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RESEARCH ARTICLE

ஜ, ஜ Near Semi Rings

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Abstract

Objectives: In the widened area of near semi rings, this article speaks about the interventions of a specific sub structure. **Methods:** Heading towards a more unique functionality, the chosen sub structure is intervened with regularity, homomorphism and also checked for weak commutative idempotency. **Findings:** The desired sub-structure is descried to have been preserved under homomorphism and is also found to be isomorphic to sub direct products of sub-directly irreducible sub structures. **Novelty:** Few descriptions and differences on weak commutative regularity and regular idempotency has been expounded. Keen and expanded utterance of Pierce Decomposition near semi ring functionalities have made the desired near semi ring to be closer to nil but not exactly a deeper nil near semi ring.

Keywords: Regularity; Homomorphism; Weak commutativity; Idempotency; Subdirectly irreducible; Pierce decomposition

1 Introduction

Functionalities in a vast detention always highlights the enormous outcomes imposed in a generic and specific study over the sub structure. The growth of semi near ring phase has been and is also being observed as one of the fast-growing branches of algebra. Impressed over the homologous sub structures, roughly inculcated elements are found to have its own form and description. Throughout this discussion, near semi ring also represents a semi near ring structure.

2 Methodology

Depending upon the coherent and cognitive usage of the below definitions and preliminary conditions, the desired results have been formulated.

Definition 2.1 [1] **:** T is said to be weak commutative if abc = acb for all a, b, c from T.

Definition 2.2 [2] **:** An element is said to be an idempotent if $e^2 = e$ for all e from the semi near ring. The set of all idempotents is denoted by E.

Definition 2.3 [3] **:** A mapping f: T $\longrightarrow$ T' is said to be a homomorphism, where T and T' are semi near rings, when the following conditions are satisfied:

(i) f(m+n) = f(m) + f(n)
(ii) f(mn) = f(m)f(n)
where m, n belongs to T.

Definition 2.4[4] **:** T is sub-commutative whenever there exists a subset F $\neq$ {0} of T such that xF = Fx for all x from T.

Definition 2.5[5] **:** A semi near-ring T is a non-empty set together with two binary operations '+' and '·' such that, it follows to be a group with respect to addition and a semi group with respect to multiplication along with the right distributivity.

Definition 2.6[6] **:** T is said to be cancellative if ea = eb(ae = be) implies a = b for all a, b from T.

Definition 2.7[7] **:** A near semi ring is said to be regular if aba = a for all a, b from T.

3 Results and Discussion

Delineation 3.1 : Every near semi ring with the detention x = xax where the elements are induced from the commutative semi near ring, comes out to be delineated as "ஜ" semi near ring. And, when the triplet equates itself to the central element[6] it is "ஜ" semi near ring.

Theorem 3.2

E is always non-zero in every ஜ sub commutative near semi ring.

Proof:

Taking T to be the ஜ near semiring, xax = x for every x and a from ஜ.
Then, $(ax)^2$ = ax(ax)
= a(xax)
= ax
Also, $(xa)^2$ = (xa)xa
= (xax)a
= xa
(ie), ax & xa are the idempotents in ஜ whence E $\neq$ {0}

Theorem 3.3

Every regular near semi ring obliges to be a ஜ semi near ring.

Proof:

Taking ஜ to be the regular semi near ring, for every a from ஜ, there exists b from ஜ such that aba = a and let x = bab. And, xax = (bab)a(bab)
= b(aba)bab
= b(a)bab
= b(aba)b
= bab
= x
Thus, xax = x making it a ஜ semi near ring.

Theorem 3.4

In a cancellative regular ஜ semi near ring, $a^n = a^{n+2}$ for all n $\geq$ 1.

Proof:

Let ஜ be the cancellative regular semi near ring, then, a = xax where a and x are from ஜ.
Now, a = xax
= x(axa)x
= (xax)ax
= aax
= a(axa)x
= aa(xax)
= aa(a)
= a^3
Thus, a = a^3 which implies a^2 = e.
Continuing this way, we get, $a^n = a^{n+2}$ for all n $\geq$ 1.
This completes the proof.

Theorem 3.5

T is a ஜ semi near ring if and only if every, a in ஜ can be written as

a = u + v where u in ஐ$_0$ and v in ஐ$_c$ &
u = x_0(nx+m) – x_0ax_c
v = $x_0ax_c+x_c$
x = x_0+x_c from ஐ$_0 \oplus$ ஐ$_c$
x_0 n ε ஐ$_0$ & x_c m ε ஐc
Further-more, u ε ஐ$_0$ & v ε ஐ c.
Proof:
Let a ε ஐ, then, there exists x ε ஐ such that a = xax
Using Pierce Decomposition from [2], a = n+m and x = x_0+x_c
Where x ε ஐ , n, $x_0 \varepsilon$ ஐ$_0$ & m, $x_0 \varepsilon$ ஐ$_c$
Now, a = (x_0+ x_c) (n + m)x
= (x_0+ x_c) (nx + mx)
= (x_0+ x_c) (nx + m)
= x_0(nx + m) + x_c(nx + m)
= x_0(nx + m) + x_c
= x_0(nx + m) – x_0ax_c+ x_0ax_c+ x_c
= u + v
Where u = x_0(nx + m) – $x_0$ax &
v = x_0ax_c+ x_c
Now, u0 = [x_0(nx + m) - x_0ax_c] 0
= x_0(nx+m)0 – x_0ax_c0
= x_0(nx0+m0) – x_0ax_c0
= x_0(nx0+m) – x_0ax_c
= x_0(nx_c+mx_c) – x_0ax_c
= x_0x_c[n+m-a]
= x_0x_c[a-a]
= 0
Thus, u ε ஐ$_0$
And, v0 = [x0axc + xc]0
= x_0ax_c0 + x_c0
= x_0ax_c+ x_c
= v
Therefore, v ε ஐ$_c$
Thus, a = u+v where u ε ஐ$_0$ & v ε ஐ$_c$
Conversely, let us assume that for all a in ஐ with a = u+v where u in ஐ$_0$, v in u ஐ$_c$ with
u = x_0(nx+m) – x_0ax_c
v = x_0ax_c+ x_c
For ஐto be a ஐ semi near ring,
Consider a = u + v
= x_0(nx+m) - x_0ax_c+ x_0ax_c+ x_c
= x_0(nx+mx) + x_c
= x_0(nx+m)x + x_c
= xax + $x_c$ax
= (x_0+x_c) ax
= xax
Thus, for every a in ஐ, a = xax and this makes ஐ to be a ஐ semi near ring.
Theorem 3.6
Homomorphic image of a ஐ semi near ring is also so.
Proof:
Let f: ஐ $\longrightarrow$ ஐ' be a homomorphism such that f(x) = x, for all x in ஐ.
f(xax) = f(x) f(a) f(x)
= xax
= x

= f(x)
Thus, f(xax) = f(x)
Thus, homomorphism preserves the structure of a ஐ semi near ring.

Theorem 3.7

Every ஐ semi near ring is isomorphic to a subdirect product of sub-directly irreducible ஐ semi near ring.

Proof:

In general, every semi near ring is isomorphic to a sub direct product of sub directly irreducible semi near ring.
And, each semi near ring is a homomorphic image of a semi near ring under the projection map Π_i.
Now, the proof follows from Theorem 3.5

Theorem 3.8

A Weak commutative regular semi near ring is a ஐ semi near ring.

Proof:

Let ஐ be the weak commutative regular semi near ring.
Then, for all a in ஐ, there exists b in ஐ such that aba = a and let x = ab.
Then, xax = (ab)a(ab)
= (aba)ab
= a(ab)
= aba
= a
Thus, it becomes a ஐ semi near ring.

4 Conclusion

The solicit acquaintances of the most important sub structure of the near semi ring has been formulated. This paper discusses the enormous efficiency of a semi near ring structure to undergo its own morphological changes to bind along with certain properties like possession of the Insertion of Factors Property (IFP). And also, the liberty of the chosen near semi ring to remain itself when compared and convened with homomorphism and also sub direct product of sub directly irreducible near semi ring structure. Enormous classifications of the procured structure is discussed and also paved the way for further developments in this sector in mere future.

References

1) Arasi KBD, Valli SRV, Kumari SRSR. On Regative Semigroups. *International Journal of Creative Research Thoughts*. 2023;11(6):f196–f201. Available from: https://ijcrt.org/papers/IJCRT2306596.pdf.
2) Manikandan G, Perumal R. Mate and mutual mate functions in a seminearring. *AIMS Mathematics*. 2020;5(5):4974–4982. Available from: https://doi.org/10.3934/math.2020318.
3) Radha D, Lakshmi BMR. The Role of Weak Commutativity in Boolean Like Near Ring. *International Multidisciplinary Innovative Research Journal*. 2021;V(2):68–80. Available from: https://journals.vvvcollege.org/assets/journalsnew/may2021/article10.pdf.
4) Uma S. A Note on Ideals in Sub-Stable Near-Rings. In: 1ST INTERNATIONAL CONFERENCE ON MATHEMATICAL TECHNIQUES AND APPLICATIONS: ICMTA2020;vol. 2277, Issue 1 of AIP Conference Proceedings. 2020;p. 1–3. Available from: https://pubs.aip.org/aip/acp/article/2277/1/110011/1026557/A-note-on-ideals-in-sub-stable-near-rings.
5) Senthil S, Perumal R. MID-units in Right Duo-seminearrings. *Mathematics and Statistics*. 2023;11(5):816–819. Available from: https://www.hrpub.org/download/20230830/MS7-13492410.pdf.
6) Chibeti S, Kyapwanyama I. An Introduction to the Theory of Matrix Near-Rings. *Advance in Pure Mathematics*. 2023;13(2):71–95. Available from: https://doi.org/10.4236/apm.2023.132004.
7) Sowjanya M, Rao AG, Radharani T, Padmaja V. Results on r-regular near-rings. *International Journal of Mathematics and Computer Science*. 2020;15(4):1327–1336. Available from: http://ijmcs.future-in-tech.net/15.4/R-Sowjanya.pdf.