2. To calculate the average system length and mean waiting time under the specified parameters. 3. To investigate and test the impact of system characteristics on the anticipated system length as well as mean waiting times. **Method:** MATLAB software is used for the Runge-Kutta method to calculate probabilities and system constants. **Findings:** An increase in Type-I arrival rate λ_1 and Type-II arrival rate λ_2 will cause both mean waiting times and queue length to increase, and they are reduced with an increase in service rates μ_1 and μ_2 of Type-I and Type-II classes. A rise in parameters such as special service rate μ_3, probability of balking and reneging will result in a reduction in both mean waiting times and queue length. **Novelty:** This article covers a batch arrival finite Markovian queueing system with multi-server support and consumer impatience. In real time, all arrivals need not enter the queue at the same arrival rates, as it is based on the needs of the customer. In view of this, we have assumed two types of customers along with two different rates of arrival and state-dependent service. We did our work in transient mode, as time can influence the waiting lines. Also determined the system's performance indices and demonstrated the impact of the input parameters on the system's constants.

Keywords: Batch Arrival; Customer Impatience; Two Class Customers; State Dependent Service; Multi Server Facility

1 Introduction

Operations research includes queueing theory, which is used to design the best strategies for delivering services subject to given constraints. Queueing models are a crucial tool for modeling and performance evaluation. Facilities or enterprises can provide services more efficiently by using queues. Queuing systems focus on customer-server equilibrium. Creating a queue benefits the server and the customer in the best way possible. Designing a mathematical model using the provided arrival and service rates is the goal of the queueing analyst. Batch arrival queueing systems are frequently used in various real-world settings, including computer networks, communication, and manufacturing and production systems. Researchers often use a bulk arrival queueing model to analyze this system because it can evaluate system performance, such as when a production or manufacturing system won't start until a certain number of raw materials have accumulated during an idle period. In a priority queue, A multi-server queueing approach provided key inputs to Madhu Jain et al.[1] developed a multi-server finite queueing model with balking under F-policy customer admission control. They concluded that deploying extra servers minimizes clients' baulking behavior, shortening the line of customers waiting during peak hours. S. Dissa et. al.[2] presented a model for inventory of a process that has perishable types of two commodities that arrive with two independent Poisson processes. They have obtained performance measures using the matrix geometric method also performed sensitivity analysis. The matrix geometric approach is used to analyze the model's steady-state behavior, and computed various system performance measures. Bara Kim et al.[3] analyzed an M/M/m queue with two non-preemptive customers. For each class, they calculated the first two moments' queued consumers. Serjio et al.[4] showed a new method to design the picking process for orders present in the queue. The statistical tests corroborated the significance of the results with statistical tests. Customer impatience has emerged as one of the most important factors in queue modeling due to the potential damage it could do to the system's finances and reputation. Much study is being done to maximize resources while avoiding losses. Xudong Chai et al.[5] used an infinite-dimensional non-homogeneous quasi-birth-and-death process to study customer impatience with flexible matching mechanisms. Many researchers contributed to this field, including R. R. Das[6], Sridhar et al.[7], and Ramadevi[8]. Carlos Franco et al.[9] proposed three objective functions for COVID-19 vaccination based on the open Jackson Network to address patient scheduling and capacity planning. Wei Sun,XumengXie,Zhiyuan Zhang and Shiyong Li[10] investigated The equilibrium balking behavior of customers and some effective regulation measures in a multi-server queueing system with a threshold policy

The system can have customers with different arrival rates. Most of the existing literature on multi-server queues with some other parameters is confined to steady analysis. Works on jobs of different classes are also few. As in real-time situations, time is an important factor, so we focused on the transient analysis of multi-server queues with bulk and two-class customers. As a result, we used the R-K approach to thoroughly analyze the transient behavior of the M_X/M/C queue with two types of arrivals and state-dependent service rates in the present work.

2 Methodology

2.1 Formatting of Mathematical Components

We investigated a finite Markovian queue with a multi-channel facility, heterogenous arrivals, state-dependent service, and transient customer impatience under the following assumptions:

1. The system's capacity is defined as s (finite)
2. The number of service channels is c
3. An arrival process is Poisson-compound. Using rates λ_1. and, λ_2., a probability density function exists for independent, identically distributed random batches of customers with size X: {a_n: a_n = P(X=n), n$\geq$1} Batch-based.
4. The mean Type-I customers arrival rate is λ_1
5. Type II customers have a mean arrival rate of $\lambda 2$
6. The mean service rate of servers serving Type-I customers is a μ_1
7. The mean service rate of servers for Type-II customers is $\mu 2$
8. Servers' mean service rate when customers at least "c" is μ_3
9. Customer balking probability is b Reneging parameter is ξ
10. $\pi_{x,y}^{(t)}$ Represents the probability that Type-I customers number "x" and "y" customers of Type-II at time point "t".

Two consumer classes have mean arrival rates $\lambda_1 \, and \, \lambda_2$ in a finite system having "c" services. FIFO mode with mean service rates of $\mu_1 \, and \, \mu_2$. However, when the number of consumers in the system of two classes combined reaches a certain level, the "c" service will be offered at a rate μ_3 regardless of consumer status. Impatience with parameters may cause customers to back out b and ξ .

The following differential equations calculate probabilities:

$$\frac{d\pi_{0,0}^{(t)}}{dt} = -(\lambda_1 + \lambda_2)\, b\pi_{0,0}^{(t)} + \mu_1 \pi_{1,0}^{(t)} + \mu_2 \pi_{0,1}^{(t)} \tag{1}$$

$$\frac{d\pi_{x,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1)\,\pi_{x,0}^{(t)} + \lambda_1 b\sum_{i=1}^{x} a_i\pi_{x-1,0}^{(t)} + (x+1)\,\mu_1\pi_{x+1,0}^{(t)} + \mu_2\pi_{x,1}^{(t)};\ 1 \le x \le c-1 \quad (2)$$

$$\frac{d\pi_{0,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + y\mu_2)\,\pi_{0,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y} a_i\pi_{0,y-1}^{(t)} + (y+1)\mu_2\pi_{0,y+1}^{(t)} + \mu_1\pi_{1,y}^{(t)};\ 1 \le y \le c-1 \quad (3)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1 + y\mu_2)\,\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=1}^{x-1} a_i\pi_{x-1,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y-1} a_i\pi_{x,y-1}^{(t)} + (x+1)\,\mu_1\pi_{x+1,y}^{(t)} + (y+1)\,\mu_2\pi_{x,y+1}^{(t)};\quad x\ and\ y \ne 0\ and\ x+y \le c-1 \quad (4)$$

$$\frac{d\pi_{c,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + c\mu_1)\,\pi_{c,0}^{(t)} + \lambda_1 b\sum_{i=1}^{c-1} a_i\pi_{c-1,0}^{(t)} + (c\,\mu_1+\xi)\pi_{c+1,0}^{(t)} + \mu_2\pi_{c,1}^{(t)} \quad (5)$$

$$\frac{d\pi_{0,c}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + c\mu_2)\,\pi_{0,c}^{(t)} + \lambda_2\, b\sum_{i=1}^{c-1} a_i\pi_{0,c-1}^{(t)} + (c\,\mu_2+\xi)\pi_{0,c+1}^{(t)} + \mu_1\pi_{1,c}^{(t)} \quad (6)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + 2\left(c\,\mu_3 + (x+y-c)\,\xi\right))\,\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=1}^{x-1} a_i\pi_{x-1,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y-1} a_i\pi_{x,y-1}^{(t)} + (c\,\mu_3 + (x+y+1-c)\xi)\pi_{x+1,y}^{(t)} + +(c\ \mu_3 + (x+y-1-c)\xi)\pi_{x,y+1}^{(t)};\ x\ and\ y \ne 0\ and\ x+y = c \quad (7)$$

$$\frac{d\pi_{x,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + \left(c\,\mu_3 + (x-c)\,\xi\right))\pi_{x,0}^{(t)} + \lambda_1 b\sum_{i=1}^{x-1} a_i\pi_{x-1,0}^{(t)} + (c\ \mu_3 + (x+1-c)\xi)\pi_{x+1,0}^{(t)} + +(c\ \mu_3 + (x-1-c)\xi)\pi_{x,1}^{(t)};\ y=0\ and\ c+1 \le x \le s-1 \quad (8)$$

$$\frac{d\pi_{0,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + \left(c\,\mu_3 + (y-c)\,\xi\right))\pi_{0,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y-1} a_i\pi_{0,y-1}^{(t)} + (c\ \mu_3 + (y+1-c)\xi)\pi_{1,y}^{(t)} + +(c\ \mu_3 + (y-c+1)\xi)\pi_{0,y+1}^{(t)};\ x=0\ and\ c+1 \le y \le s-1 \quad (9)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + 2c\mu_3)\,\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=1}^{x-1} a_i\pi_{x-1,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y-1} a_i\pi_{x,y-1}^{(t)} + (c\ \mu_3 + (x+y-c)\,\xi)\,\pi_{x+1,y}^{(t)} + (c\ \mu_3 + (x+y-c)\,\xi\,\pi_{x,y+1}^{(t)};x\ and\ y \ne 0\ and\ c+1 \le x+y \le s-1 \quad (10)$$

$$\frac{d\pi_{0,s}^{(t)}}{dt} = -(c\ \mu_3 + (s-c)\xi)\pi_{0,s}^{(t)} + \lambda_2 b\sum_{i=1}^{s-1} a_i\pi_{0,s-1}^{(t)} \quad (11)$$

$$\frac{d\pi_{s,0}^{(t)}}{dt} = -(c\ \mu_3 + (s-c)\xi)\pi_{s,0}^{(t)} + \lambda_1 b\sum_{i=1}^{s-1} a_i\pi_{s-1,0}^{(t)} \quad (12)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -(2c\,\mu_3 + (s-c)\,\xi)\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=1}^{x-1} a_i\pi_{x-1,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y-1} a_i\pi_{x,y-1}^{(t)};\ x\ and\ y \ne 0\ and\ x+y = s \quad (13)$$

2.2 Performance Measures and contribution

The following performance measures are computed to understand the system:

1. Average number of customers in the system $\left(L_S^{(t)}\right)$
2. Average waiting time in the system $\left(W_S^{(t)}\right)$

3 Results and Discussion

MATLAB software is used for the Runge-Kutta method to calculate probabilities and system constants for the given values of the parameters. These values are obtained by verifying the traffic intensity condition. They are shown in Tables 1, 2, 3, 4, 5, 6 and 7.

The model parameters are considered as given below:

$$S=6, c=4,\ \ \lambda_1=.01,\ \lambda_2=.02, \mu_1=.03,\ \mu_2=.04,\ \mu_3=.05,\ b=0.7, \xi=0.55$$

The time instances are taken as follows:$t_1=0.5,\ t_2=1.0,\ t_3=1.5,\ t_4=2.0$

Table 1. Effect of λ_1

Parameter (λ_1)	**t**	t_1	t_2	t_3	t_4
0.01	$L_S^{(t)}$	0.000265814	0.000515596	0.000750633	0.000971886
	$W_S^{(t)}$	0.037020814	0.071512251	0.103684934	0.133700979
0.011	$L_S^{(t)}$	0.000531623	0.001031173	0.001501227	0.001943709
	$W_S^{(t)}$	0.070793178	0.136755154	0.198289136	0.255705332
0.012	$L_S^{(t)}$	0.000603642	0.001170855	0.001704566	0.002206962
	$W_S^{(t)}$	0.076128477	0.147064193	0.213241129	0.274993085
0.013	$L_S^{(t)}$	0.000823149	0.001596621	0.002324408	0.003009494
	$W_S^{(t)}$	0.103811697	0.200542637	0.290784626	0.374992848

Table 1: Effect of λ_1

Finding: It is noted that an increase in Type-I arrival rate $\lambda 1$ will cause both mean waiting times and queue length to increase.

Table 2. Effect of λ_2

Parameter (λ_2)	t	t_1	t_2	t_3	t_4
0.02	$L_S^{(t)}$	0.000265814	0.000515596	0.000750633	0.000971886
	$W_S^{(t)}$	0.037020814	0.071512251	0.103684934	0.133700979
0.021	$L_S^{(t)}$	0.000396776	0.000770081	0.001121592	0.001452549
	$W_S^{(t)}$	0.053572806	0.103605282	0.150351247	0.194006139
0.023	$L_S^{(t)}$	0.000398206	0.000772305	0.00112459	0.001456663
	$W_S^{(t)}$	0.054926508	0.106086647	0.153848248	0.198476761
0.024	$L_S^{(t)}$	0.00041168	0.000798616	0.00116245	0.001504512
	$W_S^{(t)}$	0.057030152	0.110174951	0.159704623	0.205844571

Table 2 : Effect ofλ_2

Finding: An increase in Type-II arrival rate λ_2 is observed to cause a rise in both mean waiting times and queue length.

Table 3. Effect of μ_1

Parameter (μ_1)	T	t_1	t_2	t_3	t_4
0.03	$L_S^{(t)}$	0.000265814	0.000515596	0.000750633	0.000971886
	$W_S^{(t)}$	0.037020814	0.071512251	0.103684934	0.133700979
0.031	$L_S^{(t)}$	0.0002315200	0.0004490867	0.0006538221	0.0008465636
	$W_S^{(t)}$	0.0327836048	0.0633290273	0.0918228605	0.1184087054
0.032	$L_S^{(t)}$	0.0002057993	0.0003992039	0.0005812117	0.0007525673
	$W_S^{(t)}$	0.0291411944	0.0562933456	0.0816223544	0.1052559015
0.033	$L_S^{(t)}$	0.0001800777	0.0003493172	0.0005085921	0.0006585544
	$W_S^{(t)}$	0.0254987309	0.0492574446	0.0714213395	0.0921021672

Table 3: Effect of μ_1

Finding: Here, it is noted that both mean waiting times and queue length will decrease with an increase in Type-I service rate μ_1.

Table 4. Effect ofμ_2

Parameter (μ_2)	t	t_1	t_2	t_3	t_4
0.04	$L_S^{(t)}$	0.0002658139	0.0005155957	0.0007506326	0.0009718858
	$W_S^{(t)}$	0.0370208138	0.0715122512	0.1036849344	0.1337009789
0.041	$L_S^{(t)}$	0.0002315215	0.0004490923	0.0006538340	0.0008465835
	$W_S^{(t)}$	0.0327836866	0.0633293562	0.0918235941	0.1184099863
0.042	$L_S^{(t)}$	0.0002058020	0.0003992139	0.0005812328	0.0007526027
	$W_S^{(t)}$	0.0291413399	0.0562939302	0.0816236579	0.1052581770
0.043	$L_S^{(t)}$	0.0001800812	0.0003493303	0.0005086199	0.0006586008
	$W_S^{(t)}$	0.0254989218	0.0492582116	0.0714230495	0.0921051520

Table 4: Effect ofμ_2

Finding: Here, it is noted that both mean waiting times and queue length will decrease with an increase in Type-II service rate μ_2.

Table 5. Effect ofμ_3

Parameter (μ_3)	t	t_1	t_2	t_3	t_4
0.05	$L_S^{(t)}$	0.0002658139	0.0005155957	0.0007506326	0.0009718858
	$W_S^{(t)}$	0.0370208138	0.0715122512	0.1036849344	0.1337009789
0.051	$L_S^{(t)}$	0.0002571298	0.0004985342	0.0007254682	0.0009388685
	$W_S^{(t)}$	0.0364090979	0.0702987544	0.1018782148	0.1313084448
0.052	$L_S^{(t)}$	0.0000257020	0.0000498103	0.0000724514	0.0000937196
	$W_S^{(t)}$	0.0036392189	0.0070232862	0.0101733264	0.0131055855
0.053	$L_S^{(t)}$	0.0000025691	0.0000049767	0.0000072356	0.0000093553
	$W_S^{(t)}$	0.0003637533	0.0007016720	0.0010158890	0.0013080441

Table 5: Effect ofμ_3

Finding: Here, it is noted that a rise in the special service rate μ_3 will result in a reduction in both mean waiting times and queue length.

Table 6. Effect of b

Parameter (b)	t	t_1	t_2	t_3	t_4
0.01	$L_S^{(t)}$	0.000265814	0.000515596	0.000750633	0.000971886
	$W_S^{(t)}$	0.037020814	0.071512251	0.103684934	0.133700979
0.011	$L_S^{(t)}$	0.000311255	0.00060373	0.000878933	0.001137989
	$W_S^{(t)}$	0.044074767	0.085138536	0.123442235	0.159178987
0.012	$L_S^{(t)}$	0.000370414	0.000718467	0.001045956	0.00135422
	$W_S^{(t)}$	0.052451847	0.101318931	0.146900024	0.189425094
0.013	$L_S^{(t)}$	0.000434716	0.000843176	0.00122749	0.001589232
	$W_S^{(t)}$	0.061557171	0.118905552	0.172395963	0.222298453

Table 6 **:** Effect of b

Finding: Queue length and mean waiting times will both rise in response to a decline in the probability of balking.

Table 7. Effect of ξ

Parameter (ξ)	t	t_1	t_2	t_3	t_4
0.1	$L_S^{(t)}$	0.000265814	0.000515596	0.000750633	0.000971886
	$W_S^{(t)}$	0.037020814	0.071512251	0.103684934	0.133700979
0.11	$L_S^{(t)}$	0.000282652	0.00054777	0.000796892	0.001031138
	$W_S^{(t)}$	0.040018658	0.077225553	0.11187518	0.144158817
0.12	$L_S^{(t)}$	0.000308011	0.000596402	0.000867042	0.001121257
	$W_S^{(t)}$	0.043602885	0.084058844	0.121675605	0.156679358
0.13	$L_S^{(t)}$	0.000333318	0.000644869	0.000936889	0.001210933
	$W_S^{(t)}$	0.04717879	0.090865306	0.131426685	0.16912725

Table 7: Effect of ξ

Finding: Longer queues and longer mean wait times result from a decline in the reneging rate.

4 Conclusion

This paper details a bulk arrival queuing system with some real world features of two-types of customers with different arrival rates served by multi-servers with dependent service on the state of system.as time factor can influence the dynamics of the system,we have obtained transient probabilities of the system and obtained system constants such as system length and mean waiting time. Multiple parameter sensitivity analysis on system constants has also been shown with a numerical illustration. Arrival rates increased length and waiting time, whereas balking and service rates decreased them.

This work can be extended by considering a cost function since the second important goal of any system is to minimize cost and waiting time. Also, the number of servers needed to optimize total cost can be calculated. Also, steady state constants can be derived.

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