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RESEARCH ARTICLE

Total Regular Domination on a Litact Graph

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Abstract

Objectives: This work's primary goal is to ascertain the new domination parameter known as total regular domination on a litact graph. **Methods**: Several results on total regular domination on a litact graph are obtained in the present study, both in terms of different G parameters (vertices, edges, diameter, maximum degree, and many more) and domination parameters (connected domination, total domination, edge domination, and many more). A few fundamental definitions, findings, and the ideas of several dominations parameters have been used in this. **Findings**: The current study aims to determine some relations between total regular domination on a litact graph and graph G's different parameters, and domination parameters. Furthermore, results comparable to Nordhaus-Gaddum's were also established. **Novelty**: By merging the two distinct parameters total domination and regular domination, a new domination parameter known as total regular domination has been introduced. Using total regular domination on a litact graph, certain results were produced in this study, with respect to various parameters and domination parameters of the original graph G, but not into itself. This type of task has never been completed before.

Keywords: Graph; Litact Graph; Total Domination Number; Regular Domination Number; Total Regular Domination Number

1 Introduction

The current study exclusively considers simple, finite, non-trivial, connected and undirected graphs. Since Cockayne first proposed the idea of total domination in graph theory, other researchers have thoroughly examined it. It was E. Sampath Kumar who initially introduced the idea of regular domination number. The goal of this study is to combine two domination parameters, total domination and regular domination, to provide a new parameter on a litact graph termed the total regular domination number on a litact graph.

It is evident that $\gamma(G) \leq \gamma_t(G)$, if G holds a degree of one or more. Additionally, G possesses a total dominating set. Since all of the graphs taken into consideration in this study are presumed to contain no isolated vertices, unless otherwise indicated, the 'Total domination number' is defined.

This study is to introduce a new parameter 'Total regular domination number of a litact graph', $\gamma_{tr}(m(G))$, to derive few results of $\gamma_{tr}(m(G))$, regarding several domination parameters of G, like edge domination and total domination, and so on of G, as well as various parameters of G, like its vertices, edges, and so on.

The authors of the current study also cited few articles, Renuka Lakshmi et. al[1] for the study of "Inverse litact domination in graphs", Muddebihal et. al[2] for the study of "Co-regular Total Domination in Graphs", Renuka Lakshmi et. al[3] for the study of "Total Strong Litact Domination in Graphs" and Swati Mallinath et. al[4] for the study of "Split Domination in Lict Subdivision Graph of a Graph".

In reference to the graph's order, Nordhaus and Gaddum provided one result in their article as product and sum of G's and $\overline{G}$'s chromatic number. Subsequently, other writers have put forth many results on the product and sum of numerous other parameters of G, such as the domination, connected domination, total domination numbers and many more.

2 Methodology

We obtained the following results by using the fundamental definitions and a few basic results that are mentioned below.

Definition 2.1: The points in a **'litact graph'** $m(G)$ are made up of the edges and cut vertices of G. If edges and cut vertices are 'incident' or 'adjacent' in G, then the two vertices in $m(G)$ are 'adjacent'.

Definition 2.2: If each point in $V - D$ is connected to a point in D, then D is considered **'dominating set'** of $G(V,\ E)$. A **'domination number of** G**'**, represented by $\gamma(G)$, is 'minimal cardinality' of such set D.

Definition 2.3: A 'dominating set' $D \subseteq V(G)$ is regarded as a **'total dominating set'** in G, if there is no point of degree zero in $\langle D \rangle$. A **'total domination number of G'**, represented by $\gamma_t(G)$, is 'smallest cardinality' of such set D. If G is a 'nontrivial graph', then $\gamma_t(G) \geq 2$.

Definition 2.4: If each point in G holds equal degree, then G is considered **'regular'**. If $\langle D \rangle$ is regular, then a 'dominating set D' is considered a **'regular dominating set'**. **'Regular domination number of** G**'**, represented by $\gamma_r(G)$, is 'minimal cardinality' of such set D.

Definition 2.5: A 'dominating set', $D \subseteq V(G)$ is considered as **'total regular dominating set',** if every point in $\langle D \rangle$ holds degree at least one and every point in $\langle D \rangle$ has the equal degree. **'Total regular domination number of G'**, represented by $\gamma_{tr}(G)$, is the 'smallest cardinality' of such set D.

Figure 1 below shows a graph G along with its litact graph $m(G)$ and 'total regular domination number' $\gamma_{tr}(m(G)) = 2$.

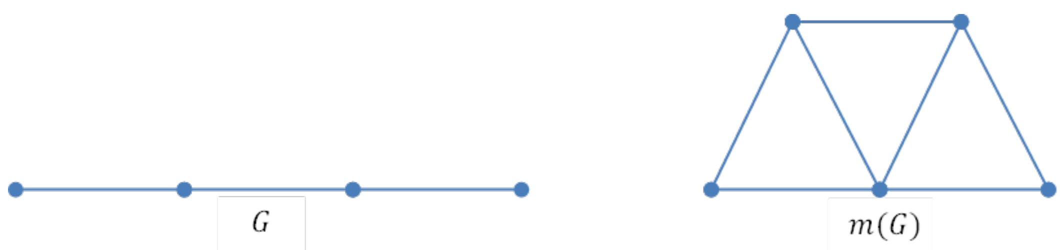

Fig 1. A graph G **and it's litact graph** $m(G)$

3 Results and Discussion

The purpose of this study is to ascertain certain relationships between domination parameters and the various characteristics of graph G, as well as total regular domination on a litact graph. Moreover, Nordhaus-Gaddum-like outcomes were also established. Total regular domination is a new domination parameter that was created by combining the two separate factors, total domination and regular domination. In this work, certain outcomes were obtained in relation to different parameters and domination parameters of the original graph G, but not into itself, using total regular domination on a litact graph. No one has ever done like this before.

Numerous domains, such as 'network design', 'project management', 'job assignments', 'graph colouring', 'social network analysis', and many more, can benefit from this topic, which offers many advantages.

All the following results are hold for any graph on which total regular domination is found and here a non-trivial graph G taken into account.

This section evaluates a litact graph's total regular domination number for a few standard graphs, such as cycle graph, star graph.

Proposition-1: For any cycle graph C_n, $\gamma_{tr}(m(C_n)) = \begin{cases} 2, if\ n = 3, 4 \\ n, if\ n \geq 5 \end{cases}$

Proposition-2: For any star graph $K_{1,n}$, $\gamma_{tr}\left(m\left(K_{1,n}\right)\right) = 2$.

Because of Harary F, we have the following finding, which is used in our subsequent result.

Theorem-A: If G is any $(p,\ q)-$graph, then

$\alpha_0(G) + \beta_0(G) = p = \alpha_1(G) + \beta_1(G)$.

The following result relates $\gamma_{tr}(m(G))$, $\alpha_0(G)$, $\beta_0(G)$, $\alpha_1(G)$ and $\beta_1(G)$.

Theorem-3.1: If G is any $(p,\ q)-$graph except a star graph, then

i) $\left\lceil \frac{\alpha_0(G)+\beta_0(G)}{4} \right\rceil \leq \gamma_{tr}(m(G))$

ii) $\left\lceil \frac{\alpha_1(G)+\beta_1(G)}{4} \right\rceil \leq \gamma_{tr}(m(G))$

iii) $\gamma_{tr}(m(G)) \leq p$

Proof: Examine the next two instances.

Case 1: If $G \cong K_{1,n}$, it would be simple to confirm that $\gamma_{tr}\left(m\left(K_{1,n}\right)\right) = 2$.

Case 2: If $G \not\cong K_{1,n}$, take $E = \{e_i; 1 \leq i \leq m\}$ be the G's edge set and $C = \left\{c_j; 1 \leq j \leq k\right\}$ where $j < i$, is G's cut vertices set such that $|E(G)| = q$ and $|C(G)| = s$. According to the definition of a litact graph, $E \cup C$ translates into a set of points in a litact graph. Assume that $D \subseteq V'(m(G))$ and $\langle D \rangle$ has no isolated vertices and that all of its vertices have the same degree. Therefore, D forms a 'Total regular dominating set in $m(G)$', so that, $|D| = \gamma_{tr}(m(G))$. Then, it is simple to confirm that, $\left\lceil \frac{p(G)}{4} \right\rceil \leq |V(m(G))| = |E(G) \cup C(G)| = q + s \leq |D|$. Now, applying Theorem-A, we obtain $\left\lceil \frac{\alpha_0(G)+\beta_0(G)}{4} \right\rceil \leq \gamma_{tr}(m(G))$ and $\left\lceil \frac{\alpha_0(G)+\beta_0(G)}{4} \right\rceil \leq \gamma_{tr}(m(G))$. Now, $\gamma_{tr}(m(G)) \leq p$ is quite obvious.

Next, we establish the relationship between $\gamma_{tr}(m(G))$ and $\gamma_c(G)$.

Theorem-3.2: If 'G is a graph', then $\left\lfloor \frac{\gamma_c(G)}{2} \right\rfloor < \gamma_{tr}(m(G)) + 1$.

Proof: Suppose $S \subseteq V(G)$ is a connected dominating set with $|S| = \gamma_c(G)$. Let X represents smallest dominating set of $m(G)$, and $Y \subseteq V(m(G)) - X$, and $Y' \subseteq Y$ such that $\left\langle X \cup Y' \right\rangle$ has no isolated point in $m(G)$ and every point in $\left\langle X \cup Y' \right\rangle$ is of equal degree. Therefore, $\left\langle X \cup Y' \right\rangle$ forms a smallest $\gamma_{tr}-$set in $m(G)$ so that $\left| X \cup Y' \right| = \gamma_{tr}(m(G))$. then we are able to conclude that $|S| < 2(\left\langle X \cup Y' \right\rangle + 1)$ based on the ideas of connected and total dominations. It follows that,

$\frac{|S|}{2} < \gamma_{tr}(m(G)) + 1$. Since, $\left\lfloor \frac{\gamma_c(G)}{2} \right\rfloor \leq \frac{\gamma_c(G)}{2} < \gamma_{tr}(m(G)) + 1$

$\Longrightarrow \left\lfloor \frac{\gamma_c(G)}{2} \right\rfloor < \gamma_{tr}(m(G)) + 1$.

The following theorem yields lower and upper bounds for $\gamma_{tr}(m(T))$.

Theorem-3.3: For any tree T with $p \geq 3$ vertices, $2 \leq \gamma_{tr}(m(T)) < q + l$ where l represents end points of T.

Proof: Suppose $D = \{e_i\}; 1 \leq i \leq n$ is the $\gamma'-$set of a tree T. A dominating set X of $m(T)$ is obtained from the litact graph definition, which corresponds to the edges of D. Assume that the set of vertices that are neighbours of the elements of X is $V_1 = V(m(T)) - X$. Then, suppose $\langle X \cup Y \rangle$ becomes a $\gamma_{tr}-$set, if $Y \subseteq V_1$. Therefore, $\gamma_{tr}(m(T)) = |X \cup Y| = |D_1|$. With regard to a total domination, we have $2 \leq \gamma_{tr}(m(T))$ and it is true that, $q + l$ exceeds $|D_1|$, therefore, $2 \leq \gamma_{tr}(m(T)) < q + l$.

The next theorem establishes an additional result for $\gamma_{tr}(m(T))$.

Theorem-3.4: If T is any $(p,\ q)-$tree where $p > 2$ vertices, then

$\gamma_{tr}(m(T)) - 1 < q(T) + diam(T)$.

Proof: Suppose T represents any tree with p points and q edges. There is an isolated vertex in its litact graph $m(T)$ if $p = 2$. For this reason, consider $p > 2$. Given a tree T with $q = p - 1$ edges, $p = q + 1$ is obtained. Suppose $X = \{e_i; 1 \leq i \leq k\} \subseteq E(T)$ is the edge set, which constitutes the longest path between any two vertices of T with length $|X| = diam(T)$. Now, from Theorem 3.1, we have

$\gamma_{tr}(m(G)) \leq p$

$\Longrightarrow \gamma_{tr}(m(T)) \leq p = q + 1$

$\Longrightarrow \gamma_{tr}(m(T)) - 1 \leq q < q + diam(T)$.

The following inequality relates $\gamma_{tr}(m(G))$, $p(G)$ and $\triangle(G)$.

Theorem-3.5: If 'G is any $(p,\ q)-$graph', then $\gamma_{tr}(m(G)) \geq \left\lfloor \frac{p(G)-1}{\triangle(G)} \right\rfloor$.

Proof: As usual, $p(G)$ represents vertices of G and $\triangle(G)$ represents vertex's maximal degree. Assume $X \subseteq V(m(G))$ is the dominating set in $m(G)$ and $V' = V(m(G)) - X$ then $V' \in N(X)$. Assuming $Y \subseteq V'$ and $Y \in N(X)$, the 'total dominating set of $m(G)$' is $X \cup Y$. Additionally, if each point in $\langle X \cup Y \rangle$ has the same degree, then $\langle X \cup Y \rangle$ forms a $\gamma_{tr}-$set in $m(G)$.

In addition, if $L = \{v_i; 1 \le i < n\} \subset \{v_j; 1 \le j \le n\}$ represents the collection of all 'non-end vertices in G', then at least one vertex $v \in L$ with the highest degree $\triangle(G)$ exists, so that $|X \cup Y|.\triangle(G) \ge p(G) - 1 \Longrightarrow \gamma_{tr}(m(G)) \ge \left\lfloor \frac{p(G)-1}{\triangle(G)} \right\rfloor$.

We now have the following outcome, which shows how $\gamma_{tr}(m(G))$, $\gamma_t(G)$, $p(G)$ and $\alpha_0(G)$ are related.

Theorem-3.6: If 'G is any $(p, q)-$graph', then $\gamma_{tr}(m(G)) + \gamma_t(G) \le p + 2\alpha_0$.

Proof: Assume that $A \subseteq V(G)$ is the smallest set of points that covers all of G's edges and ensures that $|A| = \alpha_0(G)$. Assume further that 'D is a dominating set of G' and that $D \cup B$ represents the 'Total dominating set of G', where $B \in N(D)$ such that $B \subseteq V - D$. That is, $\gamma_t(G) = |D \cup B|$. Suppose dominating set $X \subseteq V(m(G))$, and $V' = V(m(G)) - X$ then $V' \in N(X)$. Assuming $Y \subseteq V'$ and $Y \in N(X)$, then $\langle X \cup Y \rangle$ forms a γ_t-set in $m(G)$. Additionally, $\langle X \cup Y \rangle$ forms a $\gamma_{tr}-$set in $m(G)$ if all of its vertices have the same degree. Hence, it is obvious that $|X \cup Y| \cup |D \cup B| \le p + 2|A| \Longrightarrow \gamma_{tr}(m(G)) + \gamma_t(G) \le p + 2\alpha_0$.

The following outcome for $\gamma_{tr}(m(G))$, $q(G)$ *and* $\beta_1(G)$ is then obtained.

Theorem-3.7: If 'G is any $(p, q)-$graph', then $\gamma_{tr}(m(G)) < 2(q + \beta_1)$.

Proof: Assume that the maximal independent set of edges in G is $E_1 = \{e_i\}_{i=1}^{n} \subset E(G)$. Then E_1 is G's edge dominating set. Let V_1 be the collection of points of $m(G)$ which are incident to E_1's edges. Furthermore, let V_2 be the collection of G's vertices that coincide with any edge of E_1. Whenever V_2 is a null set, the vertex set $D \subseteq V(m(G))$ in which vertices of $\langle D \rangle$ consists of equal degree, forms a 'total regular dominating set' in $m(G)$, corresponding to the edges of E_1, with $|D| = 2\beta_1(G) < 2(q + \beta_1) \Longrightarrow \gamma_{tr}(m(G)) < 2(q + \beta_1)$. If not, since E_1 is an 'edge dominating set of G', $\langle V_2 \rangle$ is independent. The $\gamma_{tr}-$set of $m(G)$ is therefore formed by $S \subseteq V(m(G))$, such that $|S| < 2(q + \beta_1)$.

Another result for $\gamma_{tr}(m(G))$ found in the subsequent theorem.

Theorem-3.8: If 'G is any graph' with 3 or more points, then

$\gamma_{tr}(m(G)) \le diam(G) + 2\gamma'(G)$.

Proof: Assume V represent the 'set of points in G' and E represent the 'set of edges in G'. Suppose two points a, b such that $dist(a, b) = diam(G)$, the distance between them. The edge 'dominating set of G' is $S \subset E(G)$ so that $\gamma'(G) = |S|$. According to litact graph description, $V(m(G)) = E(G) \cup C(G)$, where $C(G)$ is a collection of G's cut vertices. Furthermore, assume, without losing generality, that $D \subset V(m(G))$ forms a $\gamma_{tr}-$set in $m(G)$ so that $\gamma_{tr}(m(G)) = |D|$. Given the aforementioned characteristics, it is straightforward to confirm that

$|D| \le diam(G) + 2|S| \Longrightarrow \gamma_{tr}(m(G)) \le diam(G) + 2\gamma'(G)$.

The relationship between $\gamma_{tr}(m(G))$, $\gamma_c(G)$ and $\gamma_t(G)$ is shown below.

Theorem-3.9: If 'G is any graph', then $\gamma_{tr}(m(G)) < 2\gamma_c(G) + \gamma_t(G)$.

Proof: Let $A \subseteq V(G)$ is the smallest 'connected dominating set' in G and $\gamma_c(G) = |A|$. Suppose $B \subseteq V(G) - A$ and $B_1 \subseteq B$. Since there are no isolated points in $\langle A \cup B_1 \rangle$, it forms a 'total dominating set' in G. Hence, $\gamma_t(G) = |A \cup B_1|$. Similarly, let 'D be a smallest dominating set of $m(G)$' and $D_1 \subseteq V(m(G)) - D$. If $D_2 \subseteq D_1$ and each vertex of $\langle D \cup D_2 \rangle$ has equal degree then $D \cup D_2$ creates a smallest total regular dominating set in $m(G)$ so that

$|D \cup D_2| < 2|A| + |A \cup B_1| \Longrightarrow \gamma_{tr}(m(G)) < 2\gamma_c(G) + \gamma_t(G)$.

We need the following theorems to establish our next results.

Theorem-B[5]**:** If 'G is any graph with order n', then,

$\gamma_t(S(G)) \le \left\lfloor \frac{3n-\alpha(G)}{2} \right\rfloor$.

Theorem-C[6]**:** If 'G is any graph with order n', then,

$\gamma_t(T(G)) \ge \gamma_t(G)$.

Theorem-D[7] **:** If 'T is a tree of order $n \ge 3$', then $\gamma_{tm}(T) \le \left\lfloor \frac{2n}{3} \right\rfloor$.

Theorem-E[8] **:** If 'G is any graph with order n', then,

$\left\lceil \frac{2n}{3} \right\rceil \le \gamma_t(M(G))$.

Corollary-3.10: If 'G is any graph of order n',

$\gamma_{tr}(m(G)) + \gamma_t(S(G)) < 2(q + \beta_1) + \left\lfloor \frac{3n-\alpha(G)}{2} \right\rfloor$.

Proof: This outcome is derived from Theorem-3.7 and Theorem-B.

Corollary-3.11: If 'G is any graph of order n',

$\gamma_{tr}(m(G)) + \gamma_t(T(G)) \ge \gamma_t(G) + \left\lfloor \frac{p(G)-1}{\triangle(G)} \right\rfloor$.

Proof: Using Theorem-3.5 and Theorem-C, the proof is easily obtained.

Corollary-3.12: For any tree T of order $n \ge 3$,

$\gamma_{tr}(m(T)) - 1 + \gamma_{tm}(T) < q(T) + diam(T) + \left\lfloor \frac{2n}{3} \right\rfloor$

Proof: Proof can be obtained from Theorem-3.4 and Theorem-D.

Corollary-3.13: If 'G is any graph of order n',

$\gamma_{tr}(m(G)) + \left\lceil \frac{2n}{3} \right\rceil \le \gamma_t(M(G)) + diam(G) + 2\gamma'(G)$.

Proof: This outcome is derived from Theorem-3.8 and Theorem-E.

Finally, results of the Nordhaus-Gaddum type are determined.

Theorem-3.14: For any '$(p,\ q)-$ connected graphs G and $\bar{G}$',

i) $\gamma_{tr}(m(G))+\gamma_{tr}\left(m\left(\bar{G}\right)\right)<2(p+1)$

ii) $\gamma_{tr}(m(G)).\gamma_{tr}\left(m\left(\bar{G}\right)\right)<p^2$.

Proof: The aforementioned results clearly apply to any $(p,\ q)$ connected graphs G and $\bar{G}$.

4 Conclusion

A major component of encryption is graph characteristics. The dominating set of a graph is used to quickly break the code. The concept of a new domination parameter called Total regular domination has been introduced in a litact graph. The information retrieval system permits the dominating set and elements of total regular dominating sets to stand alone in order to promote communication. Additionally, it can be used in a variety of contexts, including "social network analysis," "project management," "job assignments," "graph colouring," and "network design." Here we investigate and verify the results on total regular domination in the litact graph of a graph. We also demonstrate some outcomes between certain other domination parameters of a graph G and this innovative domination parameter. When it comes to putting domination theory into practice, these results are really beneficial. The findings can be further developed, and readers can examine more extensive uses of the parameters in future research.

References

1) Lakshmi AR, Vani M, Majeed A. Inverse litact domination in graphs. *Materials Today: Proceedings*. 2022;65(Part 8):3552–3557. Available from: https://dx.doi.org/10.1016/j.matpr.2022.06.147.
2) Muddebihal MH, Mandarvadkar PH. Co-regular Total Domination in Graphs. *International Journal of Applied Information Systems (IJAIS)*. 2020;12(29):16–19. Available from: https://www.ijais.org/archives/volume12/number28/muddebihal-2020-ijais-451852.pdf.
3) Lakshmi AR, Vani M, Ramprasad C, Kumar JV. Total Strong Litact Domination in Graphs. *International Advanced Research Journal in Science, Engineering and Technology*. 2022;9(1):230–235. Available from: https://iarjset.com/wp-content/uploads/2022/02/IARJSET.2022.9140.pdf.
4) Mallinath KS, Shailashree. Split Domination in Lict Subdivision Graph of a Graph. *European Chemical Bulletin*. 2023;12(Special Issue 4):7719–7726. Available from: https://www.eurchembull.com/uploads/paper/582189fa258bab7b63d328151057b24a.pdf.
5) Sigarreta JM. Total Domination on Some Graph Operators. *Mathematics* . 2021;9(3):1–9. Available from: https://doi.org/10.3390/math9030241.
6) Bermudo S. Total Domination on Tree Operators. *Mediterranean Journal of Mathematics*. 2023;20(1):1–16. Available from: https://dx.doi.org/10.1007/s00009-022-02236-7.
7) Kazemi AP, Kazemnejad F, Moradi S. Total mixed domination in graphs. *AKCE International Journal of Graphs and Combinatorics*. 2022;19(3):229–237. Available from: https://dx.doi.org/10.1080/09728600.2022.2111240.
8) Kazemnejad F, Pahlavsay B, Palezzato E, Torielli M. Total domination number of middle graphs. *Electronic Journal of Graph Theory and Applications*. 2022;10(1):275–288. Available from: https://dx.doi.org/10.5614/ejgta.2022.10.1.19.