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RESEARCH ARTICLE

On Micro Pre-Neighborhoods in Micro Topological Spaces

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Abstract

Objective: The basic objective of this article is to introduce and investigate the properties of micro pre-neighborhoods, micro pre-interior, micro pre-closure and micro pre-limit points. **Methods:** Using the concept of micro pre-open, micro pre-closed and micro pre-derived sets the result of the paper is obtained. **Findings:** In this article the properties and characterization of pre-open sets and pre-closed sets were opted on Micro topological space. **Novelty:** The relation between the characterization of micro pre-open sets, micro pre-closed sets and micro pre-derived sets were shown.

AMS Subject Classifications: 54B05

Keywords: Micro topology; Micro pre-interior; Micro pre-closure; Micro pre-neighborhoods; Micro pre-limit points

1 Introduction

Levine's introduction of generalized closed sets in 1970, providing a foundational framework for subsequent developments. Lellis Thivagar, further expanded this framework with the introduction of nano topology, utilizing approximations and boundary regions of a subset of a universe using an equivalence relation on it to define nano closed sets, nano-interior and nano-closure. The exploration of weak forms of nano open sets, such as nano α-open sets, nano semi-open sets, nano pre-open sets, and nano β-open sets, was undertaken by many authors adding layers of complexity to the existing theories.

In 2019, Chandrasekar[(1)], introduced the concept of micro topology which is a simple extension of nano topology, with a focus on micro pre-open and semi-open sets. Chandrasekar and Swathi[(2)], introduced Micro α-open sets. In 2020, Hariwan Z. Ibrahim[(3)], introduced Micro β-open sets in Micro topological spaces. In 2020, Reem O. Rasheed and Taha H. Jasim[(4)] introduced the concept of micro α-open sets and micro α-continuous function. In 2018, Sathishmohan P, et al.,[(5)] introduced nano pre-limit points. In 2023, Sindhu M, et al.,[(6)] introduced separation axioms on semi pre and semi generalized pre-Kasag topological space. In 2023, Viplavanjali KS et al.,[(7)] introduced the concept of separation axioms on generalized semi and semi generalized Kasag topological space, where Kasaj topology is an extension of Micro topology.

The aim of this paper is to introduce and investigate the properties of micro pre-neighbourhoods, micro pre-interior, micro pre-closure and micro pre-limit points using the concept of micro pre-open sets and micro pre-derived sets.

2 Preliminaries

Definition 2.1. [1]

Let U be the Universe R be an equivalence relation on U and τ_R(X) = {U, ∅, L_R(X), U_R(X), B_R(X)}, where X ⊆ U. τ_R(X) satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$.
2. The union of elements of any sub collection of τ_R(X) is in τ_R(X).
3. The intersection of the elements of any finite sub collection of τ_R(X) is in τ_R(X).

That is, τ_R(X) forms a topology on U is called the nano topology on U with respect to X. {U, τ_R(X)} is called the nano topological space.

Definition 2.2. [1]

Let {U, τ_R(X)} is a nano topological space here μ_R(X) = {{N ∪(N'∩μ)}: N, N' ∈ τ_R(X)} and called it Micro topology of τ_R(X) by μ where $\mu \notin \tau_R$(X).

Definition 2.3. [1]

The Micro topology μ_R(X) satisfies the following axioms.

1. U and $\emptyset \in \mu_R(X)$.
2. The union of elements of any sub collection of μ_R(X) is in μ_R(X).
3. The intersection of the elements of any finite sub collection of μ_R(X) is in μ_R(X).

Then μ_R(X) is called Micro topology on U with respect to X. The triplet (U, τ_R(X), μ_R(X)) is called Micro topological space and the elements of μ_R(X) are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. [1]

The Micro closure of a set A is denoted by Mic-cl(A) and is defined as Mic-cl(A) = ∩{B : B is Micro closed and A ⊆ B}. The Micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) = ∪{B : B is Micro open and A ⊇ B}.

Definition 2.5. [1]

Let (U, τ_R(X), μ_R(X)) be a Micro topological space and A ⊆ U. Then A is called Micro pre-open if A ⊆ Mic-int(Mic-cl(A)). The complement of a Micro pre-open is called a Micro pre-closed and it is denoted by Mic-PF(U).

Definition 2.6. [8]

A point x ∈ X is said to be a limit point of A if every neighborhood of x intersects A in some point other than x itself.

Definition 2.7. [8]

The set of all limit points of A is called the derived set of A and it denoted by D(A).

3 Methodology

In this article, we first study the micro pre-neighborhoods, where we found out some neighborhoods for pre-open sets and pre-closed sets in micro topological space. Then, with the help of micro pre-open sets, we found out the properties of the micro pre-interior and the relation between the micro-interior and the micro pre-interior. Next, with the help of micro pre-closed sets, we found the micro pre-closure. Similar to the interior, here we have found the relationship between the micro-closure and the micro pre-closure. Further, with the help of micro pre-derived sets, we found out the properties of micro pre-limit points.

4 Result and Discussion

4.1 Micro pre-neighborhoods

In this section, we define and study the notions of micro pre-neighborhoods, micro pre-interior and micro pre-closure in micro topological spaces and obtain some of its basic properties.

Definition 4.1.1. A subset M_x ⊂ U is called a micro pre-neighborhood of a point x ∈ u iff there exists A ∈ Mic-PO(U) such that x ∈ A ⊂ M_x and a point x is called micro pre-neighborhood point of the set A.

Definition 4.1.2. The family of all micro pre-neighborhoods of the point x $\in$ U is called micro pre-neighborhoods.

Example 4.1.3. Let U = {a, b, c, d}, U\R = {{a, b},{c},{d}}, X = {a}, τ_R(X) = {U,$\emptyset$, {a, b}}, μ = {b, c} and μ_R(X) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {a, b, c}}.

Mic-PO(U) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}}. Then,

MicP-nbd(a) = {U, {a}, {a, b}, {a, d}, {a, c}, {a, b, c}, {a, c, d}}.

MicP-nbd(b) = {U, {b}, {a, b}, {b, c}, {a, b, c}, {b, c, d}}.

MicP-nbd(c) = {U, {c}, {a, c}, {b, c}, {a, b, c}, {b, c, d}, {a, c, d}}.

MicP-nbd(d) = {U, {a, d}, {b, c, d}, {a, c, d}}.

Lemma 4.1.4. Let B_i/i $\in$ I be a collection of micro pre-open sets in a micro topological space U, then $\cup B_i \in$ Mic-PO(U).

Definition 4.1.5. The union of all micro pre-open sets which are contained in A is called the micro pre-interior of A and is denoted by Mic_{pint}(A) or by Mic-PA_*. As the union of micro pre-open sets is micro pre-open, Mic-PA_* is micro pre-open always.

Example 4.1.6. Let U = {a, b, c, d}, U\R = {{a, b},{c},{d}}, X = {a}, τ_R(X) = {U,$\emptyset$, {a, b}}, μ = {b, c} and μ_R(X) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {a, b, c}}.

Mic-PO(U) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}}. Take A = {a, b}. Then Mic-PA_* = {a, b}.

Definition 4.1.7. The intersection of micro pre-closed sets containing a set A is called the micro pre-closure of A and is denoted by Mic_{pcl}(A) or by Mic-PA^*.

Example 4.1.8. Let U = {a, b, c, d}, U\R = {{a, b}, {c}, {d}}, X = {a}, τ_R(X) = {U,$\emptyset$, {a, b}}, μ = {b, c} and μ_R(X) = {U, $\emptyset$, {b}, {a, b}, {b, c}, {a, b, c}}.

Mic-PF(U) = {U, $\emptyset$, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {c, d, a}}. Take A = {a, b}. Then Mic-PA^* = U.

Lemma 4.1.9. Let A and B be subsets of a space U. Then the following hold for the micro pre-closure operator.

1. A $\subseteq$ Mic-PA^*.
2. Mic-$PA^*\subseteq$ Mic-PB^* if A $\subset$ B.
3. Mic-P$(A^*)^*$ = Mic-PA^*.
4. Mic-PA^* is a micro pre-closed set in U.

Lemma 4.1.10. For every subset W $\subset$ U, we have the following.

1. Mic-P$(U-W)^*\subseteq$ U $-$ Mic-PW_*.
2. Mic-P$(U-W)_*\subseteq$ U $-$ Mic-PW^*.

Corollary 4.1.11. Intersection of any two micro pre-closed subsets is micro pre-closed.

Proof: Let A, B $\in$ Mic-PF(U). Then we have $Mic_{cl}[Mic_{int}(A\cap B)] = Mic_{cl}[Mic_{int}(A)\cap Mic_{int}(B)] \subset Mic_{cl}(Mic_{int}(A)) \cap Mic_{cl}(Mic_{int}(B)) \subset A\cap B$. Thus, A $\cap$ B in Mic-PF(U).

Theorem 4.1.12. A subset of a space U is micro pre-open iff it is a micro pre-neighborhood of each of its points.

Proof: Let G $\subset$ U be a micro pre-open set. Then by definition, it is clear that G is a micro pre-neighborhood of each of its points, since for every, x $\in$ G $\subset$ G and G is micro pre-open.

Conversely, suppose G is a micro pre-neighborhood of each of its points. Then for each x $\in$ G, there exists $S_x \in$ Mic-PO(x) such that $S_x \subset$ G. Then, G = $\bigcup_{X\in G} S_X$. Since each S_x is micro pre-open it follows that G is micro pre-open by Lemma 4.1.4.

Lemma 4.1.13. Let A be a subset in a space U. A point x $\in$ U is in the micro pre-interior of A iff there is a G $\in$ Mic-PO(x) such that G $\subset$ A.

Proof: Suppose x $\in$ Mic-PA_*. By definition of Mic-PA_*, there exists G $\in$ Mic-PO(x) such that x $\in$ G and G $\subset$ A. Hence there is G $\in$ Mic-PO(x), such that G $\subset$ A. Conversely, suppose G $\in$ Mic-PO(x), such that G $\subset$ A. Then

x $\in$ G $\subset$ Mic-PA_*. Hence x $\in$ Mic-PA_*.

Definition 4.1.14. A point x $\in$ U is called a micro pre-interior point of A $\subset$ U if x $\in$ Mic-PA_*.

Lemma 4.1.15. Let U be a space and A $\subset$ U and x $\in$ U. Then x is a micro pre-interior point of A iff A is a micro pre-neighborhood of x.

Remark 4.1.16. Since every micro-open set is micro pre-open, every micro-interior point of a set A $\subset$ U is a micro pre-interior point of A. Thus, Mic_{int}(A) $\subseteq$ Mic-PA_*.

Theorem 4.1.17. Let U be a space and A $\subset$ U. Then Mic-PA_* is the largest micro pre-open subsets of U contained in A.

Proof: To prove Mic-PA_* is the largest micro pre-open set contained in A. In other words, to show that Mic-PA_* contains any other micro pre-open set which is contained in A. Now, assume that U is any micro pre-open set with U $\subset$ A. Let x $\in$ U. Then by definition x $\in$ U $\subset$ A. Therefore, A is a micro pre-neighborhood of x $\in$ U. This shows that x is a micro pre-interior point of A. Then x ∈ Mic-PA$_*$ by Lemma 4.1.13 as x ∈ U implies x ∈ Mic-PA$_*$. Thus, U ⊂ Mic-PA$_*$ and Mic-PA$_*$ is micro pre-open. Therefore, Mic-PA$_*$ contains every micro pre-open set U contained in A and hence Mic-PA$_*$ is the largest micro pre-open set contained in A.

Theorem 4.1.18. In a micro topological space U, A is micro pre-open iff A = Mic-PA$_*$.

Proof: Suppose A = Mic-PA$_*$. As Mic-PA$_*$ is micro pre-open set, by hypothesis, A is micro pre-open. Next suppose that A is micro pre-open. Then A is a micro pre-open set contained in A. But Mic-PA$_*$ is the largest micro pre-open set contained in A by Theorem 4.1.17. Therefore, A ⊂ Mic-PA$_*$. But Mic-PA$_*$ ⊂ A always. Hence, A = Mic-PA$_*$.

Lemma 4.1.19. If A ⊂ B, then Mic-PA$_*$ ⊆ Mic-PB$_*$.

Note 4.1.20. Mic-PA$_*$ = Mic-PB$_*$ does not imply that A = B. This is shown by the following.

Example 4.1.21. Let U = {a, b, c, d}, U\R = {{a, b}, {c, d}}, X = {b, c}, τ_R(X) = {U,∅, {b, c}}, μ = {a} and μ_R(X) = {U, ∅, {a}, {b, c}, {a, b, c}}.

Mic-PO(U) = {U, ∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}. Take A = {b, c} and B = {b, c, d}. Then we obtain, Mic-PA$_*$ = {b, c} = Mic-PB$_*$. But A $\neq$ B.

Note 4.1.22. Mic-PA* = Mic-PB* does not imply that A = B. This is shown by the following.

Example 4.1.23. Let U = {a, b, c, d}, U\R = {{a, b}, {c, d}}, X = {b, c}, τ_R(X) = {U,∅, {b, c}}, μ = {a} and μ_R(X) = {U, ∅, {a}, {b, c}, {a, b, c}}.

Mic-PO(U) = {U, ∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, {a, c, d}, {a, b, d}}. Mic-PF(U) = {U, ∅, {b, c, d}, {a, c, d}, {a, b, d}, {c, d}, {a, d}, {b, d}, {d}, {b}, {c}} Take A = {b, c} and B = {b, c, d}. Then we obtain, Mic-PA* = {b, c} = Mic-PB*. But A $\neq$ B.

Lemma 4.1.24. Let A and B be subsets of U. Then,

1. Mic-PA$_*$ ∪Mic-PB$_*$ ⊆ Mic-P(A ∪B)$_*$.
2. Mic-P(A ∩B)$_*$ ⊆ Mic-PA$_*$ ∪Mic-PB$_*$.

Proof: It follows by Lemma 4.1.19.

Theorem 4.1.25. Let A and B be subsets of U. Then,

1. Mic-PA*∪Mic-PB* ⊆ Mic-P(A ∪B)*.
2. Mic-P(A ∩B)* ⊆ Mic-PA$_*$ ∪Mic-PB*.

Proof:

1. Since A ⊂ A ∪ B and B ⊂ A ∪ B. Then by Lemma 3.7(2), we obtain that Mic-PA* ⊆ Mic-P(A ∪B)* and Mic-PB* ⊆ Mic-P(A ∪B)*. It follows that Mic-PA*∪Mic-PB* ⊆ Mic-P(A ∪B)*.
2. Mic-P(A ∩B)* ⊆ Mic-PA* and Mic-P(A ∩B)* ⊆ Mic-PB*. Hence, it follows that Mic-P(A ∩B)* ⊆ Mic-PA*∩ Mic-PB*.

Example 4.1.26. Let U = {a, b, c, d}, U\R = {{a, b}, {c}, {d}}, X = {a},

τ_R(X) = {U, ∅, {a, b}}, μ = {b, c} and μ_R(X) = {U, ∅, {b}, {a, b}, {b, c}, {a, b, c}}.

Mic-PF(U) = {U, ∅, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}. Take, A = {a} and B = {d} then A ∪ B = {a, d}. Then Mic-PA* = {a}, Mic-PB* = {d} and Mic-P(A ∪ B)* = {a, d}. It follows that Mic-PA*∪ Mic-PB* = Mic-P(A ∪ B)*.

Example 4.1.27. Let U = {a, b, c, d}, U\R = {{a, b}, {c}, {d}}, X = {a},

τ_R(X) = {U, ∅, {a, b}}, μ = {b, c} and μ_R(X) = {U, ∅, {b}, {a, b}, {b, c}, {a, b, c}}.

Mic-PF(U) = {U, ∅, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}. Take, A = {a} and B = {a, d} then A ∩ B = {a}. Then Mic-PA* = {a}, Mic-PB* = {a, d} and Mic-P(A ∩ B)* = {a}. It follows that Mic-P(A ∩ B)* = Mic-PA*∩ Mic-PB*.

4.2 Micro pre-limit point

In this section, we define the micro pre-limit point of a subset of space U and obtain some of its basic properties.

Definition 4.2.1. A point x ∈ U is said to be a micro pre-limit point of A iff for each U ∈ Mic-PO(U), U ∩ (A – {x}) $\neq$∅.

Remark 4.2.2. Since every micro-open set is micro pre-open, it follows that every micro pre-limit point of A is a micro limit point of A.

Definition 4.2.3. The set of all micro pre-limit points of A is said to be the micro pre-derived set of A and in denoted by Mic-PD(A).

Note 4.2.4. By Remark 4.2.2, it follows that Mic-PD(A) ⊆ Mic-D(A), where Mic-D(A) is the micro derived set of A. But in general, converse does not hold.

Example 4.2.5. Let U = {a, b, c, d}, U\R = {{a, c}, {b, d}}, X = {a, c}, τ_R(X) = {U, ∅, {a, c}}, μ = {b} and μ_R(X) = {U, ∅, {b}, {a, c}, {a, b, c}}. Then,

Mic-PO(U) = {U, ∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, {b, c, d}, {a, b, d}}.

For A = {b, c}, Mic-D(A) = {a, d} and Mic-PD(A) = {d}. Hence, Mic-PD(A) ⊂ Mic-D(A). For A = {b, d}, Mic-D(A) = {d} and Mic-PD(A) = {d}. Hence, Mic-PD(A) = Mic-D(A). Therefore, Mic-PD(A) ⊆ Mic-D(A).

Lemma 4.2.6. A is micro pre-closed set iff it contains the set of its micro pre-limit points.

Proof: By definition, $U - A$ is micro pre-open set since A is micro pre-closed. Thus, A is micro pre-closed iff each point of $(U - A)$ has a micro pre-neighborhood contained in $(U - A)$ iff no point of $(U - A)$ is micro pre-limit point of A, or equivalently that A contains each of its micro pre-limit points.

Lemma 4.2.7. Let A and B be subsets of a space U and A ⊂ B. Then, A ⊂ B implies Mic-PD(A) ⊆ Mic-PD(B).

Proof: Let x ∈ U be a micro pre-limit point of A. Then by definition, there exists x ∈ Mic-PO(U) such that U ∩ (A – {x}) $\neq \emptyset$ and hence it follows that U ∩ (B – {x}) $\neq \emptyset$, i.e., x is a micro pre-limit point of B. Thus, the micro pre-derived set of A is subset or equal to the micro pre-derived set of B.

Theorem 4.2.8. Let A and B be subsets of a space U. Then we have the following properties.

1. Mic-PD(∅) = ∅.
2. x ∈ Mic-PD(A) implies x ∈ Mic-PD(A – {x}).
3. Mic-PD(A) ∪ Mic-PD(B) ⊆ Mic-PD(A ∪ B).
4. Mic-PD(A ∩ B) ⊆ Mic-PD(A) ∩ Mic-PD(B).

Proof:

1. It is obvious.
2. Let x ∈ Mic-PD(A). Then x is a micro pre-limit point of A. That is, every micro pre-neighborhood of x contains at least one point of A other than x. It means that every micro pre-neighborhood of x contains at least one point other than x of A – {x} and therefore x ∈ Mic-PD(A – {x}).
3. It follows by Lemma 4.2.7.
4. It follows by Lemma 4.2.7.

Lemma 4.2.9. Let U be a space and A be subset of U. Then A ∪ Mic-PD(A) is a micro pre-closed set.

Proof: Let x ∉ A ∪ Mic-PD(A). This implies x ∉ A and x ∉ Mic-PD(A). Since x ∉ Mic-PD(A), there exists a micro pre-open neighborhood M_x of x which contains no point of A other than x. But x ∉ A. So M_x contains no point of A, which implies $M_x \subset U - A$. Again, M_x is a micro pre-open neighborhood of each of its points. But as M_x does not contain any point of A, no point of M_x can be a micro pre-limit point of A. Therefore, no point of M_x can belong to Mic-PD(A). This implies that $M_x \subset U -$ Mic-PD(A). Hence, it follows that x ∈ $M_x \subset (U - (A \cup$ Mic-PD(A))). Therefore, A ∪ Mic-PD(A) is micro pre-closed.

Remark 4.2.10. Comparing the results of Lemma 4.2.9 and Lemma 4.1.9(1), one can easily write that, A ∪ Mic-PD(A) is micro pre-closed iff Mic-PA* = A ∪ Mic-PD(A). It is obvious that Mic-PA* ⊂ Mic-cl(A). The converse may be false as shown by the following.

Example 4.2.11. Let U = {a, b, c, d}, U\R = {{a, c}, {b, d}}, X = {a, c}, τ_R(X) = {U, ∅, {a, c}}, μ = {c} and μ_R(X) = {U, ∅, {c}, {a, c}}, $\mu_R{}^c$(X) = {U, ∅, {a, b, d}, {b, d}}.

Mic-PO(U) = {U, ∅, {c}, {b, c}, {c, d}, {a, c}, {a, b, c}, {b, c, d}, {a, c, d}}.

Mic-PF(U) = {U, ∅, {a}, {b}, {d}, {a, b}, {b, d}, {a, d}, {a, b, d}}.

Take, A = {a} then we obtain, Mic-PA* = {a} and Mic-cl(A) = {a, b, d} which shows that Mic-PA* ⊂ Mic-cl(A). If we take, A = {b, d} then we obtain, Mic-PA⋆ = {b, d} = Mic-cl(A). So, Mic-PA* ⊆ Mic-cl(A).

Theorem 4.2.12. Mic-PA$_*$ ⊆ A − Mic-PD$(U - A)$.

Proof: Let x ∈ A − Mic-PD$(U - A)$. Then x ∈ A and x ∉ Mic-PD$(U - A)$. Since x ∉ Mic-PD$(U - A)$, there exists X ∈ Mic-PO(x) with X ∩ $(U - A)$ = ∅. Hence, x ∈ X ⊂ A, which implies x ∈ Mic-PA$_*$. Conversely, let x ∈ Mic-PA$_*$. Then x ∉ Mic-PD$(U - A)$, for Mic-PA$_*$ is micro pre-open neighborhood of x and, Mic-PA$_*$ ∩ $(U - A)$ = ∅. Also, Mic-PA$_*$ ⊂ A. This shows that x ∈ A. Hence Mic-PA$_*$ ⊆ A − Mic-PD$(U - A)$.

Remark 4.2.13. Using Lemma 4.1.10 one can write that Mic-PA$_*$ = $U - (U - A)^*$.

5 Conclusion

In this paper, we introduced the notion of micro pre-neighborhoods and micro pre-limit points by employing the concept of micro pre-open sets, micro pre-derived sets elucidating various associated properties. Our intent is to further elaborate on these findings in forthcoming research endeavors, with a particular focus on exploring practical applications.

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References

1) Chandrasekar S. On Micro Topological Spaces. *Journal of new theory*. 2019;26:23–31. Available from: https://dergipark.org.tr/en/pub/jnt/issue/42082/506329.
2) Chandrasekar S, Swathi G. Micro-α-open sets in Micro Topological Spaces. *International Journal of Research in Advent Technology*. 2018;6(10):2633–2637. Available from: https://ijrat.org/downloads/Vol-6/oct-2018/Paper%20ID-610201820.pdf.
3) Ibrahim HZ. Micro β-open sets in Micro topology. *General Letters in Mathematics*. 2020;8(1):8–15. Available from: https://www.researchgate.net/publication/341030725.
4) Rasheed RO, Jasim TH. On Micro-α-Open Sets and Micro-α- Continuous Functions in Micro Topological Spaces. In: Imam Al-Kadhum International Conference for Modern Applications of Information and Communication Technology (MAICT) ;vol. 1530 of Journal of Physics: Conference Series. 2020;p. 1–6. Available from: https://iopscience.iop.org/article/10.1088/1742-6596/1530/1/012061/pdf.
5) Sathishmohan P, Rajendran V, Kumar CV, Dhanasekaran PK. More on nano pre-neighborhoods in nano topological spaces. *Journal of Applied Science and Computation*. 2018;5(10):899–907. Available from: https://kongunaducollege.ac.in/sites/kongunaducollege.ac.in/files/NAAC/1177791117ls_1.pdf.
6) Sindhu M, Sathishmohan P, Rajalakshmi K. A Note on Separation Axioms in Kasaj Topological Spaces. *Indian Journal Of Science And Technology*. 2023;16(42):3764–3770. Available from: https://dx.doi.org/10.17485/ijst/v16i42.1508.
7) Viplavanjali KS, Sathishmohan P, Prakash E, Rajalakshmi K. Some Kinds of Separation Axioms in Kasaj Topological Spaces. *Indian Journal Of Science And Technology*. 2023;16(41):3575–3582. Available from: https://dx.doi.org/10.17485/ijst/v16i41.1509.
8) Munkers JR. Topology. 2nd ed. 2022, editor;Pearson Education. 2022. Available from: http://www.alefenu.com/libri/topologymunkres.pdf.