INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Semimedial Near Rings

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OPEN ACCESS

Received: 26-04-2024

Accepted: 20-05-2024

Published: 10-06-2024

Citation: Dhivya C , Radha D (2024) Semimedial Near Rings. Indian Journal of Science and Technology 17(24): 2478-2481. https://doi.org/10.17485/IJST/v17i24.1403

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: To define the term 'semimediality' in the near ring and to demonstrate its presence with an example and to employ several attributes in near rings to analyse its features. **Methods:** Semimedial near rings are examined in terms of their characteristics using commutativity, distributivity and regular property. The distinction is observed using the concept of homomorphism. **Findings:** Every near ring which is weak commutative is proved to be semimedial and the converse exists with some additional axioms. The semimedial near ring's anti-homomorphic image is also observed. Additionally, the correlation between the semimedial near ring and the reduced property is identified. **Novelty:** This study provides a novel method to medial near ring, which is the semimedial near ring.

Keywords: Identity; Left Self Distributive; Nilpotent; Regular; Weak Commutative

1 Introduction

Rings are generalized into near rings. Generalizing several ring theories into near rings is a natural phenomenon. Ignoring the commutativity of addition and one distributive property in the ring leads it to become a near ring. When the left distributive law does not exist, we acquire a right near ring. Gunter Pilz's [1] "Near Rings" is a unique compilation of research results on near rings. Numerous authors have contributed to different aspects of near rings [2–5].

$xyzt = xzyt$ is an identity that has been developed in the field of quasi groups. These near rings were referred to as medial near rings in 1987 by Pellegrini Manara S [6]. Subsequently, mediality in semigroups, seminear rings, fuzzy algebra and other related fields has been investigated by numerous researchers [7]. In 2009, Fitore Abdullahu and Abdullah Zejnullahu [8] established the notion of semimediality in semigroups. This motivates the introduction of semimediality to the near ring's domain.

In this paper, $\aleph^1$ is termed as a right near ring having two or more elements and, in the group, $(\aleph^1, +)$, '0' corresponds to the identity element and $u.v$ is expressed as uv for any two elements u, v in $\aleph^1$.

2 Methodology

Below are some preliminary definitions that are essential to the following outcomes.

Notation 2.1 [1]

η indicates the class of all near rings.

Definition 2.2 [1]

Let $\aleph^1 \in \eta$. A normal subgroup $\mathfrak{I}$ of $(\aleph^1, +)$ is called an **ideal** of $\aleph^1 \left(\mathfrak{I} \trianglelefteq \aleph^1\right)$ if

i) $\mathfrak{I}\aleph^1 \subseteq \mathfrak{I}$.

ii) $\mathfrak{n}\left(\mathfrak{n}^{'} + i\right) - \mathfrak{n}\mathfrak{n}^{'} \in \mathfrak{I}$ for all $\mathfrak{n}, \mathfrak{n}^{'} \in \aleph^1$ and for all $i \in \mathfrak{I}$.

Definition 2.3 [1]

Let $\mathfrak{I} \trianglelefteq \aleph^1$. Then we know that $\aleph^1/\mathfrak{I} = \{\mathfrak{n} + \mathfrak{I} | \mathfrak{n} \in \aleph^1\}$ is called a **factor near ring (or) quotient near ring**.

Definition 2.4 [9]

Let $\aleph^1$ and $\aleph_1$ be near rings. A mapping $g : \aleph^1 \longrightarrow \aleph_1$ is called a (**near ring homomorphism**) if

i) $g(r+s) = g(r) + g(s)$ and

ii) $g(rs) = g(r)\, g(s)$ for all $r, s \in \aleph^1$.

Definition 2.5 [10]

Let $\aleph^1$ and $\aleph_1$ be two near rings. A function $g : \aleph^1 \longrightarrow \aleph_1$ is called an **anti-homomorphism** if the following conditions are satisfied:

(i) $g(s+r) = g(r) + g(s)$

(ii) $g(sr) = g(r)g(s)$ for all $r, s \in \aleph^1$.

Definition 2.6 [11]

$\aleph^1$ is stated as a **weak commutative near ring** if it has the property that $rst = rts$ for every $r, s, t \in \aleph^1$.

Definition 2.7 [11]

A near ring $\aleph^1$ is considered to be **reduced** if $\aleph^1$ has no nonzero nilpotent elements.

Definition 2.8 [12]

An element $\mathfrak{n} \in \aleph^1$ is termed as a **nilpotent element** if there is a positive integer $\mathfrak{a}$ such that $\mathfrak{n}^{\mathfrak{a}} = 0$.

Definition 2.9 [12]

$\aleph^1$ is defined to be **regular** if for every $u \in \aleph^1$ there exists $v \in \aleph^1$ such that $u = uvu$.

Definition 2.10 [13]

i) $\aleph^1$ is considered to be **left self-distributive near ring** if it has the property that $rst = rsrt$ for all $r, s, t \in \aleph^1$.

ii) $\aleph^1$ is considered to be **right self-distributive near ring** if it has the property that $rst = rtst$ for all $r, s, t \in \aleph^1$.

iii) Near rings that satisfy the conditions of being both left self-distributive and right self-distributive are referred to as **self-distributive**.

3 Results and Discussion

Definition 3.1

$\aleph^1$ is defined to be a **left semimedial near ring** if it fulfils the condition that $x^2yz = xyxz$ for all $x, y, z \in \aleph^1$.

Definition 3.2

$\aleph^1$ is defined to be a **right semimedial near ring** if it fulfils the condition that $zyx^2 = zxyx$ for all $x, y, z \in \aleph^1$.

Definition 3.3

$\aleph^1$ that satisfy the conditions of being both left semimedial and right semimedial is referred to as **semimedial near ring**.

Proposition 3.4

A Semimedial near ring's homomorphic image is also a semimedial near ring.

Proof:

Obvious.

Corollary 3.5

The quotient near ring $\aleph^1/\mathfrak{I}$ is a semimedial near ring whenever $\aleph^1$ is semimedial near ring.

Proof:

Let $g : \aleph^1 \longrightarrow \aleph^1/\mathfrak{I}$ be the canonical near ring homomorphism given by $g(y) = \mathfrak{I} + y$. Clearly g is onto. The proof is followed by Proposition 3.4.

Proposition 3.6

Anti-Homomorphic image of a semimedial near ring is also a semimedial near ring.

Proof:

Let $g:\aleph^1 \longrightarrow \aleph_1$ be an onto, anti-homomorphism where $\aleph^1$ is a semimedial near ring. Let $p^{'}, q^{'}, r^{'} \in \aleph_1$. Now $(p^{'})^2 q^{'} r^{'} = (g(p))^2 g(q)g(r) = g(p^2)g(q)g(r) = g(rqp^2)$ (Since g is an anti-homomorphism) $= g(rpqp)$ (Since $\aleph^1$ is semimedial near ring) $= g(p)g(q)g(p)g(r)$ (Since g is an anti-homomorphism) $= p^{'}q^{'}p^{'}r^{'}$. That is $(p^{'})^2 q^{'} r^{'} = p^{'}q^{'}p^{'}r^{'}$. Hence, $\aleph_1$ is a left semimedial near ring.

Similarly, $\aleph_1$ is a right semimedial near ring. Therefore, $\aleph_1$ is a semimedial near ring.

Proposition 3.7

Semimediality prevails for every commutative near rings.

Proof:

Let $\aleph^1$ be a commutative near ring. Then for all $p, q \in \aleph^1$, $pq = qp$. Therefore, $p(pq) = p(qp)$. This implies $p^2q = pqp$. Let $r \in \aleph^1$. Then $(p^2q)r = (pqp)r$. This implies $p^2qr = pqpr$. Hence, $\aleph^1$ is left semimedial.

Again, by the definition of commutativity we have, $qp = pq$ for all $p, q \in \aleph^1$. Let $r \in \aleph^1$. Then $r(qp) = r(pq)$. Therefore, $r(qp)p = r(pq)p$. This implies $rqp^2 = rpqp$. Thus, $\aleph^1$ is right semimedial. Hence, $\aleph^1$ is semimedial.

Proposition 3.8

Every weak commutative near ring is semimedial.

Proof:

Let $\aleph^1$ be a weak commutative near ring and let $p, q, r \in \aleph^1$. Now $pqpr = (pqp)r = (ppq)r = p^2qr$. Hence, $\aleph^1$ is left semimedial.

Similarly, it holds right semimediality. Therefore, $\aleph^1$ is semimedial.

Remark 3.9

Above Proposition 3.8 is not true in its contrary, as the case below demonstrates.

The Klein's four group is represented by $\aleph^1 = \{0, a, b, c\}$. In $\aleph^1$, define multiplication as follows:

Table 1. Multiplication Table

·	0	a	b	c
0	0	0	0	0
a	0	a	b	a
b	0	0	0	0
c	0	a	b	a

Fig 1.

Then $(\aleph^1, +, .)$ is a near ring (Pilz[1], scheme 11 of p.408), which is semimedial but not weak commutative (Since $abc \neq acb$).

Proposition 3.10

Every semimedial near ring which is commutative and has a left identity is also weakly commutative.

Proof:

Let $\aleph^1$ be a semimedial near ring and let $\mathfrak{e} \in \aleph^1$ be the left identity. Let $p, q, r \in \aleph^1$. Then $pqr = \mathfrak{e}pqr = (p^{-1}p)(pqr) = p^{-1}(ppqr) = p^{-1}(p^2qr) = p^{-1}(p^2rq)$ [Since $\aleph^1$ is commutative] $= p^{-1}(pprq) = (p^{-1}p)(prq) = \mathfrak{e}(prq) = prq$. That is $pqr = prq$. Hence, $\aleph$ is weak commutative.

Proposition 3.11

A Semimedial regular near ring which is non-trivial is reduced.

Proof:

Let $\aleph^1$ be a semimedial near ring. If $m \in \aleph^1$ is a nilpotent, then $m^{\mathfrak{a}} = 0$ for some positive integer $\mathfrak{a}$. Since $\aleph^1$ is regular, $m = mzm$ for some $z \in \aleph^1$. Now $mz = (mzm)z = (mz)(mz) = (mz)^2 = [(mzm)zm]z = (mz)(mz)(mz) = (mz)^3 = \cdots = (mz)^{\mathfrak{a}}$. That is $mz = (mz)^{\mathfrak{a}}$. Since $\aleph^1$ is semimedial, $(mz)^{\mathfrak{a}} = m^{\mathfrak{a}}z^{\mathfrak{a}} = 0z^{\mathfrak{a}} = 0$. That is $(mz)^{\mathfrak{a}} = 0$. This implies $mz = 0$. Then $m = mzm = 0m = 0$. This implies $m = 0$. Thus, $\aleph^1$ has no nonzero nilpotent elements. Hence, $\aleph^1$ is reduced.

Proposition 3.12

A left self-distributive near ring $\aleph^1$ is left semimedial if and only if $\aleph^1$ fulfills the equation $pq^2 = p^2q^2$ for all $p, q \in \aleph^1$.

Proof:

Let $\aleph^1$ be a left self distributive near ring. Then $pqr = pqpr$ for all $p, q, r \in \aleph^1$. Since $\aleph^1$ is left semimedial, then for all $p, q, r \in \aleph^1$, $p^2qr = pqpr$. Therefore, $pqr = p^2qr$. Replace r by q. Then $pqq = p^2qq$. This implies $pq^2 = p^2q^2$.

Conversely, we have, $pq^2 = p^2q^2$ for all $p, q \in \aleph^1$. Using in left self distributivity definition, proof follows.

4 Conclusion

Semimedial near ring is the subject of this investigation. We provide this concept based on the substructures of mediality and examine the commutativity on semimedial near ring for a near ring $\aleph^1$. With the help of this definition, we have demonstrated a few findings that shed light on the ideas of semimediality in near rings and advance the development of the structural theorem. Future developments could lead to the application of these similar semimedial structures in cryptography as well as seminear rings. It is intended that the concepts and discoveries offered in this paper would inspire and enhance subsequent studies on semimedial near rings and results in their progress in real-world applications.

References

1) Pilz G. Near-rings : the theory and its applications. North-Holland mathematics studies : a series of tutorials in pure and applied mathematics;Amsterdam, Netherlands. North Holland. 1983. Available from: https://search.worldcat.org/title/near-rings-the-theory-and-its-applications/oclc/632905791.
2) Kuznetsov YA. Introduction to Dynamical Systems. In: Elements of Applied Bifurcation Theory;vol. 112 of Applied Mathematical Sciences. Springer, Cham. 2023;p. 1–46. Available from: https://doi.org/10.1007/978-3-031-22007-4_1.
3) Shamim MS, Ali SS. An Analysis on Derivation in Near-Rings and its Generalizations. *International Journal of Advance Research and Innovative Ideas in Education*. 2022;8(4):1789–1798. Available from: http://ijariie.com/AdminUploadPdf/AN_ANALYSIS_ON_DERIVATION_IN_NEAR_RINGS_AND_ITS_GENERALIZATIONS_ijariie17971.pdf.
4) Elavarasan B, Muhiuddin G, Porselvi K, Jun YB. Hybrid structures applied to ideals in near-rings. *Complex & Intelligent Systems*. 2021;7(3):1489–1498. Available from: https://dx.doi.org/10.1007/s40747-021-00271-7.
5) Kandasamy WBV, Kandasamy I, Smarandache F. Semi-Idempotents in Neutrosophic Rings. *Mathematics* . 2019;7(6):1–7. Available from: https://doi.org/10.3390/math7060507.
6) Manara SP. On Medial Near-Rings. *North-Holland Mathematics Studies*. 1987;137:199–209. Available from: https://doi.org/10.1016/S0304-0208(08)72303-X.
7) Atagun AO, Kamaci H, Tastekin İ, Sezgin A. P-properties in Near-rings. *Journal of Mathematical and Fundamental Sciences*. 2019;51(2):152–167. Available from: https://doi.org/10.5614/j.math.fund.sci.2019.51.2.5.
8) Abdullahu F, Zejnullahu A. Notes on semimedial semigroups. *Commentationes Mathematicae Universitatis Carolinae*. 2009;50(3):321–324. Available from: https://eudml.org/doc/33317.
9) Radha D, Dhivya C. On Some Characterizations of CM' (2,2) Near Ring. *Science, Technology and Development*. 2021;10(11):104–111. Available from: https://drive.google.com/file/d/1B-yOsDDa9DPKC462aQf_dbM6JuZTnqRn/view.
10) Rao KC, Venkataraman S. Anti-Homomorphisms in Near-Rings. *Journal of Institute of Mathematics & Computer Sciences*. 2008;21(2):83–88. Available from: https://www.researchgate.net/profile/Swaminathan-Venkataraman/publication/267430650_Anti-homomorphisms_in_near-rings/links/5a1b9652aca272df080f1e7d/Anti-homomorphisms-in-near-rings.pdf.
11) Dhivya C, Radha D. The Role of Boolean and Pseudo Commutativity in Near Rings. *International Journal of Mathematics Trends and Technology*. 2022;68(9):34–38. Available from: https://dx.doi.org/10.14445/22315373/ijmtt-v68i9p506.
12) Soundararajan D. Drugs of the Future. In: American Heart Association (AHA) Scientific Sessions - 2022 Annual Meeting;vol. 48, No. 2. Portico. 2023;p. 141–147. Available from: https://dx.doi.org/10.1358/dof.2023.48.2.3521848.
13) Birenmeier G, Heatherly H. Left self distributive near-rings. *Journal of the Australian Mathematical Society Series A Pure Mathematics and Statistics*. 1990;49(2):273–296. Available from: https://dx.doi.org/10.1017/s144678870003055x.