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RESEARCH ARTICLE

# Analyzing the Non-compositionality of Conceptual Combinations

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## Abstract

**Objectives:** This research is first of its kind to investigate the application of quantum mechanics and related mathematical models to cognitive science, specifically focusing on the challenge of modeling how the human mind assigns meaning to combinations of words. The primary objective is to classify whether given word combinations are compositional or non-compositional. **Methods:** The author introduces the "WWW method," which leverages the World Wide Web (WWW) as an experimental tool in place of human subjects, which makes this method cheaper and handy in comparison to the earlier methods which used human subjects for spoken and sign language studies and robots as non-human subjects. The "WWW method" involves checking the adherence to Bell inequalities and marginal selectivity. The term "meaning bound" is introduced to quantify the extent to which two words influence each other's meaning within a combination. This concept proves instrumental in devising a method for assessing compositionality using the WWW method. **Findings:** When the WWW serves as the information space, the verification of whether a word combination adheres to Bell inequalities entails determining if the meaning bound for the word combination (AB & C) exceeds that of its constituent parts (A & C, B & C) within the conceptual combination AB. Marginal selectivity is evaluated by establishing a vector model for grammatically framed word combinations, utilizing the WWW to collect data. If the word combination 'A then B' yields an equivalent number of hits as 'B then A,' it signifies that the word combination AB satisfies marginal selectivity. **Novelty:** Consequently, this research presents an accessible approach to identifying entangled pairs of word combinations using the WWW method, eliminating the need for human subject experiments. The earlier studies which include human subjects are costly and cannot be replicated anytime-anywhere, for obvious reasons.

**Keywords:** Quantum cognition; Marginal selectivity; Compositionality; WWW method; CHSH inequality

## 1 Introduction

The study explores quantum computers' myriad applications across diverse life domains, underscoring quantum mechanics' transformative impact[(1)]. Quantum game theory emerges as a potent tool, surpassing traditional game theory in solving real-world challenges[2]. Games serve as arenas where intellectual engagement intersects with competitive drive, with cognitive prowess enhancing winning probabilities[3]. Human decision-making, intrinsic to every facet of life, navigates complexities blending emotions, logic, and reason[4].

Compositionality is that meaning of a complex word can be understood by breaking it into smaller parts. Earlier works have studies whether compositionality is strictly obeyed or not by taking into account human subjects, considering both spoken and signed languages[5]. On the other hand, in this study author has performed the study on WWW without taking any human subjects. Thus, providing the easiest and user- friendly way to perform the compositionality test.

The paper[6] attempt to study whether non-human agents such as robots also understand and obey compositionality. In the present study, the author checked whether conceptual combinations obey compositionality using WWW. WWW is a non-human subject, thus presenting a way cheaper method. And the results obtained are astonishing that non-human subjects also obey compositionality.

In[7], to be able to completely mimic human intelligence, Artificial Intelligence needs to learn to identify contexuality. In the present study, the author used WWW as an experimental tool to study whether a conceptual combination obeys compositionality. Thus, user can easily train Google Bard, ChatGPT, etc. to perform this study.

A given word can be understood by breaking it into parts and combining the meaning obtained from each of the parts. For example:

'a red hat' = 'a' + 'red' + 'hat.'

'The girl is playing football' = 'the' + 'girl' + 'is' + 'playing' + 'football'.

'PET' is described as 'furry and warm,' a typical 'FISH' is 'greyish,' but a typical 'PET FISH' is a 'GUPPY,' which is neither 'furry and warm' nor 'greyish.'

Many times, the meaning of a combined word differs from the meanings of its constituent words. A typical

'APPLE CHIP' could refer to CHIPS made from the fruit 'APPLE' or integrated circuit (IC) 'CHIPS' manufactured by 'Apple Inc.'

If $A_1$, $A_2$, $B_1$, and $B_2$ are bivalent random variables with joint distributions Pr $(A_i, B_j)$, where $i, j \in \{1, 2\}$, then the necessary and sufficient condition for a joint distribution Pr $(A_1, A_2, B_1, B_2)$ is that the following system of inequalities is satisfied:

$-1 \leq \Pr(A_1,B_1) + \Pr(A_1,B_2) - \Pr(A_2,B_1) + \Pr(A_2,B_2) - \Pr(A_1) - \Pr(B_2) \leq 0$

$-1 \leq -\Pr(A_1,B_1) + \Pr(A_1,B_2) + \Pr(A_2,B_1) + \Pr(A_2,B_2) - \Pr(A_2) - \Pr(B_2) \leq 0$

$-1 \leq \Pr(A_1,B_1) + \Pr(A_1,B_2) + \Pr(A_2,B_1) - \Pr(A_2,B_2) - \Pr(A_1) - \Pr(B_1) \leq 0$

$-1 \leq \Pr(A_1,B_1) - \Pr(A_1,B_2) + \Pr(A_2,B_1) + \Pr(A_2,B_2) - \Pr(A_2) - \Pr(B_1) \leq 0$

Where Pr $(A_i, B_j)$ is shorthand for Pr $(A_i=+1, B_j=+1)$, Pr $(A_i)$ for Pr $(A_i=+1)$ and Pr $(B_j)$ represents Pr $(B_j=+1)$, $i, j \in \{1,2\}$. But[8] [9] used CHSH inequality which uses expectation values instead of probabilities.

$-2 \leq C(A_1,B_1) + C(A_1,B_2) + C(A_2,B_1) - C(A_2,B_2) \leq 2$

$-2 \leq C(A_1,B_1) - C(A_1,B_2) + C(A_2,B_1) + C(A_2,B_2) \leq 2$

$-2 \leq C(A_1,B_1) + C(A_1,B_2) - C(A_2,B_1) + C(A_2,B_2) \leq 2$

$-2 \leq -C(A_1,B_1) + C(A_1,B_2) + C(A_2,B_1) + C(A_2,B_2) \leq 2$

C $(A_i, B_j)$ is a correlation function that computes the expectation value of the product $A_i$ $B_j$; $i,j \varepsilon \{1, 2\}$. The correlations can be computed from the matrix of probabilities, e.g.,

$C(A_1, B_1) = (\Pr(A_1, B_1) + \Pr(A_1', B_1')) - (\Pr(A_1, B_1') + \Pr(A_1', B_1))$.

Two concepts A and B can be analyzed using four random variables $\{A_1, A_2\}$ and $\{B_1, B_2\}$ $\varepsilon$ {+1, -1} (Table 1).

**Table 1. The table explains the dominant and subordinate senses of words**

| | Value | Priming is done with | Sense interpreted |
|---|---|---|---|
| $A_1$ | +1 | Dominant | Dominant |
| $A_1$' | -1 | Dominant | NOT Dominant |
| $A_2$ | +1 | Subordinate | Subordinate |
| $B_2$' | -1 | Subordinate | NOT Subordinate |

- **Joint probability distribution**- The author models using variables $A_1$, $A_2$, $B_1$, and $B_2$-$\Pr_{A1,\ A2,\ B1,\ B2}$. The author gets 16 joint probability distributions corresponding to the responses of the subject (Table 2).

**Table 2. The table depicting sixteen probability values corresponding to the responses of subjects**

| | | | **B** | | | |
|---|---|---|---|---|---|---|
| | | | **$B_1$** | | **$B_2$** | |
| | | | +1 | -1 | +1 | -1 |
| **A** | **$A_1$** | +1 | $P_1$ | $P_2$ | $P_5$ | $P_6$ |
| | | -1 | $P_3$ | $P_4$ | $P_7$ | $P_8$ |
| | **$A_2$** | +1 | $P_9$ | $P_{10}$ | $P_{13}$ | $P_{14}$ |
| | | -1 | $P_{11}$ | $P_{12}$ | $P_{15}$ | $P_{16}$ |

”Marginal selectivity is analogous to the Causal communication constraint of the Special Theory of Relativity. The Causal communication constraint is mathematically expressed as follows:

$P(A_j = m) = P(A_j = m, B_1 = n) = P(A_j = m, B_2 = n)$

This equation asserts that the probability of obtaining $A_j = m$ is independent of which measurement ($B_1$ or $B_2$) is performed on part B.

In contrast, the probability for $B_k = n$ depends on which measurement ($A_1$ or $A_2$) is applied to part A.

For any given system, if two conditions are met:

**(a)** a change of state of the system is provoked by the measurement, and

**(b)** there is a lack of knowledge about the measurement process, then a violation of Bell inequalities occurs[(10)]. The study explores quantum computers’ myriad applications across diverse life domains, underscoring quantum mechanics’ transformative impact[(8)]. Quantum game theory emerges as a potent tool, surpassing traditional game theory in solving real-world challenges[(9)].

Games serve as arenas where intellectual engagement intersects with competitive drive, with cognitive prowess enhancing winning probabilities[(10)].

## 2 Methodology

As an example, consider the case of ’APPLE CHIP’ in a free association experiment – ’APPLE CHIPS.’ The associated probabilities are as follows:”

$P_1$= Pr ($A_1$,$B_1$)

$P_4$= Pr ($A_1$’,$B_1$’)

$P_2$= Pr ($A_1$,$B_1$’)

$P_3$= Pr ($A_1$’,$B_1$)

Where random variables corresponding to events are

$A_1$: -APPLE interpreted as a fruit

$A_2$:- APPLE as a computer

$B_1$:- CHIP as a food item

$B_2$:- CHIP as a circuit

Two cases arise-

**(a) Perfect Correlation**- Priming with fruit sense of APPLE and food sense of CHIP.

$P_1$ - APPLE interpreted fruit and CHIPS as food

$P_4$ - APPLE, not fruit, and CHIPS, not food

$P_1+P_4=1$

$P_3+P_2=0$

$P_1$, $P_2$, $P_3$, and $P_4$ are the probabilities associated with individual cases.

**(b) Perfect Anti-correlation-**

”Priming is employed with the fruit sense of ’APPLE’ and the electronic sense of ’CHIP.’

$P_3$ - ’APPLE’ is not interpreted as fruit, and ’CHIP’ is interpreted as an electronic circuit.

$P_2$ - ’APPLE’ is interpreted as fruit, and ’CHIP’ is not interpreted as a circuit.

For a given conceptual combination, three cases may arise:

(a) If Marginal selectivity fails, then the conceptual combination is non-compositional.

(b) If Marginal selectivity holds, but the Bell/CH/CHSH inequality fails, then the conceptual combination is non-compositional (Entangled).

(c) If Marginal selectivity holds, and the Bell/CH/CHSH inequality also holds, then the conceptual combination is compositional.

Let's discuss each case:
**(c) If Marginal selectivity** fails, then the conceptual combination is non-compositional.
Example: For APPLECHIPS Free association experiment (Table 3) –

**Table 3. Probability values for the APPLECHIP and TOASTGAG association experiment**

| | | | | | |
|---|---|---|---|---|---|
| **APPLE CHIP** | | | | | |
| | | **$B_1$** | | **$B_2$** | |
| | | +1 | -1 | +1 | -1 |
| **$A_1$** | +1 | $P_1$=0.94 | $P_2$=0.06 | $P_5$=0 | $P_6$=0.75 |
| | -1 | $P_3$=0 | $P_4$=0 | $P_7$=0.25 | $P_8$=0 |
| **$A_2$** | +1 | $P_9$=0 | $P_{10}$=0.35 | $P_{13}$=0.47 | $P_{14}$=0 |
| | -1 | $P_{11}$=0.65 | $P_{12}$=0 | $P_{15}$=0 | $P_{16}$=0.53 |
| **TOAST GAG** | | | | | |
| | | **$B_1$** | | **$B_2$** | |
| | | +1 | -1 | +1 | -1 |
| **$A_1$** | +1 | $P_5$=0.50 | $P_6$=0.4375 | $P_1$=0.625 | $P_2$=0.375 |
| | -1 | $P_7$=0.0625 | $P_8$=0 | $P_3$=0 | $P_4$=0 |
| **$A_2$** | +1 | $P_{13}$=0.29 | $P_{14}$=0 | $P_9$=0.07 | $P_{10}$=0.21 |
| | -1 | $P_{15}$=0.29 | $P_{16}$=0.42 | $P_{11}$=0.57 | $P_{12}$=0.14 |

Checking for Marginal selectivity:
$P_1+P_2=1.00$
$P_5+P_6=0.75$
$(P_1+P_2) - (P_5+P_6) = 0.25 \neq 0$
Marginal selectivity is not obeyed.
a) If Marginal selectivity holds and Bell/CH/CHSH inequality fails, then the conceptual combination is non-compositional (entangled):
**Example:** For BATTERY CHARGE Free association experiment –
To check for marginal selectivity:
Diff $(A_1)$ =0.067
Diff $(A_2)$ =0.048
Diff $(B_1)$ =0.116
Diff $(B_2)$ =0.120
Marginal selectivity approximately holds.
Checking for Bell/CH/CHSH inequality:
$C(A_1,B_1)=0$
$C(A_1,B_2)= 0.025$
$C(A_2,B_1)=-0.057$
$C(A_2,B_2)= 0.42$
BATTERY CHARGE slightly violates the CHSH inequalities (CHSH= 2.01), so it would be deemed "Entangled".
If Marginal selectivity and Bell/CH/CHSH inequality hold, then the conceptual combination is compositional:
**Example:** Free association experiment for word combination "TOAST GAG" (Table 3)
To check for marginal selectivity
$P_1+P_2=0.93$
$P_5+P_6=1.00$
$\text{Diff}(A_1) = [(P_1+P_2) - (P_5+P_6)] = -0.07 \sim 0$
$P_9+P_{10}=0.29$
$P_{13}+P_{14}=0.28$
$\text{Diff}(A_2) = [(P_9+P_{10}) - (P_{13}+P_{14})] = 0.01$
$P_1+P_3=0.5625$
$P_9+P_{11}=0.580$
$\text{Diff}(B_1) = [(P_1+P_3) - (P_9+P_{11})] = -0.0165$

$P_5+P_7=0.625$

$P_{13}+P_{15}=0.640$

$Diff(B_2) = [(P_5+P_7)- (P_{13}+P_{15})] = - 0.015$

As difference between the one-marginal and the other second marginal is approximately zero for four cases therefore Marginal selectivity holds. Checking for Bell/CH/CHSH inequality:

$C(A_1,B_1)=0$

$C(A_1,B_2)= 0.025$

$C(A_2,B_1)=-0.057$

$C(A_2,B_2)=.42$

$C(A_1,B_1) + C(A_1,B_2) + C(A_2,B_1)-C(A_2,B_2) = -2\leq 0+0.025+0.42+0.057=0.502 \leq 2$

$C (A_1,B_1) – C(A_1,B_2) + C(A_2,B_1)+C(A_2,B_2) = -2\leq 0-0.025+0.42-0.057=0.338 \leq 2$

$C(A_1,B_1) + C(A_1,B_2) – C(A_2,B_1)+C(A_2,B_2) = -2\leq 0+0.025-0.42-0.057=-0.45 \leq 2$

$-C(A_1,B_1) + C(A_1,B_2) + C(A_2,B_1)+C(A_2,B_2) = -2\leq -0+0.025+0.42-0.057=0.388 \leq 2$

The Bell/CH/CHSH inequality is satisfied in four cases, assuming that the word combination is compositional. Instead of human subjects, the author uses WWW (World Wide Web) as a reference - examining the 'Guppy effect' on WWW[11]. Conceptual combinations like 'PET FISH' tend to yield more likely interpretations when searched on the web, as in the case of 'GUPPY,' which is a particular type of American fish. These conceptual combinations, as identified by Bruza[9], are considered non-compositional as these word combinations violate the principle of semantic compositionality. However, it's important to note that the 'GUPPY' is neither a well-known PET nor a standard example of FISH. This leads to the intriguing 'GUPPY effect' on the WWW.

A violation of Bell inequalities is equivalent to a departure from classical probability theory[12]. In[11], Aerts suggests employing the analysis used in[8] and[13] to demonstrate how the Pet-Fish problem violates Bell inequalities.

Regarding 'meaning bound,' as analyzed in[9], if 'APPLE' is interpreted as a fruit, then 'CHIP' is more likely to be understood as a potato CHIP. Conversely, if 'APPLE' is seen as a computer, 'CHIP' is more likely to be interpreted as an electronic circuit. In the case of 'APPLE CHIP,' the concept of 'meaning bound' provides insight into how these two words influence each other's interpretation. When it comes to the 'GUPPY effect,' meaning bound measures the degree of correlation between 'PET' and 'GUPPY,' 'FISH' and 'GUPPY,' and 'PET FISH' and 'GUPPY.'

Furthermore, the analysis reveals that the meaning bound shows a stronger correlation between 'PET FISH' and 'GUPPY' than between 'PET' and 'GUPPY' or 'FISH' and 'GUPPY.' This implies that 'PET FISH' is more likely to be interpreted as 'GUPPY' than the likelihood of 'PET' or 'FISH' being interpreted as 'GUPPY.' The author can define meaning bound as a representation of the degree of relatedness between two words in a conceptual combination. Mathematically, the author calculates the meaning bound between words A and B by dividing the relative weight of word B concerning A by the absolute weight of word B."

$M(A,B) = P(A,B) \div \{P(A)P(B)\} = [n(A,B) * n(WWW)] \div [n(A)n(B)]$,

where n(A) and n(B) are the number of web pages containing the words A and B, respectively,

n(A, B) is the number of web pages containing the words A and B, and

n(WWW) is the total number of web pages on WWW.

P(A) Represents the probability that the word A would be present on WWW.

$P(A) = n(A) \div n(WWW)$

There could be three types of meaning bounds:

(a) Attractive meaning bound: $M > 1$,

(b) Repulsive meaning bound: $M < 1$, and

(c) No meaning bound.

If the measure of the presence of a word 'B' in the 'meaning context' of another word 'A' equals the measure of the overall presence of this word 'B,' this would indicate that there is no specific 'meaning bound' between 'A' and 'B.' The author can also define the meaning bound in terms of conditional probabilities.

$M (A, B) = P (A|B) \div P (A) = P (B|A) \div P (B)$,

where conditional probability is given by

$P(A|B) = n(A, B) \div n(B)$

**Venn diagram:** For P(A|B), the shaded white portion represents the probability that pages containing 'B' also contain 'A.'

**Conjunction fallacy**[14]: According to Fuzzy Set Theory, the membership of an item in the intersection of two fuzzy sets is the minimum of its membership in either one. However, violations occur: Just like Guppy, a goldfish is neither a typical pet nor a typical fish.

$M_{pet}(goldfish) = M_{fish}(goldfish) = 0.5$

However, the goldfish is a typical pet fish; let's say $M_{pet\text{-}fish}$(goldfish)=0.9. These violations of classical fuzzy set theory can be seen in two forms:

**Overextension:**

$M_{A \wedge B}(x) > min(m_A(x), m_B(x))$

"Overextension occurs when the meaning of a combination of two words is more likely to occur than that of the individual words."

**Underextension:**

$M_{A \vee B}(x) < max(m_A(x), m_B(x))$

When the meaning of a combination of two words is less likely to occur than that of the individual words, the conjunction fallacy occurs when people judge the conjunction of two events as more likely than one of the two events taken separately[10].

**Overextension effects ( Example):** Let A = pet, B = fish, AB = pet fish, C = goldfish; in terms of conditional probability, overextension results in:

P (C|AB) >P (C|A) and/or P (C|AB) >P (C|B)

**Meaning bound:**

P (C|AB)/P (C|A) = M(C, AB)/M(C, A) = $n_{AB,C}$ * $n_A$ / $n_{A,C}$ *$n_{AB}$>1[10]

When attempting to translate[9]'s paper using the World Wide Web (WWW), the author aims to fulfill the objectives: 'to find possibly entangled pairs of word combinations.' As per Bruza's analysis[9], conceptual combinations that adhere to marginal selectivity but violate Bell inequalities are considered entangled.

## 3 Results and Discussion

In this study, the World Wide Web (WWW) has been used as an information space instead of using human subjects. The word combinations on the WWW and the number of hits represent probabilities associated with that word, denoted as 'n (word to be searched) as shown in the Table 4.

**Table 4. Verifying if[9]'s compositionality definition obeys the WWW experiment**

| November 26, 2016 | With quotes | Meaning bound (M)(with quotes) | Without quotes | Meaning bound (M)(Without quotes |
|---|---|---|---|---|
| 1- non-compositional | | | | |
| Pet | 1,34,00,00,000 | | | |
| Fish | 93,50,00,000 | | | |
| Guppy | 1,05,00,000 | | | |
| "Pet fish" | 5,27,000 | | 9,64,00,000 | |
| Pet guppy | 4,57,000 | 1.786425017768301 | | |
| Fish guppy | 6,23,000 | 3.490196078431373 | | |
| "Pet fish" guppy | 1,37,000 | 173.9405439595193 | 4,51,000 | 24.50602647698083 |
| 2- non-compositional | | | | |
| APPLE | 2,09,00,00,000 | | | |
| CHIP | 49,70,00,000 | | | |
| "APPLECHIP" | 11,50,000 | | 5,68,00,000 | |
| Food | 3,39,00,00,000 | | | |
| Apple food | 29,90,00,000 | 2.32106815711846 | | |
| CHIP food | 10,80,00,000 | 3.525578248250565 | | |
| "APPLECHIP" food | 69,700 | 0.9833269206104912 | 2,22,00,000 | 6.341144210395114 |
| 3-entangled | | | | |
| Battery | 69,10,00,000 | | | |
| CHARGE | 1,20,00,00,000 | | | |
| Crime | 71,40,00,000 | | | |
| "BatteryCHARGE" | 4,72,000 | | 11,50,00,000 | |
| Battery crime | 2,77,00,000 | 3.087921130825702 | | |
| CHARGE crime | 24,90,00,000 | 15.98389355742297 | | |
| "BatteryCHARGE" crime | 1,56,000 | 25.45933627688363 | 1,03,00,000 | 6.899281451711119 |
| 4- non-compositional | | | | |

*Continued on next page*

*Table 4 continued*

| | | | | |
|---|---|---|---|---|
| Boxer | 19,40,00,000 | | | |
| Bat | 41,30,00,000 | | | |
| Sports | 3,86,00,00,000 | | | |
| "Boxer bat" | 1,150 | | 6,88,000 | |
| Boxer sports | 3,88,00,000 | 2.849740932642487 | | |
| Bat sports | 14,10,00,000 | 4.86456987291272 | | |
| "Boxer bat" sports | 945 | 11.70871817977022 | 4,46,00,000 | 923.6805639233643 |
| 5-non-compositional | | | | |
| Boxer animal | 1,59,00,000 | 3.363979073703647 | | |
| Bat animal | 3,29,00,000 | 3.269668606121933 | | |
| "Boxer bat" animal | 1,280 | 45.68462037637897 | 48,60,000 | 289.9383894481083 |
| Animal | 1,34,00,00,000 | | | |
| 6-non-compositional | | | | |
| Spring | 1,59,00,00,000 | | | |
| Plant | 91,00,00,000 | | | |
| Manufacturing unit | 22,90,00,000 | | | |
| "Spring plant" | 4,00,000 | | 15,70,00,000 | |
| Spring manufacturing unit | 3,00,00,000 | 4.531597594133641 | | |
| Plant manufacturing unit | 6,55,00,000 | 17.28729785498344 | | |
| "Spring plant" manufacturing unit | 48,400 | 29.06113537117904 | 1,62,00,000 | 24.78235474091175 |
| 7-non-compositional | | | | |
| Spring green | 4,57,00,000 | 0.1209500849321265 | | |
| Plant green | 36,60,00,000 | 1.692492664183559 | | |
| "Spring plant" green | 3,84,000 | 4.039785768936496 | 8,85,00,000 | 2.372087583272823 |
| Green | 13,07,00,00,000 | | | |
| 1- compositionality | | | | |
| Toast | 12,70,00,000 | | | |
| GAG | 11,00,00,000 | | | |
| Food | 3,39,00,00,000 | | | |
| "Toast GAG" | 849 | | 7,29,000 | |
| Toast food | 9,32,00,000 | 11.90625508094674 | | |
| GAG food | 1,50,00,000 | 2.212389380530973 | | |
| "Toast GAG" food | 382 | 7.299929467602003 | 7,30,000 | 16.24644419356536 |
| 2- compositionality | | | | |
| Toast wedding | 3,17,00,000 | 9.467825142546837 | | |
| GAG wedding | 9,87,000 | 0.3403448275862069 | | |
| "Toast GAG" wedding | 171 | 7.639819666138662 | 5,49,000 | 28.56534695615155 |
| Wedding | 1,45,00,00,000 | | | |
| 3- compositionality | | | | |
| BILL | 1,04,00,00,000 | | | |
| SCALE | 81,40,00,000 | | | |
| Currency | 63,50,00,000 | | | |
| "BILL SCALE" | 7,750 | | 17,30,00,000 | |
| BILL Currency | 7,88,00,000 | 6.562689279224712 | | |
| SCALE currency | 7,98,00,000 | 8.491168333688019 | | |
| "BILL SCALE" currency | 1,560 | 17.43459486918974 | 1,68,00,000 | 8.411087342405899 |

By analyzing the observed data, the author may formulate the following hypotheses:

- **Hypothesis 1:** If M(AB, C)>M(B, C) and M(AB, C)>M(A, C) or M(AB, C)<M(B, C) and M(AB, C)<M(A, C), then the conceptual combination is non-compositional.
- **Hypothesis 2:** If either M (A, C) or M(B, C) is greater than M(AB, C), the conceptual combination is compositional. Thus, the author has a method to determine if a conceptual combination is compositional.

Figure 1, represents the meaning bound of each conceptual combination. And in the end of this paper, both of the hypotheses have been verified.

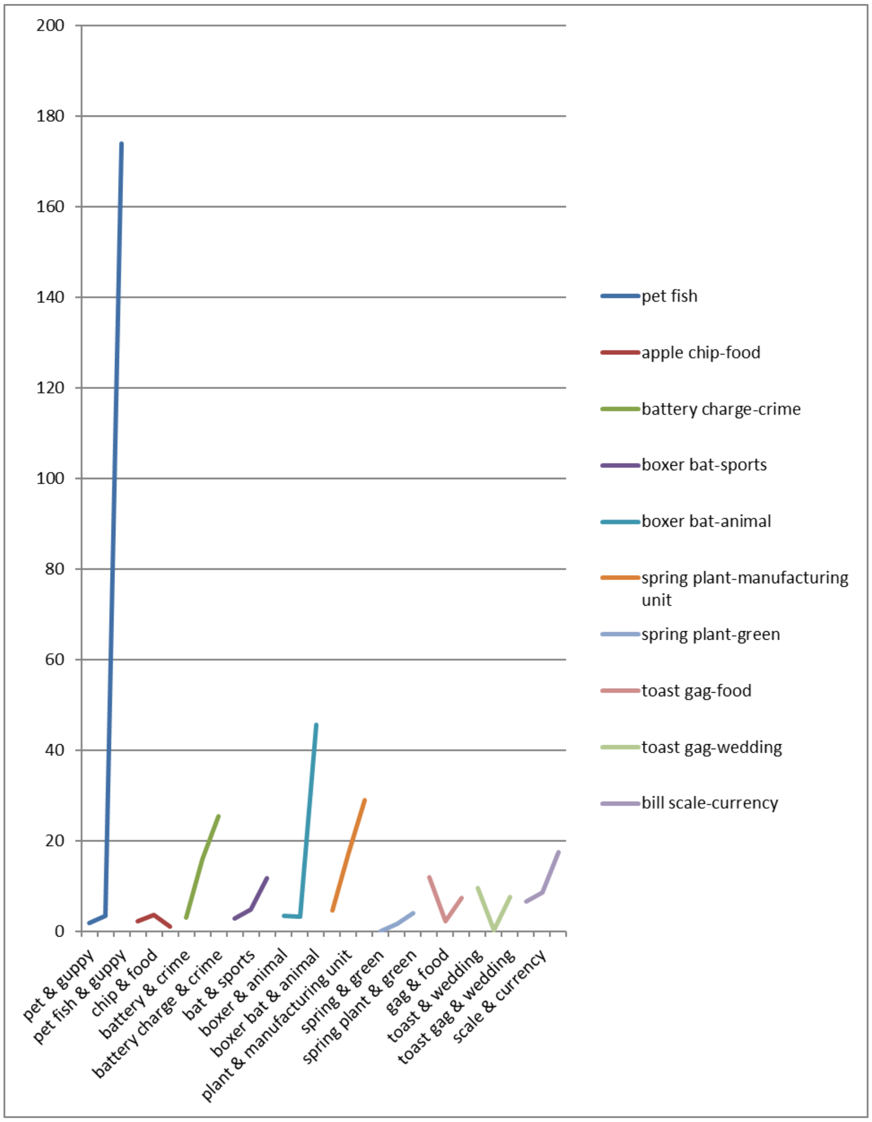


**Fig 1. Each curve represents the meaning bound of a word combination on the Y-axis. Non-compositional word combinations are those with monotonic plots, while compositional ones exhibit non-monotonic behavior**

**Table 5. Compositionality and marginal probability analysis for TOAST GAG**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Compositionality for TOAST GAG** | | | | | | | | |
| **Nov-25** | | **For toast GAG (compositional)** | | | | | | |
| | | | **GAG** | | | | | |
| | | **Choke($B_1$)** | | | **Joke($B_2$)** | | | |
| | | 1 | -1 | | 1 | -1 | | |
| **Jam($A_1$)** | 1 | 6,36,000 | 14,90,000 | $A_1B_1$+$A_1B_1$’ = 21,26,000 | 8,95,000 | 9,23,000 | $A_1B_2$+$A_1B_2$’ = 18,18,000 | Diff($A_1$) = 3,08,000 |

*Continued on next page*

*Table 5 continued*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Toast** | -1 | 8,29,000 | 5,30,000 | $A_1$'$B_1$+$A_1$'$B_1$' = 13,59,000 | 8,50,000 | 4,41,000 | $A_1$'$B_2$+$A_1$'$B_2$' = 12,91,000 | DIFF($A_1$') = 68,000 |
| | | $A_1B_1$+$A_1$'$B_1$ = 14,65,000 | $A_1B_1$'+$A_1$'$B_1$' = 20,20,000 | | $A_1B_2$+$A_1$'$B_2$ = 17,45,000 | $A_1B_2$'+$A_1$'$B_2$' = 13,64,000 | | |
| **Speech($A_2$)** | 1 | 13,60,000 | 37,80,000 | $A_2B_1$+$A_2B_1$' = 51,40,000 | 4,65,000 | 21,60,000 | $A_2B_2$+$A_2B_2$' = 26,25,000 | DIFF($A_2$) = 25,15,000 |
| | -1 | 7,84,000 | 5,19,000 | $A_2B_1$+$A_2B_1$' = 13,03,000 | 16,60,000 | 4,37,000 | $A_2B_2$+$A_2B_2$' = 20,97,000 | DIFF($A_2$') = - 7,94,000 |
| | | $A_2B_1$+$A_2B_1$' = 21,44,000 DIFF($B_1$) = - 6,79,000 | A2$B_1$+$A_2B_1$' = 42,99,000 DIFF($B_1$') = - 22,79,000 | | $A_2B_2$+$A_2$'$B_2$ = 21,25,000 DIFF($B_2$) = - 380000 | $A_2B_2$'+$A_2$'$B_2$' = 25,97,000 DIFF($B_2$') = - 1233000 | | |

| | **Diff($A_1$)** | **Diff($A_2$)** | **Diff($B_1$)** | **Diff($B_f$)** |
|---|---|---|---|---|
| TOAST-GAG (compositional) | 3,08,000 | 25,15,000 | -6,79,000 | -3,80,000 |
| APPLE-CHIP (non-compositional) | 12,80,000 | 22,32,00,000 | -1,03,89,000 | -23,15,50,000 |
| BILL-SCALE (compositional) | 3,00,00,000 | 2,10,000 | 2,20,80,000 | 3,25,70,000 |
| BATTERY-CHARGE (entangled) | 1,67,10,000 | 19,03,000 | 1,50,24,000 | 49,55,000 |

Finding if a similar method exists for marginal selectivity- Marginal selectivity is given by[9]

Diff($A_1$) = [Pr($A_1$,$B_1$)+Pr($A_1$,$B_1$')] - [Pr($A_1$,$B_2$) + Pr($A_1$,$B_2$')] ~ 0

Furthermore, by[11], if the measure of the presence of a word 'B' in the 'meaning context' of another word 'A' equals the measure of the overall presence of 'B', this would indicate that there is no specific 'meaning bound' between 'A' and 'B'. M(A,B)=1 would imply that n(B)/n(WWW)=n(A,B)/n(A). Thus, the author may use M=1 as a constraint for the satisfaction of marginal selectivity. The author has not been able to achieve the desired result for marginal selectivity over the WWW until now, as indicated in the table in Section 3.2.1. Consequently, the author attempted a variant: searching if 'APPLE fruit with CHIP-potato' yields approximately the same number of hits as 'APPLE fruit with CHIP-circuit' (Table 5).

However, it does not provide a method to differentiate the words that obey Marginal selectivity from those that do not. In grammatical analysis[14], specifically Pre-group grammar, the author suggests considering the grammar of the sentence instead of solely focusing on the subject, object, or adjective. For instance, in the sentence 'JAMES SHOOTS BALLOONS,' while in Bruza's[9] analysis the author would have only considered 'James balloons' as a conceptual combination,[14] suggests also considering 'Shoots.' By using Pre-group grammar (Table 6):

**Pre group:** is a partially ordered monoid, each element p $\varepsilon$ Pre group satisfies

$p^l . p \leq 1 \leq p.p^l$ and $p.p^r \leq 1 \leq p^r.p$

Where $p^l$ is the left adjoint, and $p^r$ is the right adjoint. JAMES SHOOTS BALLOONS can be reduced as

$N\ (n^r s n^l)\ n \leq 1.sn^l n \leq 1.s.1 \leq s$

In Pre-group grammar, parts of speech have unique representations: 'n' stands for noun, 's' for declarative sentence, 'nrsnl' for transitive verb, and 'nnl' for adjective.

Mapping Pre-group grammar to vector spaces involves the following steps:

1-Define 'N' and 'S' as vector spaces for 'n' and 's,' respectively.

- For vector spaces over $\mathbb{R}$, 'N' has only one adjoint, denoted as 'N*', which is isomorphic to 'N.'
- Therefore, 'nr' and 'nl' belong to 'N,' while 'sr' and 'sl' belong to 'S.

2-Composites of words are formed by tensor product $\otimes$

JAMES SHOOTS BALLOONS = $n\ (n^r s n^l)\ n = N \otimes N \otimes S \otimes N \otimes N$

3-Reductions
$E^r$: $nn^r \leq 1$, $\varepsilon^l$: $n^l n \leq 1$
$E$: $N \otimes N \rightarrow \mathfrak{R}$ :: $\sum_{i,j} c_{ij} v_i \otimes w_j \rightarrow \sum_{i,j} c_{ij} \langle v_i | w_j \rangle$
4-Type introductions:
$H^r$: $1 \leq n^r n$, $\eta^l$: $1 \leq nn^l$
$H$: $\mathfrak{R} \rightarrow N \otimes N$:: $1 \rightarrow \sum_i e_i \otimes e_i$
5- Meaning of sentences computed by calculating the cosine of the angle between the vector representations, given by the degree of synonymy
Sim (A, B) = $s_a . S_b / || s_a |||| s_b ||$
**Frobenious algebra:** A vector space V $\varepsilon$ $\mathfrak{R}$:
$\Delta$:: $v \rightarrow v \otimes v$
$\iota$:: $v \rightarrow 1$
$M$:: $v \otimes v \rightarrow \delta_{ij}$ v
$\zeta$:: $1 \rightarrow \sum_i v$
**For swap map:**
$\sigma$: $X \otimes Y \rightarrow Y \otimes X$
$\Sigma$ o $\Delta$ = $\Delta$ and $\mu$ o $\sigma$ = $\mu$
$M$ o $\Delta$ = 1
Understanding the algebras through examples- James shoots balloons
Shoots – a transitive verb
Shoots = $\sum_{ijk} c_{ijk} e_i \otimes s_j \otimes e_k$ $\varepsilon$ $N \otimes S \otimes N$
James shoots balloons = $\varepsilon_n \otimes 1_S \otimes \varepsilon_n$ (James $\otimes$ shoots $\otimes$ balloons)
= $\sum_{ijk} c_{ijk} \langle James | e_i \rangle \otimes s_j \otimes \langle e_k | balloons \rangle$ = $\sum_j \sum_{ik} c_{ijk} \langle James | e_i \rangle \langle e_k | balloons \rangle s_j$
**A Noun phrase**
James who shoots balloons
Who – interrogative pronoun (Table 6)

**Table 6. Decomposing a sentence**

| James | Who | Shoots | Balloons |
|---|---|---|---|
| N | $N^r nsn$ | $N^r sn^l$ | N |

= $(\mu_n \otimes \iota_s \otimes \varepsilon_n)$(James$\otimes$shoots$\otimes$balloons)
Thus, James who shoots balloons = James $\odot$ (shoots X balloons)
Where $\odot$ - element-wise multiplication
X – matrix multiplication
Shoots – matrix representing verb shoot
**a) PET FISH**: They may consider this sentence in two ways:
1- Pet as an adjective that modifies the meaning of the noun fish:
Pet = $nn^l$,
Matrix for pet = $\sum_{ij} p_{ij}\ e_i \otimes e_j$
pet fish = $\sum_{ij} p_{ij}\ e_i \langle e_j | fish \rangle$ = $pet_{adj} \odot fish_{noun}$
Sim(pet X fish, goldfish) > sim(fish, goldfish)
2- Fish which is a pet = fish $\odot$ (is X pet)

Experiment[14] involves using the grammar 'fish $\odot$ (is X pet)' with vector representations of 'fish,' 'pet,' and the matrix of the verb 'to be.' The author can also create a grammar for the conceptual combination 'APPLE CHIP' to check it for marginal selectivity.

A- CHIP(circuit)$\odot$ (of X APPLE (banana))
B- CHIP(circuit)$\odot$ (of X APPLE (computer))

As per the definition of marginal selectivity, the probability of 'CHIP' as a circuit should not be affected whether 'APPLE' is a banana or a computer.

**Hypothesis:** Ideally, the matrices obtained from steps 1 and 2 should have identical respective entries if marginal selectivity is obeyed. Otherwise, the entries must be different.

The World Wide Web (WWW) serves as an information space. Instead of relying on human subjects, the author utilizes the word combination 'WWW' and the number of hits to represent probabilities associated with that word, denoted as 'n (word to be searched).'

**Experimental Data:**

1-For APPLECHIP

A- CHIP (circuit)⊙ (of X APPLE (banana))

B- CHIP (circuit)⊙ (of X APPLE (computer)) (Table 7)

**Table 7. Search result counts and analysis table for APPLE CHIP**

| **Search result counts** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **28 November - For APPLECHIP** | **APPLE** | **CHIP** | **APPLE CHIP food** | **APPLE CHIP computer** | **APPLE CHIP Tom Jerry** | **APPLE CHIP nail art** | **APPLE CHIP gulab jamun** | **APPLE CHIP shin chan** |
| "APPLE (noun) food" | 3,47,000 | 87,500 | 1,31,000 | 76,800 | 6,560 | 22,900 | 1,390 | 113 |
| CHIP (noun) food | 7,66,000 | 72,800 | 7,66,000 | 17,600 | 1,210 | 5,060 | 217 | 24 |
| APPLE (adjective) CHIP food | 6,950 | 9 | 6,960 | 2 | 1 | 2 | 0 | 0 |
| APPLE (noun) computer | 93,30,000 | 4,22,000 | 28,90,000 | 5,00,000 | 3,33,000 | 43,000 | 627 | 3,750 |
| CHIP (noun) computer | 42,200 | 3,20,000 | 40,200 | 42,400 | 6,260 | 1,320 | 4 | 692 |
| APPLE (adjective) CHIP computer | 456 | 454 | 201 | 452 | 2 | 1 | 0 | 0 |

| **Analysis table** | | | | |
|---|---|---|---|---|
| | **APPLE Banana** | **APPLE computer** | **CHIP circuit** | **CHIP potato** |
| "APPLE (noun) food" | 1,32,000 | 34,000 | 7,04,000 | 6,01,000 |
| CHIP (noun) food | 7,60,000 | 17,000 | 8,19,000 | 22,600 |
| APPLE (adjective) CHIP food | 4 | 3 | 5 | 7 |
| APPLE (noun) computer | 2,50,000 | 26,80,000 | 4,22,000 | 2,79,000 |
| CHIP (noun) computer | 22,500 | 1,56,000 | 3,21,000 | 41,000 |
| APPLE (adjective) CHIP computer | 5 | 447 | 5 | 2 |

| **Analysis table** | | | |
|---|---|---|---|
| **Of X APPLE banana** | **Of X APPLE computer** | **CHIP circuit. Of X APPLE computer** | **CHIP circuit. Of X APPLE banana** |
| 1.53463407*10^11 | 1.172371878*10^11 | 8.253498021*10^16 | 1.080382385*10^17 |
| 1.236637315*10^11 | 6.41622329*10^10 | 5.254887374*10^16 | 1.012805961*10^17 |
| 5284097810 | 126551744 | 6.32758720*10^8 | 2.642048905*10^10 |
| 3.599761555*10^12 | 7.376597723*10^12 | 3.112924239*10^18 | 1.519099376*10^18 |
| 7.57864743*10^10 | 4.700875302*10^11 | 1.508980972*10^17 | 2.432745825*10^16 |
| 155590780 | 1275106286 | 6.375531430*10^9 | 7.77953900*10^8 |

According to [9], APPLECHIP does not obey marginal selectivity, so its matrix entries in the third table are dissimilar.

2- For TOAST GAG

A- TOAST (jam)⊙ (of X GAG (choke))

B- TOAST (jam)⊙ (of X GAG (joke))(Table 8)

**Table 8. Analysis table for TOAST GAG**

| **Analysis table for TOAST GAG** | | | | |
|---|---|---|---|---|
| | **TOAST jam** | **TOAST Speech** | **GAG choke** | **GAG joke** |
| "TOAST (noun) food" | 54,800 | 1,80,000 | 1,560 | 7,470 |
| GAG (noun) choke | 1,890 | 11,400 | 1,18,000 | 2,25,000 |

*Continued on next page*

*Table 8 continued*

| | | | | |
|---|---|---|---|---|
| TOAST(adjective) GAG food | 0 | 0 | 0 | 0 |
| TOAST (noun) wedding | 11,900 | 1,31,000 | 662 | 4,470 |
| GAG (noun) joke | 1,780 | 11,500 | 29,100 | 1,96,000 |
| TOAST (adjective) GAG wedding | 7 | 0 | 0 | 0 |

**Analysis table for TOAST GAG**

| | "TOAST (noun) food" | GAG (noun) choke | TOAST (adjective) GAG food | TOAST (noun) wedding | GAG (noun) joke | TOAST (adjective) GAG wedding |
|---|---|---|---|---|---|---|
| "TOAST (noun) food" | 1,80,000 | 1,560 | 26,200 | 72,700 | 7,470 | 4,51,000 |
| GAG (noun) choke | 9,990 | 1,18,000 | 9,980 | 10,200 | 2,25,000 | 10,100 |
| TOAST(adjective) GAG food | 0 | 0 | 0 | 0 | 0 | 0 |
| TOAST (noun) wedding | 8,76,000 | 662 | 8,210 | 1,31,000 | 4,470 | 49,200 |
| GAG (noun) joke | 8,580 | 29,100 | 8,510 | 8,720 | 1,96,000 | 8,720 |
| TOAST (adjective) GAG wedding | 0 | 0 | 0 | 0 | 0 | 0 |

**Analysis table for TOAST GAG**

| Of X GAG choke | Of X GAG joke | TOAST jam. Of X GAG joke | TOAST jam. Of X GAG choke |
|---|---|---|---|
| 730384400 | 3484689000 | 1.909609572*10^14 | 4.002506512*10^13 |
| 2.04938368*10^10 | 7.07702193*10^10 | 1.337557145*10^14 | 3.873335155*10^13 |
| 0 | 0 | 0 | 0 |
| 1661475000 | 8154360000 | 9.7036884*10^13 | 1.97715525*10^13 |
| 9156557440 | 4.5066571*10^10 | 8.021849638*10^13 | 1.629867224*10^13 |
| 0 | 0 | 0 | 0 |

According to [9], TOAST GAG obeys marginal selectivity, so its matrix entries in the third table are pretty similar.

3- For BATTERY CHARGE

A- BATTERY (car)⊙ (of X CHARGE (volt))

B- BATTERY (car)⊙ (of X CHARGE (prosecute)) (Table 9)

**Table 9. Analysis table for BATTERY CHARGE**

**Analysis table for BATTERY CHARGE**

| | BATTERY car | BATTERY Assault | CHARGE volt | CHARGE prosecute |
|---|---|---|---|---|
| BATTERY car | 11,40,000 | 18,200 | 1,94,000 | 12,000 |
| CHARGE volt | 30,900 | 485 | 35,200 | 57 |
| BATTERY CHARGE dead | 5,590 | 9 | 3,990 | 3 |
| BATTERY Assault | 35,800 | 1,02,000 | 5,000 | 14,800 |
| CHARGE prosecutes | 335 | 305 | 104 | 11,500 |
| BATTERY CHARGE crime | 8 | 104 | 4 | 1 |

**Analysis table for BATTERY CHARGE**

| | BATTERY dead | CHARGE volt | BATTERY CHARGE dead | BATTERY Assault | CHARGE criminal | BATTERY CHARGE crime |
|---|---|---|---|---|---|---|
| BATTERY car | 2,06,000 | 1,94,000 | 1,75,000 | 18,200 | 72,500 | 1,44,000 |
| CHARGE volt | 2,130 | 35,200 | 2,010 | 485 | 1,340 | 382 |
| BATTERY CHARGE dead | 2,890 | 3,990 | 2,890 | 9 | 6 | 8 |
| BATTERY Assault | 64,400 | 5,000 | 64,300 | 1,02,000 | 42,900 | 59,400 |
| CHARGE criminal | 5,040 | 1,580 | 4,830 | 4,430 | 51,300 | 12,500 |

*Continued on next page*

*Table 9 continued*

| | | | | | | |
|---|---|---|---|---|---|---|
| BATTERY CHARGE crime | 9 | 4 | 8 | 104 | 108 | 176 |

**Analysis table for BATTERY CHARGE**

| **Of X CHARGE volt** | **Of X CHARGE prosecute** | **BATTERY car. Of X CHARGE volt** | **BATTERY car. Of X CHARGE prosecute** |
|---|---|---|---|
| 4.7590166*10^10 | 3.586837000*10^9 | 5.425278924*10^16 | 4.08899418*10^15 |
| 1.662845788*10^9 | 5.0160812*10^7 | 5.138193485*10^13 | 1.549969091*10^12 |
| 7.12684756*10^8 | 3.5118308*10^7 | 3.983907786*10^12 | 1.963113417*10^11 |
| 1.34408562*10^10 | 2.776287300*10^9 | 4.81182652*10^14 | 9.939108534*10^13 |
| 1.080182900*10^9 | 7.16111050*10^8 | 3.618612715*10^11 | 2.398972018*10^11 |
| 2.450656*10^6 | 2.889628*10^6 | 1.9605248*10^7 | 2.3117024*10^7 |

According to [9], BATTERY CHARGE obeys marginal selectivity, so its matrix entries in the third table are similar.

4- For BILL SCALE

A- BILL (car)⊙ (of X SCALE (weight))

B- BILL (car)⊙ (of X SCALE (fish)) (Table 10)

**Table 10. Analysis table for BILL SCALE**

**Analysis table for BILL SCALE**

| | **BILL phone** | **BILL pelican** | **SCALE weight** | **SCALE fish** |
|---|---|---|---|---|
| BILL phone | 2,94,000 | 4,320 | 30,900 | 1,14,000 |
| SCALE weight | 2,67,000 | 2,550 | 4,68,000 | 1,71,000 |
| BILL SCALE currency | 1 | 0 | 0 | 0 |
| BILL pelican | 42,300 | 10,500 | 3,840 | 6,120 |
| SCALE fish | 89,900 | 3,530 | 68,900 | 2,22,000 |
| BILL pelican aquatic | 1 | 1 | 0 | 0 |

**Analysis table for BILL SCALE**

| | **BILL phone** | **SCALE weight** | **BILL SCALE currency** | **BILL pelican** | **SCALE fish** | **BILL pelican aquatic** |
|---|---|---|---|---|---|---|
| BILL phone | 2,94,000 | 30,900 | 5,730 | 4,320 | 1,14,000 | 121 |
| SCALE weight | 2,67,000 | 4,68,000 | 1,99,000 | 2,550 | 1,71,000 | 290 |
| BILL SCALE currency | 1 | 0 | 1 | 0 | 0 | 0 |
| BILL pelican | 42,300 | 3,840 | 2,970 | 10,500 | 6,120 | 5,070 |
| SCALE fish | 89,900 | 68,900 | 69,200 | 3,530 | 2,22,000 | 2,710 |
| BILL pelican aquatic | 1 | 0 | 0 | 1 | 0 | 1 |

**Analysis table for BILL SCALE**

| **Of X SCALE weight** | **Of X-SCALE fish** | **BILL phone. Of X SCALE weight** | **BILL phone. Of X-SCALE fish** |
|---|---|---|---|
| 3.14169888*10^10 | 6.41343384*10^10 | 9.236594707*10^15 | 1.885549549*10^16 |
| 2.39065992*10^11 | 1.48443606*10^11 | 6.383061986*10^16 | 3.96344428*10^16 |
| 30900 | 114000 | 30900 | 114000 |
| 3566178000 | 6.901740000*10^9 | 1.508493294*10^14 | 2.91943602*10^14 |
| 5.03324652*10^10 | 7.13361036*10^10 | 4.524888621*10^15 | 6.413115714*10^15 |
| 34740 | 120120 | 34740 | 120120 |

According to [9], BILL SCALE obeys marginal selectivity, so its matrix entries in the third table are similar.

## 4 Conclusion

In this paper, the author has unveiled a novel and accessible method to determine the compositional nature of a given conceptual combination simply by conducting a web search instead of spending a huge amount of money on human subjects or costly non-human subjects like robots. Furthermore, the author has introduced a systematic approach to assessing whether a conceptual combination adheres to marginal selectivity and Bell inequalities, effectively identifying instances of entanglement. This innovation holds the potential to advance our understanding of human behavior through the application of quantum mechanical principles.

By pinpointing word combinations that exhibit entanglement—those that comply with marginal selectivity while defying Bell inequalities—the author has bridged the gap between semantic compositionality and Quantum Theory. The presence of entanglement in non-compositional systems mirrors the intricate behaviors observed in quantum-entangled systems, further blurring the boundaries between language and quantum phenomena.

As elucidated by Fine's theorem, the author has established the necessary and sufficient conditions for the existence of joint probability distributions, encapsulated by Bell inequalities such as Bell/CH and CHSH. Marginal selectivity serves as a critical constraint in this framework, functioning as a litmus test for the nature of conceptual combinations.

The findings can be succinctly summarized:

(a) Failure of marginal selectivity signifies non-compositionality.

(b) The simultaneous presence of marginal selectivity and failure to meet Bell inequalities indicates entanglement.

(c) When both marginal selectivity and Bell inequalities are satisfied, the conceptual combination is deemed compositional.

Thus, hypotheses 1 and 2 have been duly verified. With this groundbreaking research, the author has not only expanded the horizons of linguistic analysis but also opened the door to deeper insights into the interplay between language and quantum phenomena. The implications of this work extend far beyond the scope of this paper, offering a promising avenue for future research and a new perspective on the intricate nature of meaning in language and beyond.

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