INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

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Competing Interests: None

Comparative Study of Solutions of Fractional Order Mixed KdV Burger's Equation

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Abstract

Objective: The aim of this study is to obtain numerical solutions of fractional order mixed KdV Burger's equation using iterative methods. **Methodology:** The study is centered on a time fractional mixed KdV Burger's equation in the Caputo sense. The approximate solutions of this equation are obtained using Adomian decomposition method and Homotopy Perturbation method. Also, a Python code is developed to obtained numerical solutions and simulate graphically. **Finding:** The obtained solutions are compared with the exact solution and presented graphically using Python program for better analysis. **Novelty:** These methods shall be used to obtain approximate solution of such type of non-linear fractional order differential equations.

Keywords: Fractional Differential Equation; Caputo Derivative; ADM; HPM; KdV Burger equation; Python

1 Introduction

In recent decades, many researchers have made considerably good efforts to find solutions of fractional order partial differential equations. It is observed that many real physical phenomenon are represented using fractional order partial differential equations. In last two decades it has been found that fractional order partial differential equations have tremendous applications in different branches of engineering like electrochemistry, material science, plasma physics, thermodynamics etc.[1,2]. Since many fractional order non-linear differential equations are not easy to solve, hence we prefer numerical methods like finite difference method[3], Adomian Decomposition Method[4–6] and Homotopy Perturbation Method[7] which gives approximate solutions.

The KdV equation was initially developed by Korteweg and de Vries in 1895 to describe the phenomena of nonlinear waves. Further, in 1969, Su and Gardner derive Kdv Burger's equation to study different physical properties of wave propagation. Many researchers studied numerical methods like the Homogeneous Balance Method, Special Truncated Expansion Method, Exp-function Method, Variational Iterations Method, Element-Free Galerkin Method and New Decomposition Method to find the exact

solutions of a compound KdVB equation [7–11]. Fractional derivatives offer a simpler yet more comprehensive description of physical phenomena compared to conventional integer-order derivatives, rendering them more effective and versatile in various applications. KdV Burger's equation particularly represents the study of the nature of different types of waves on water surfaces, effects of dispersion, and dissipation in the propagation of waves in elastic tubes filled with liquid and describes many similar processes in dynamical systems and applied Physics [10].

In this paper, we study following type of nonlinear time fractional mixed KdV Burger's equation [12]

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0; \; t \geq 0, \; 0 < \alpha \leq 1. \tag{1}$$

2 Methodology

2.1 Basic definitions

Definition 1. Caputo derivative with fractional order $\alpha > 0$ is given by

$$D^\alpha h(t) = \frac{1}{\Gamma(l-\alpha)} \int_a^t \frac{h^{(l)}(\tau)}{(t-\tau)^{\alpha+1-l}} d\tau \quad (l-1 < Re(\alpha) \leq l, l \in N) \tag{2}$$

where α is positive real number and function h initially takes value as a.

Definition 2. For integer $l \geq \alpha$, Caputo derivative operator with time fractional order $\alpha > 0$ is defined as

$$D_t^\alpha h(y,t) = \frac{\partial^\alpha h(y,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(l-\alpha)} \int_0^t (t-\tau)^{l-\alpha-1} \frac{\partial^l h(y,\tau)}{\partial \tau^l} d\tau & if \quad l-1 < \alpha < l \\ \frac{\partial^l h(y,t)}{\partial t^l} & if \quad \alpha = l \in N \end{cases} \tag{3}$$

Definition 3. The Riemann-Liouville integral operator with fractional order $\alpha \geq 0$ for a function $h \in C_p, p \geq -1$ is defined as

$$J^\alpha h(y) = \frac{1}{\Gamma(\alpha)} \int_0^y (y-t)^{\alpha-1} h(t) dt \quad \alpha > 0, \; y > 0 \tag{4}$$

Following are some properties:

1. For $h \in C_p, \; p \geq -1, \; \alpha, \beta \geq 0, \; \delta \geq -1$
2. $J^0 h(y) = h(y), \; J^\alpha y^\delta = \frac{\Gamma(\delta+1)}{\Gamma(\alpha+\delta+1)} y^{(\alpha+\delta)}$
3. $J^\alpha J^\beta h(y) = J^{\alpha+\beta} h(y), J^\alpha J^\beta h(y) = J^\beta J^\alpha h(y)$
4. If $l-1 < \alpha \leq l, \; l \in N$ and $h \in C_p^l, \; p \geq -1$ then
5. $D^\alpha J^\alpha h(y) = h(y)$
6. $J^\alpha D^\alpha h(y) = h(y) - \sum_{q=0}^{l-1} h^q(0^+) \frac{y^q}{q!}, y > 0$

2.2 The Fractional Homotopy Perturbation Method

The standard form of Equation (1) in an operator form can be written as

$$D_t^\alpha u + N_x u + L_x u = 0, \; 0 < \alpha \leq 1, \; t > 0, \tag{5}$$

where, $D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$, $N_x = \frac{\partial}{\partial x}$ and $L_x u = -a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3}$.

According to the method, we use simple homotopy as follows.

$$(1-p)D_t^\alpha u + p(D_t^\alpha u + N_x u + L_x u) = 0, \; 0 \leq p \leq 1, \tag{6}$$

where p is called an embedding parameter.

If we take $p = 0$, then we get $D_t^\alpha u = 0$, which is easily solvable.

If we take $p = 1$, it becomes equation in operator form Equation (5). This type of conversion is called deformation with D_t^α and $D_t^\alpha u + N_x u + L_x u$ as homotopic.

The parameter p is used to elaborate the solution with the form [5]

$$u = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + p^4 u_4 + ... \tag{7}$$

Now, for $p = 1$ we get the approximate solution as

$$u = u_0 + u_1 + u_2 + u_3 + u_4 + ... \tag{8}$$

Substituting the above equations in homotopy and finding terms having equal powers of p, we get a series of equations easily with the help of Python software.

For simplicity, we write some of the equations as follows:

$D_t^\alpha u = 0$

$D_t^\alpha u_1 + u_0 u_{0x} + L_x u_0 = 0$

$D_t^\alpha u_2 + u_0 u_{1x} + u_1 u_{0x} + L_x u_1 = 0$

$D_t^\alpha u_3 + u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x} + L_x u_2 = 0$

$\vdots$

Now, on both sides of the above equations, we will apply the operator J^α to obtain

$u_0 = h_0(x)$

$u_1 = h_1(x) - h^1(x) \frac{t^\alpha}{\Gamma(\alpha+1)}$

$u_2 = h_2(x) - h^2(x) \frac{t^\alpha}{\Gamma(\alpha+1)} + h^3(x) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$

$u_3 = h_3(x) - h^4(x) \frac{t^\alpha}{\Gamma(\alpha+1)} + h^5(x) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - h^6(x) \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$

where,

$h_q(x) = u_q(x,0), \quad \sum_{q=0}^{\infty} h_q(x) = h(x).$

$h^1 = h_0 h_{0x} - a h_{0xx} + b h_{0xxx}$

$h^2 = [h_0 h_{1x} + h_1 h_{0x}] - a h_{1xx} + b h_{1xxx}$

$h^3 = [h_0 h_x^1 + h^1 h_{0x}] - a h_{xx}^1 + b h_{xxx}^1$

$h^4 = [h_0 h_{2x} + h_2 h_{0x} + h_1 h_{1x}] - a h_{2xx} + b h_{2xxx}$

$h^5 = [h_0 h_x^2 + h^2 h_{0x} + h_1 h_x^1 + h^1 h_{1x}] - a h_{xx}^2 + b h_{xxx}^2$

$h^6 = [h_0 h_x^3 + h^3 h_{0x} + h^1 h_x^1 \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2}] - a h_{xx}^3 + b h_{xxx}^3$

As per the survey of different perturbation methods, it is observed that the use of approximate solutions having low order gives higher accuracy.

2.3 The Fractional Adomian Decomposition Method

The standard form of Equation (1) in an operator form is given by

$$D_t^\alpha u + N_x u + L_x u = 0, \quad 0 < \alpha \le 1, t > 0 \tag{9}$$

where, $D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$, $N_x = \frac{\partial}{\partial x}$ and $L_x u = -a \frac{\partial^2 u}{\partial x^2} + b \frac{\partial^3 u}{\partial x^3}$

After applying the operator J^α on both sides of the above equations, we get

$$u(x,t) = h(x) - J^\alpha \phi(u) - J^\alpha L_x u, \ where \ \phi(u) = u u_x \tag{10}$$

In decomposition method, we assume that the solution of $u(x,t)$ is given in the series form by

$u(x,t) = \sum_{q=0}^{\infty} u_q(x,t)$

where the term $\phi(u)$ is nonlinear and decomposes into the infinite series of polynomial as follows

$\phi(u) = \sum_{q=0}^{\infty} A_q(u_0, u_1, ..., u_q)$

where A_q represents Adomian Polynomials and are calculated with the formula

$$A_q = \frac{1}{q!} \left[\frac{d^q}{d\lambda^q} \phi \left(\sum_{q=0}^{\infty} \lambda^q u_q \right) \right]_{\lambda=0} = \frac{1}{q!} \left[\frac{d^q}{d\lambda^q} \left(\left(\sum_{q=0}^{\infty} \lambda^q u_q(x,t) \right) \left(\sum_{q=0}^{\infty} \lambda^q u_q(x,t) \right)_x \right) \right] \tag{11}$$

After substituting all the series, Equation (10) becomes

$$\sum_{q=0}^{\infty} u_q(x,t) = h(x) - J^\alpha \left(\sum_{q=0}^{\infty} A_q \right) - J^\alpha L_x \left(\sum_{q=0}^{\infty} u_q(x,t) \right)$$

The recurrence relation is developed as follows

$$u_0 = u(x,0) = h(x)$$

$$u_{q+1} = -J^\alpha (A_q) - J^\alpha L_x (u_q), \quad q = 0,1,2,\dots \tag{12}$$

Hence, the approximate solution will take the form

$$u = u_0 + u_1 + u_2 + u_3 + \dots \tag{13}$$

2.4 Numerical Example

We consider mixed KdV Burger's equation with time-fractional order as follows

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0 \qquad\qquad 0 < \alpha \leq 1 \tag{14}$$

with the initial condition,

$$u(x,0) = \frac{3}{25}\frac{a^2}{b} \operatorname{sech}\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right] \tag{15}$$

for $\alpha = 1$, we have the exact solution as follows [13]

$$u(x,t) = \frac{3}{25}\frac{a^2}{b} \operatorname{sech}\left[\frac{a}{10b}\left(x - \frac{6}{25}\frac{a^2}{b}t\right)\right]^2 + \frac{6}{25}\frac{a^2}{b}\left(1 - tanh\left[\frac{a}{10b}\left(x - \frac{6}{25}\frac{a^2}{b}t\right)\right]\right)$$

2.4.1 Solution by Homotopy Perturbation Method

Here, for simplicity, the method is illustrated with the initial four terms, and we can write

$f(x) = f_0(x) + f_1(x) + f_2(x) + f_3(x).$

Assume, initial condition $u(x,0) = u_0$

Then we will get

$u_0 = f_0(x) = \frac{3}{25}\frac{a^2}{b}\operatorname{sech}\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right]$

$u_1 = -g_1(x)\frac{t^\alpha}{\Gamma(\alpha+1)}$

$u_2 = g_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$

$u_3 = -g_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}.$

where,

$g_1 = f_0 f_{0x} - a f_{0xx} + b f_{0xxx}$

$g_2 = [g_1 f_{0x} + f_0 g_{1x}] - a g_{1xx} + b g_{1xxx}$

$g_3 = [g_2 f_{0x} + g_1 g_{1x}\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2} + f_0 g_{2xx}] - a g_{2xx} + b g_{2xxx}$

We obtain,

$u_0 = \frac{3}{25}\frac{a^2}{b}\operatorname{sech}\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right]$

$u_1 = (18(t^\alpha \cosh(\frac{1}{10} x) + t^\alpha \sinh(\frac{1}{10} x)))/(3125 \cosh(\frac{1}{10} x)^3 \Gamma(\alpha+1))$

$u_2 = \dfrac{54\left(2t^{2\alpha}cosh(\frac{1}{10} x)^2 + 2t^{2\alpha}cosh(\frac{1}{10} x)sinh(\frac{1}{10} x) - 3t^{2\alpha}\right)}{390625\,cosh(\frac{1}{10} x)^4 \Gamma(2\alpha+1)}$

and so on.

Thus, the approximate solution is of the form

$u(x,t) = u_0 + u_1 + u_2 + \dots$

$$\therefore u(x,t) = \frac{3}{25}\frac{a^2}{b}\operatorname{sech}\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right] + \frac{18\left(t^\alpha cosh\left(\frac{1}{10}x\right) + t^\alpha sinh\left(\frac{1}{10}x\right)\right)}{3125\,cosh\left(\frac{1}{10}x\right)^3 \Gamma(\alpha+1)}$$

$$+ \frac{54\left(2t^{2\alpha}cosh\left(\frac{1}{10}x\right)^2 + 2t^{2\alpha}cosh\left(\frac{1}{10}x\right)sinh\left(\frac{1}{10}x\right) - 3t^{2\alpha}\right)}{390625\,cosh\left(\frac{1}{10}x\right)^4 \Gamma(2\alpha+1)} + \dots \tag{16}$$

2.4.2 Solution by Adomian Decomposition Method

According to the method, we have

$u_0 = u(x,0) = v_0$

$v_1 = -J^\alpha \left(A_0 - a(u_0)_{xx} + b(u_0)_{xxx} \right)$

$v_2 = -J^\alpha \left(A_1 - a(u_1)_{xx} + b(u_1)_{xxx} \right)$

$v_3 = -J^\alpha \left(A_2 - a(u_2)_{xx} + b(u_2)_{xxx} \right)$

$\vdots$

We obtain

$v_0 = \frac{3}{25}\frac{a^2}{b} sech\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right]$

$v_1 = \frac{18\left(t^\alpha cosh(\frac{1}{10}x) + t^\alpha sinh(\frac{1}{10}x)\right)}{3125\, cosh(\frac{1}{10}x)^3 \Gamma(\alpha+1)}$

$v_2 = \frac{54\left(2\,t^{2\alpha}cosh(\frac{1}{10}x)^2 + 2\,t^{2\alpha}cosh(\frac{1}{10}x)sinh(\frac{1}{10}x) - 3\,t^{2\alpha}\right)}{390625\, cosh(\frac{1}{10}x)^4 \Gamma(2\alpha+1)}$

$\vdots$

Thus the approximate solution will take the form

$v(x,t) = v_0 + v_1 + v_2 + ...$

$$\therefore v(x,t) = \frac{3}{25}\frac{a^2}{b} sech\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - tanh\left(\frac{a}{10b}x\right)\right] + \frac{18\left(t^\alpha cosh\left(\frac{1}{10}x\right) + t^\alpha sinh\left(\frac{1}{10}x\right)\right)}{3125\, cosh\left(\frac{1}{10}x\right)^3 \Gamma(\alpha+1)}$$

$$+ \frac{54\left(2\,t^{2\alpha}cosh\left(\frac{1}{10}x\right)^2 + 2\,t^{2\alpha}cosh\left(\frac{1}{10}x\right)sinh\left(\frac{1}{10}x\right) - 3\,t^{2\alpha}\right)}{390625\, cosh\left(\frac{1}{10}x\right)^4 \Gamma(2\alpha+1)} + ... \tag{17}$$

We observed that, from Equations (16) and (17), the approximate solutions obtained by ADM and HPM are precisely same.

3 Result and Discussion

In these section we present numerical solutions of nonlinear time fractioned mixed KdV Burger's equation given in Equations (14) and (15) obtained by Adomian Decomposition Method and Homotopy Perturbation Method and it is compared with the exact solution [12].

In the following Table 1, we validate these solutions and find that they agree with the exact solution, confirming the accuracy of both the Adomian Decomposition Method and the Homotopy Perturbation Method.

Table 1. Comparison of approximate solution with exact solution for $\alpha = 1$, $t = 10$

x	Exact solution	Approximate solution	Absolute error
-5	0.463496	0.463450	0.000046
-4	0.457283	0.457167	0.000115
−3	0.449153	0.448962	0.000191
−2	0.438742	0.438488	0.000254
−1	0.425726	0.425436	0.000289
0	0.409864	0.409582	0.000282
1	0.391060	0.390832	0.000229
2	0.369403	0.369264	0.000139
3	0.345186	0.345156	0.000031
4	0.318904	0.318978	0.000074
5	0.291209	0.291364	0.000154

Hence, we conclude that both the iterative methods (ADM and HPM) are equally works for highly nonlinear fractional order partial differential Equations (14) and (15). Furthermore, we plot the obtained solution for $\alpha = 1$, 0.9, $0 < t < 10$, $-60 < x <$

60 and presented graphically shown in the Figure 1. It is observed that the obtained solution is a kink type of traveling wave solution.
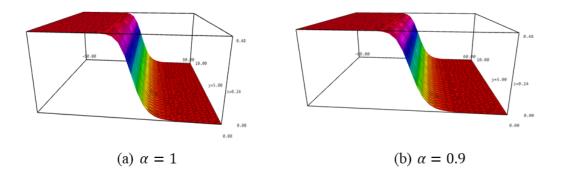

(a) $\alpha = 1$ (b) $\alpha = 0.9$

Fig 1. Graphical representation of kink type travelling wave for $-60 \le x \le 60, \;\; 0 \le t \le 1, \;\; k = 0.1$

Furthermore, Figure 2 shows the graphical representation of solutions for the range $10 \le t \le 50$. From Figure 2**(a)**-**(c)**, we observe the kink type travelling wave solution for $0 < \alpha \le 1, \; t \le 20$. Also, in Figure 2 **(d)**-**(f)**, we observe the variation in wave patterns for $0.9 \le \alpha \le 1, \; 30 \le t \le 50$.

(a) $t = 10$ (b) $t = 15$ (c) $t = 20$
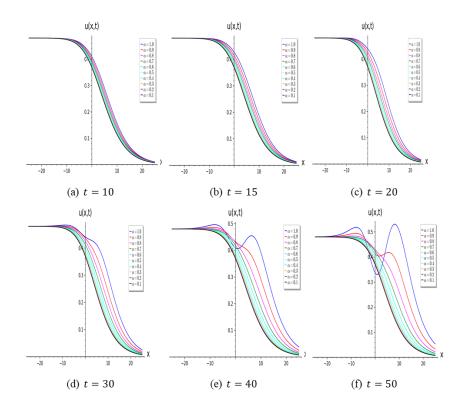
(d) $t = 30$ (e) $t = 40$ (f) $t = 50$

Fig 2. Graphical representation of solutions for different values of t

The graphical representation of solutions for the range $70 \le t \le 110$ are shown in Figure 3 and it is observed that the behavior of the traveling waves follows a specific pattern as time increases.
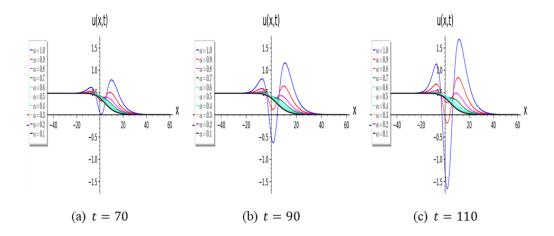

(a) $t = 70$ (b) $t = 90$ (c) $t = 110$

Fig 3. Behaviour of solution for increasing time t

4 Conclusion

This study has successfully applied the HPM and ADM to solve the time-fractional mixed KdV Burger's equation. The comparative study shows that the approximate solutions obtained by HPM and ADM are precisely the same for this modeled problem. The obtained results and the exact results of the proposed problem are closely related. The convergence related to integer order results gets confirmed through the graphical representation. 3D graphical representation remains the same for different values of t and α. For short memory period $0 < t \le 20$, we observe the fast convergence, and the obtained solutions are closer to the exact solution. For the memory period $30 \le t \le 50$, variation in the wave patterns are observed for $0.9 \le \alpha \le 1$. For long memory period $t > 70$, we observe specific pattern formation in the behavior of traveling wave solutions for $\alpha = 1, \ 0.9, \ 0.8, \ 0.7$. From the graphical solution, we observe that time fractional KdV Burger's equation gives the kink-type solutions.

References

1) Nieto JJ, Rodríguez-López R, editors. Fractional Differential Equations. MDPI - Multidisciplinary Digital Publishing Institute. 2019. Available from: https://doi.org/10.3390/books978-3-03921-733-5.
2) Balachandran K. Fractional Differential Equations. In: An Introduction to Fractional Differential Equations. Industrial and Applied Mathematics;Singapore. Springer. 2023;p. 51–85. Available from: https://doi.org/10.1007/978-981-99-6080-4.
3) Takale K, Kharde U, Takale G. Fractional Order Mathematical Model to Investigate Topical Drug Diffusion in Human Skin. *Indian Journal of Science and Technology*. 2023;16(48):4657–4666. Available from: https://doi.org/10.17485/IJST/v16i48.2113.
4) Kulkarni S, Takale K, Shaikh A. Application of adomian decomposition method to solve the fractional mathematical model of corona virus. *Journal of Mathematical and Computational Science*. 2020;10(5):1327–1339. Available from: https://scik.org/index.php/jmcs/article/view/4624.
5) Li W, Pang Y. Application of Adomian decomposition method to nonlinear systems. *Advances in Difference Equations*. 2020;2020:1–17. Available from: https://doi.org/10.1186/s13662-020-2529-y.
6) Sonawane J, Sonatakke B, Takale K. Numerical Solution of Subdiffusion Bioheat equation with Single Phase lag. *Indian Journal of Science and Technology*. 2024;17(11):955–966. Available from: https://doi.org/10.17485/IJST/v17i11.3220.
7) Zada L, Aziz I. The numerical solution of fractional Korteweg-de Vries and Burgers' equations via Haar wavelet. *Mathematical Methods in the Applied Sciences*. 2021;44(13):10564–10577. Available from: https://doi.org/10.1002/mma.7430.
8) Wei L, Wei X, Tang B. Numerical analysis of variable-order fractional KdV-Burgers-Kuramoto equation. *Electronic Research Archive*. 2022;30(4):1263–1281. Available from: https://doi.org/10.3934/era.2022066.

9) Alqahtani Z, Hagag A. A new semi-analytical solution of compound KdV-Burgers equation of fractional order. *Revista Internacional de Metodos Numericos para Calculo y Diseno en Ingenieria*. 2023;39(4):1–15. Available from: https://www.scipedia.com/public/Alqahtani_Hagag_2023b.

10) Rida SZ, Hussien HS. Efficient Computational Approach for Generalized Fractional KdV-Burgers Equation. *International Journal of Applied and Computational Mathematics*. 2020;6(6). Available from: https://doi.org/10.1007/s40819-020-00915-1.

11) Alderremy AA, Aly S, Fayyaz R, Khan A, Shah R, Wyal N. The Analysis of Fractional-Order Nonlinear Systems of Third Order KdV and Burgers Equations via a Novel Transform. *Complexity*. 2022;2022:1–24. Available from: https://doi.org/10.1155/2022/4935809.

12) Khan A, Akram T, Khan A, Ahmad S, Nonlaopon K. Investigation of time fractional nonlinear KdV-Burgers equation under fractional operators with nonsingular kernels. *AIMS Mathematics*. 2023;8(1):1251–1268. Available from: https://doi.org/10.3934/math.2023063.

13) Ganie AH, Mofarreh F, Khan A. On new computations of the time-fractional nonlinear KdV-Burgers equation with exponential memory. *Physica Scripta*. 2024;99(4):1–15. Available from: https://iopscience.iop.org/article/10.1088/1402-4896/ad2e60.