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RESEARCH ARTICLE

Comparative Study on Different Types of Energies

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Abstract

Objectives: The absolute sum of the eigenvalues is the definition of the graph's energy. In addition to discussing their relationship, this study compares the energy of the Adjacency matrix, Laplacian matrix, Signless Laplacian matrix, and Seidel matrix applied to ten distinct kinds of graphs. In this study, a correlation is established between the energy of graphs and the energy of Edge Antimagic graphs. **Methods:** The technical description of the graph's energy, $E(G) = \sum_{i=1}^{n} |\lambda_i|$. An Edge Antimagic graph's energy, identified by E_{EA} (G) =$\sum_{i=1}^{n} |\lambda_i|$, is the absolute total of its eigenvalues. **Findings**: The energy was calculated along with its link to the energy of Edge Antimagic matrix, Adjacency matrix, Laplacian matrix, Signless Laplacian matrix, and Seidel matrix. **Novelty:** Edge antimagic graphs have been applied using the concept of energy. The energy of Edge Antimagic graphs and the energy of graphs were discovered to be related. An analysis was conducted comparing Adjacency energy, Seidel energy, Laplacian energy, and Signless Laplacian energy.

Keywords: Laplacian Energy; Spectrum; Seidel Matrix; Signless Laplacian Matrix; Edge Antimagic Graphs.

1 Overview

Ivan Gutman first put forth the idea of a graph's energy in 1978. The energy of a graph has many applications in Chemistry especially in the molecular orbital theory of coupled molecules. Hartsfield and Ringel created the term "antimagic graph theory" in 1990. Graphs connect many different types of matrices. Collatz and Sinogowitz established the notion of graph spectrum in 1957. Because of the close association between the eigenvalues of a molecular graph and the Huckel orbital energy levels, the spectrum of a graph is essential in theoretical chemistry. The qualities of a graph's spectrum are connected to graph properties. The spectral graph theory is a set of properties that deals with the area of a graph. The spectrum of a graph is useful for a variety of tasks, including pattern identification, virus simulation in computer networks, and personal data security in databases. The energy between different types of graphs and Edge Antimagic graphs was examined in this article. In several cases, Ivan Gutman et al. showed the inequality$E(G) \leq LE(G)$.For every graphG $\varepsilon(S(G)) \geq \varepsilon(G)$ i.e., the energy of Seidel matrix is greater than or equal to the energy of graphs was proved by

Mohammad Reza Oboudi in 2016. In 2019, John K. Rajan and Mathew Varkey TK examined the hypoenergetic and non-hypoenergetic graphs among solitary graphs[1]. After the introduction of the idea of the energy of partial complements of graphs, it was applied to a few types of graphs, and limitations were found[2]. The energy concept was applied to a Bipolar fuzzy graph in 2021[3]. The writers P. Sumathi and N. Chandravadana revealed the Lucas Antimagic Labeling of Subdivision of Star, Shadowgraph of Star, Splitting graph of Star, and Subdivision of Bistar in 2022[4]. That same year, researchers looked at the Randic, Seidel, and Laplacian energies of the NEPS for path graph[5]. Suresh M, Ebenezer Bonyah, and Ugasini Preetha P investigated the adjacency energy and Laplacian energy of several non-simple standard graphs with self-loops in 2023[6]. A few relationships between a graph's energy and Seidel energy in terms of various graph properties and the inertia links between a graph's Seidel eigenvalues and graph eigenvalues are shown by Harishchandra S. Ramane, Ashoka K, Parvathalu B and Daneshwari Patil[7]. A research on the energy graph after the Covid-19 pandemic was done by Rupesh R Atram, and Sharad A Barde[8]. The comparison between energy and resolvent energy for multi-monad graphs was investigated[9]. In 2023, the precise value of the rainbow antimagic connection number was figured out[10]. Randic energy for face magic labeled graphs was found[11]. The applications of graph energy were described by Shiv Kumar, Prosanta Sarkar, and Anita Pal[12]. A new bound for the energy of the graph was found in 2024[13]. The authors' relationship between Randic energy, Seidel energy, and Laplacian energy of the NEPS graph is documented in the body of extant research[14]. N. Feyza Yalcin discovered the relationship between the Seidel Laplacian energy of regular graphs and the energy of graphs in 2022[15]. In this study, the authors compared the Adjacency, Seidel, Laplacian, and Signless Laplacian energies for the Star, Comb, and Caveman graphs, and they discovered a relationship between them. Similarly, the literature that was already in existence included the idea of Edge Antimagic graphs. The author examined the energy of 10 conventional graphs and the energy of Edge Antimagic graphs by applying the energy concept to the latter.

2 Methodology

2.1. Preliminaries

2.1.1. Adjacency Energy

Let *G* be a simple graph with $n$vertices and $m$edges. The Adjacency matrix of the graph *G* is given by, $A(G) = a_{ij}$, where$a_{ij} = 1$,if $v_i$is adjacent to $v_j\ or\ a_{ij} = 0$, otherwise. Adjacency energy is the adjacency matrix with the sum of all eigenvalues and is denoted by $E_A(G) = \sum_{i=1}^{n} |\lambda_i|$.

2.1.2. Laplacian Energy

Let *G* be a simple graph with *n* vertices and *m* edges. The Laplacian matrix of the graph G is given by, $L(G) = l_{ij} = -1, i \neq j$ for v_i is adjacent to v_j and zero otherwise. For $i = j,\ L(G) = l_{ij} = i^{th}$degree of vertex v_i. Laplacian energy is the Laplacian matrix with the sum of all eigenvalues. The Laplacian energy is denoted by $E_L(G) = \sum_{i=1}^{n} |\mu_i|$

2.1.3. Signless Laplacian Energy

Let *G* be a basic graph. The Signless Laplacian matrix of the graph *G* is given by, $SL(G) = {}^{+}_{ij} = 1, i \neq j$ for $v_i$is adjacent to v_j and zero otherwise. For $i = j,\ SL(G) = {}^{+}_{ij} = i^{th}$ degree of vertex v_i.The Signless Laplacian energy is denoted by $E_{SL}(G) = \sum_{i=1}^{n} |\mu_i^{+}|$.

2.1.4. Seidel Energy

Let *G* be a straightforward graph. The Seidel matrix of the graph *G* is given by $S(G) = s_{ij} = -1, i \neq j$ for v_i is corresponding to v_j and zero otherwise. If $i = j$, then$S(G) = S_{ij} = 1$. Seidel energy is the Seidel matrix with the sum of all eigenvalues. The Seidel energy is denoted by $E_S(G) = \sum_{i=1}^{n} |\sigma_i|$.

2.1.5. Energy of an Edge Antimagic Graph

The absolute total of the eigenvalues of Edge Antimagic graphs is the energy of an Edge Antimagic graph, which is represented by $E_{EA}(G) = \sum_{i=1}^{n} |\lambda_i|$.

3 Results and Discussion

3.1. Results for Star graphs

The Star Graph $S_{1,n}$ is a graph with $n-1$vertices incident with a single center vertex.

Theorem 1

The Star Graph $S_{1,n}$ satisfies the energy relation, $E_A(G) \leq E_S(G) \leq E_L(G) = E_{SL}(G)$

Proof:

Suppose there is a single vertex and m edges in a Star Graph G. Values for the Adjacency energy, Laplacian energy, Signless Laplacian energy, and Seidel energy were determined using the preliminary information from 2.1.1, 2.1.2, 2.1.3, and 2.1.4. The relation $E_A(G) \leq E_S(G) \leq E_L(G) = E_{SL}(G)$ was obtained.

Corollary 1: The above relation, $E_A(G) \leq E_S(G) \leq E_L(G) = E_{SL}(G)$ is satisfied for the following graphs: Bipartite graph, Banner graph, Eulerian graph, Hexahedral graph, and King graph. The results for the above said graphs were given in Table 1 **.**

Table 1. Comparison between different energies

S.No	Graph Name	Adjacency Energy	Laplacian Energy	Signless Energy	Laplacian	Seidel Energy
1.	Banner graph, vertices, *n*=5 or 5 graph.	5.596	9.5124	9.5124		8.7446
2.	Bipartite graph, $K_{3,3}$	6	18	18		9.9999
3.	Caveman graph, K_5 – e	23.714	60.1605	21.7163		40.7124
4.	Comb graph, $P_7 \odot K_1$	16.6276	25.9998	25.9998		38.6738
5.	Eulerian graph, (*n*=5, *m*=7)	6.3548	14	14		8.7446
6.	Hexahedral graph, vertices, *n*=6	8.9786	22	22		12.4722
7.	King graph, (*n*=6, *m*=11)	9.6568	22	22		12
8.	Ladder graph, *n*=6	16.1956	32	32		32.9575
9.	Ladder Rung graph, *n*=6	12	12	12		29.9999
10.	Star graph, $S_{1,5}$	4.4722	10	10		9.9999

3.2. Results for Comb graphs

Theorem 2

A Graph known as the Comb graph $P_n \odot K_1$ satisfies the energy relation, $E_A(G) \leq E_L(G) = E_{SL}(G) \leq E_S(G)$.

Proof:

Contemplate an $2n-1$edge and $2n$ vertex-rich Comb graph G. Values for the Adjacency energy, Laplacian energy, Signless Laplacian energy, and Seidel energy were determined using the preliminary information from 2.1.1, 2.1.2, 2.1.3, 2.1.4 and the relation $E_A(G) \leq E_L(G) = E_{SL}(G) \leq E_S(G)$ was obtained.

Corollary 2: The above relation $E_A(G) \leq E_L(G) = E_{SL}(G) \leq E_S(G)$ is satisfied for the following graphs: Ladder graph and Ladder Rung graph. The results for the above said graphs were given in Table 1.

3.3. Results for Caveman Graphs

When one edge from each clique is eliminated and used to link to a nearby clique in a central cycle, the resultant graph is known as the Caveman graph, which forms an uninterrupted loop made up of all the cliques.

Theorem 3

The Caveman graph satisfies the energy relation, $E_{SL}(G) \leq E_A(G) \leq E_S(G) \leq E_L(G)$

Proof:

Visualize a graph created in the style of K_{5-e} a Caveman. Energy values of the Adjacency matrix, Laplacian matrix, Signless Laplacian matrix, and Seidel matrix were computed using the preliminary information from 2.1.1, 2.1.2, 2.1.3, 2.1.4 and the relation $E_{SL}(G) \leq E_A(G) \leq E_S(G) \leq E_L(G)$was obtained.

The comparison among the Adjacency energy, Laplacian energy, Signless Laplacian energy, and Seidel energy of ten different graphs are listed in Table 1.

3.4. Results for Edge Antimagic Graphs

Theorem 4

Let $G\ (V,\ E)$ be any undirected and simple graph. The energy of a graph is therefore lower than that of Edge Antimagic graphs.

Proof:

Consider G to be a straightforward, undirected graph. $E\,(G) = \sum_{i=1}^{n} |\lambda_i|$according to the definition of energy: "The total of absolute eigenvalues is the graph's energy", $E_{EA}(G)\ = \sum_{n}^{i=1}\ |\lambda_i|$.. After computing these eigenvalues, we found that the energy of a graph is lesser than the energy of Edge Antimagic graphs, in accordance with the preliminary results of 2.1.5.

Theorem 5

Every complete graph attains edge antimagic.

Proof:

Let us assume that $G(V,\ E)$ is a full graph, as Figure 1 illustrates. A basic undirected graph with a single edge joining every pair of unique vertices is called a complete graph. Let $V = \{v_1\ ,\ v_2, \dots, v_n\}E = \{e_1\ ,\ e_2, \dots, e_n\}$ denote the vertices and edges of G respectively. The total of the labels of all vertices intersecting the edge is called an edge sum. Edge anti-magic labeling (EAL) is defined by $|f\,(x) + f\,(y)|$, where $f\,(x)$ is the edge value of v_i ,$f\,(y)$is the edge value of $v_j$and the edge labels that arise are distinct. Thus, every complete graph attains edge antimagic.

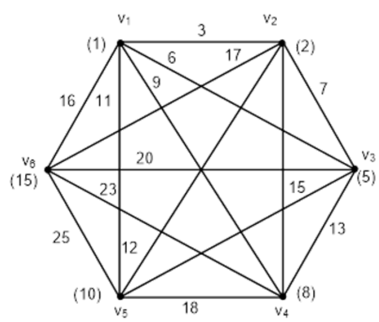

Fig 1. Edge antimagic complete graph

Example

Consider $G\,(V, E)$ be a graph where $V =\ \{v_1\ ,\ v_2, \dots, v_6\} and\ E =\ \{e_1\ ,\ e_2, \dots, e_{13}\}$be an edge antimagic graph.

Proof:

Pretend that graph G is complete. The total of the labels of all vertices intersecting the edge is called an edge sum. An edge antimagic graph is described as a graph with antimagic labeling on its edges and is specified as $|f\,(x) + f\,(y)|$. Using [Figure 1] we get $e(V_2, V_3) = 7$,$e(V_1, V_2) = 3$,$e(V_3, V_4) = 13$,$e(V_4, V_5) = 18$,$e(V_5, V_6) = 25$,$e(V_6, V_1) = 16$,$e(V_1, V_3) = 6$,$e(V_1, V_4) = 9$,$e(V_1, V_{53}) = 11$,$e(V_6, V_{12}) = 17$,$e(V_6, V_3) = 20$,$e(V_6, V_4) = 23$,$e(V_5, V_3) = 15$.

Therefore, G is an edge antimagic.

3.5. Energy of an edge antimagic Cayley graph

The adjacency of an edge antimagic Cayley graph Figure 2 is

$$A = \begin{vmatrix} 0 & 14 & 7 & 0 & 13 & 0 \\ 14 & 0 & 9 & 19 & 0 & 0 \\ 7 & 9 & 0 & 0 & 0 & 11 \\ 0 & 19 & 0 & 0 & 18 & 21 \\ 13 & 0 & 0 & 18 & 0 & 17 \\ 0 & 0 & 11 & 21 & 17 & 0 \end{vmatrix}$$

The Eigen values are 45.9787, -33.9898, -25.2148, 12.4847, 0.9723, -0.2312.

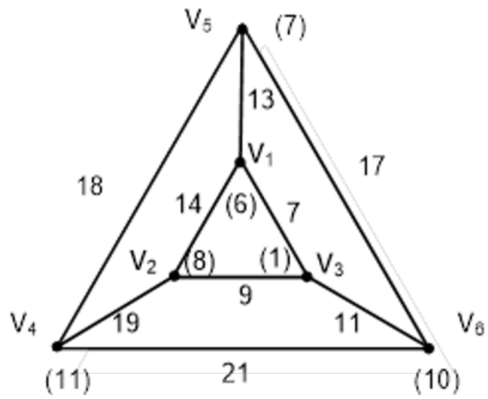

Fig 2. Edge antimagic Cayley graph

The contrast of a graph's energy and the energy of Edge Antimagic graphs of other graphs is displayed in Table 2.

Table 2. Energy of graphs and Energy of edge antimagic graphs

S. No	Graphs	E(G)	E_{EA} (G)
1.	Bipartite	6	71.442
2.	Cayley	8	118.8715
3.	Complete	9.9999	144.9786
4.	Dutch Windmill	12.0365	34.7317
5.	Eulerian	6.3548	56.2521
6.	Flower	7.7274	76.033
7.	Hexahedral	8.9786	142.5748
8.	King	9.6568	172.459
9.	Prism	12	35.222
10.	Utility	11.2916	54.9293

The authors compared the Adjacency, Seidel, Laplacian, and Signless Laplacian energies for some standard graphs, and they discovered the relation $E_A(G) \leq E_S(G) \leq E_L(G) = E_{SL}(G)$ is true for star graphs, bipartite graphs, Eulerian graphs, hexahedral graph, king graph. The relation $E_A(G) \leq E_L(G) = E_{SL}(G) \leq E_S(G)$ is true for comb graph, Ladder graph and Ladder rung graph. $E_{SL}(G) \leq E_A(G) \leq E_L(G)$ is true for caveman graphs. Similarly, theliterature that was already in existence included the idea of edge antimagic graphs. The author examined the energy of 10 conventional graphs and the energy of edge antimagic graphs and analyzed $E(G) < E_{EA}(G)$.

4 Conclusion

For roughly 10 common graphs, the energy values of adjacency matrix, laplacian matrix, signless laplacian matrix, and seidel matrix were compared, and their relationship was discovered. It was discovered that the energy of the graph is lesser than the energy of the edge antimagic graphs. The goal of an energy graph in chemistry was to find the approximate value of a molecule's total-electron energy. The applications are utilized in online games, designing circuits, electrical switching boards, safety mechanism in the construction industry and energy concepts in chemistry. The spectrum of a graph is useful for a variety of tasks, including pattern identification, virus simulation in computer networks, and personal information safety in databases.

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