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RESEARCH ARTICLE

On Distance Average Degree Independent Resolving Sets of Some Algebraic Graphs

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Abstract

Objectives: This study introduces the concept of distance average degree independent resolving sets and its dimensions for identity graphs of finite groups and order prime graphs of finite groups as a theorem in detail. **Methods:** The methodology involves first constructing the subset from the given graph, then finding the distance between each and every vertice of the graph and the subset, and also finding the average of the distance such that each and every vertice should receive a distinct code. The minimum number of vertices that satisfy this condition, their cardinality, is called the dimension. And then check various conditions like adjacency, equitability, independence, etc. in order to find various types of dimensions for the considered graphs. **Findings:** In this article, the concept of distance average degree resolving sets has been found, and further distance average degree independent resolving sets have been found, with their corresponding dimensions also found for algebraic graphs like identity graphs of finite groups and order prime graphs of finite groups. **Novelty:** The novelty in this article is the concept of distance-average-degree independent resolving sets. Particularly, this concept has been applied to the algebraic graphs, namely the identity graph and the order prime graph of a finite group. In addition to that, the concept of distance has been added to this resolving sets area, which is totally new to this area of graph theory.

AMS Subject Classification: 05C12, 05C50.

Keywords: Resolving set; Independent set; Distance average degree independent resolving set; Identity graphs; Order prime graphs

1 Introduction

In this article, first we have found the distance average resolving set and its dimensions, and then the distance average independent resolving set and its dimensions have been analyzed. The concept of resolving sets was first introduced by Slater, Harary, and Melter. Resolving sets have many applications in network discovery and verification, chemistry, and robot navigation[(1)]. After that, many resolving sets have been introduced by many mathematicians and studied for various types of graphs[(1–11)]. The concept of resolving sets is not much applied in algebraic graphs. So we planned to work with algebraic graphs like identity graphs, which were studied by Kandasamy, W.B.V., and Smarandache.[(12)], and order prime graphs, which were introduced by M. Sattanathan and Kala[(13)]. Inspiring various resolving sets and independent resolving sets, distance average degree independent sets have been introduced and studied for various identity and order prime graphs of finite groups.

Preliminaries

Definition 1.1 Resolving sets: A set of vertices S in a graph G is called a resolving set for G if, for any two vertices u, v there exists $x \in S$ such that the distances $d(u,x) \neq d(v,x)$. The minimum cardinality of a resolving set of G is called the dimension of G and is denoted $dim(G)$.

Definition 1.2 Identity graphs: Let g be a group. The identity graph $G = (V,E)$ with vertices as the elements of group and two elements $x, y \in g$ are adjacent or can be joined by an edge if $x.y = e$, where e is the identity element of g and identity element is adjacent to every other vertices in G.

Definition 1.3 Order Prime graphs: Let Γ be a finite group. The order prime graph (Γ) of a group Γ is a graph with V $((\Gamma))$ = Γ and two vertices are adjacent in (Γ) if and only if their orders are relatively prime in Γ.

2 Methodology

1. To find resolving set first choose the subset S of V(G) such that distance from V(G) to subset S. Here the distance is the maximum length between any two vertices in G.

2. Then degree has been found for every vertices.

3. Then we have to find $\Gamma(u/S) = \{d(u,\ s_1),\ d(u,\ s_2), ..., d(u,\ s_k)\}$ where $d(u,v)$ is defined by $d(u,v) = d(u,v) + \left\lceil \frac{d(u)+d(v)}{2} \right\rceil$, where $d(u)$ is the degree of the vertex u.

4. Then using the concept of adjacency, independent set concepts we check whether it satisfy those conditions or not.

5. The subset with minimum cardinality which satisfy above condition is called as distance average degree independent resolving sets and its cardinality is called as distance average degree independent metric dimension and it is denoted by $idadd(G)$.

3 Results on Distance average degree independent resolving sets for some identity graphs of finite groups

Distance average degree independent resolving sets

Let graph $G = (V,E)$. *For* $u \in V$ associate a vector with respect to a subset $S = \{s_1,\ s_2, \ldots, s_k\}$ of V by $\Gamma(u/S) = \{d(u,\ s_1),\ d(u,\ s_2), ..., d(u,\ s_k)\}$ where $d(u,v)$ is defined by $d(u,v) = d(u,v) + \left\lceil \frac{d(u)+d(v)}{2} \right\rceil$, where $d(u)$ is the degree of the vertex u. Then the subset S is said to be distance average degree resolving sets if $\Gamma(u/S) \neq \Gamma(v/S)$ for all $u, v \in V - S$. And set S is also independent set. The minimum cardinality is called as distance average degree independent metric dimension, and it is denoted by $idadd(G)$.

Theorem: 3.1 Identity graph of Z_n, $n > 3$ odd number has the $idadd(Z_n) = \frac{n-1}{2}$

Proof: Let graph $G = (Z_n,\ \oplus_n)$ for n>3 odd number

The vertex set of G is $V(G) = \{0, 1, 2, \ldots n-1\} = \left\{x_0, x_1, x_2, \ldots, x_{\frac{n-1}{2}},\ x_{\frac{n+1}{2}}, \ldots, x_{n-1}\right\}$.

The edge set of G is $E(G) = \left\{x_0 x_i,\ \ x_1 x_{n-1},\ x_2 x_{n-2}, \ldots, x_{\frac{n-1}{2}}\ \ x_{\frac{n+1}{2}}\right\}, 1 \leq i \leq n-1$.

$|V(G)| = n$; $|E(G)| = \frac{3n-3}{2}$. Let $S \subseteq V(G)$. The subset S is considered in such a way that choose $\frac{n-1}{2}$ odd vertices like $\{x_1,\ x_3,\ \ldots\}$ or $\frac{n-1}{2}$ even vertices like $\{x_2,\ x_4, \ldots\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains either even vertices like $\{x_2,\ x_4,\ x_6, \ldots\}$ or odd vertices like $\{x_1,\ x_3,\ \ldots\}$ clearly there is no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd(G) = \frac{n-1}{2}$.

Theorem: 3.2 For Identity graph of $Z_n,\ n > 4$ be an even number the $idadd\,(Z_n) = \frac{n}{2}$.

Proof: Let graph $G = (Z_n,\ \oplus_n)$ for n>4 even number

The vertex set of G is $V\,(G) = \{0, 1, 2, \ldots n-1\} = \left\{x_0, x_1, x_2, \ldots,\ x_{\frac{n}{2}}, \ldots, x_{n-1}\right\}$.

The edge set of G is $E\,(G) = \{x_0x_i,\ \ x_1x_{n-1},\ x_2x_{n-2}, \ldots, /1 \leq i \leq n-1\}$

$|V(G)| = n\,;\, |E(G)| = \frac{3n-3}{2}$

Let $S \subseteq V(G)$. The subset S is considered in such a way that choose $\frac{n}{2}$ odd vertices like $\{x_1,\ x_3,\ \ldots x_{n-3}\}$ or $\frac{n}{2}$ even vertices like $\{x_2,\ x_4, \ldots\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e., sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains either even vertices like $\{x_2,\ x_4,\ x_6, \ldots\}$ or odd vertices like $\{x_1,\ x_3,\ \ldots\}$ clearly there is no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd(G) = \frac{n}{2}$.

Theorem: 3.3 For any Identity graph of $K_4 idadd(K_4)$ is three

Proof: Let graph G =Identity graph of Klein-4 group

The vertex set of G is $V\,(G) = \{x_0,\ x_1,\ x_2, x_3\} = \{e, a, b, ab\}$.

The edge set of G is $E\,(G) = \{x_0x_i/1 \leq i \leq 3\}$.

Let $S \subseteq V(G)$. The subset S is considered in such a way choose the vertex $\{x_1,\ x_2, x_3\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains only $\{x_1,\ x_2, x_3\}$. Clearly there is no pair of adjacent vertices in set S. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd(G) = 3$.

Theorem: 3.4 Identity graph of Q_8 has $idadd(Q_8)$ to be 3.

Proof: Let graph G =identity graph of Q_8.

The vertex set of G is $V\,(G) = \{x_0,\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7\}$.

The edge set of G is $E\,(G) = \{x_0x_i,\ x_2x_3,\ x_4x_5,\ x_6x_7\ : 1 \leq i \leq 7\}$.

Let $S \subseteq V(G)$. The subset S is considered in such a way choose any three odd or even numbered vertices from $\{x_1,\ x_2, x_3,\ x_4,\ x_5,\ x_6\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains either three odd number vertices or even numbered vertices accordingly as S chosen. Clearly there is no pair of adjacent vertices in set S. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd(G) = 3$.

4 Results on Distance average degree independent resolving set of Order Prime graphs of Finite group

Theorem: 3.5 The $OP\,(\Gamma\,(Z_n))\,,\ n = 2p,\ p > 3$ under addition modulo $n = 2p$

$idadd(OP\,(\Gamma\,(Z_n))) = 2p-4$.

Proof: Let graph G=$OP(\Gamma\,(Z_n))$ and $n = 2p$, $p > 3$ and p is a prime number.

The vertex set of G is $V\,(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0,\ x_1,\ x_2, \ldots, x_{n-1}\}$.

The edge set of G is $E\,(G) = \left\{x_0x_i,\ \ x_{\frac{p}{2}}x_{\frac{p}{2}-1},\ \ x_{\frac{p}{2}}x_{\frac{p}{2}+1}\ /1 \leq i \leq n-1\right\}$.

$|V\,(G)| = n$ and $|E\,(G)| = n+1$. Let $S \subseteq V(G)$. The subset S is considered in such a way choose any $2p-5$ vertices of degree one and $x_{\frac{p}{2}-1}$ vertex. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd\,(G) = 2p-4$.

Theorem: 3.6 The $OP\,(\Gamma\,(Z_n))\,,\ n = 3p,\ p > 3$ under addition modulo $n = 3p$

$idadd(OP\,(\Gamma\,(Z_n))) = 3p-4$.

Proof: Let graph G=$OP(\Gamma\,(Z_n))$ and $n = 3p$, $p > 3$ and p is a prime number.

The vertex set of G is $V\,(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0,\ x_1,\ x_2, \ldots, x_{n-1}\}$.

The edge set of G is $E\,(G) = \left\{x_0x_i,\ \ x_{\frac{p}{3}}x_j,\ \ x_{\frac{2p}{3}}x_k/1 \leq i \leq n-1\ ;\ \frac{pj}{3} \cong 0mod3p\,;\, \frac{2pk}{3} \cong 0mod3p\right\}$. $|V\,(G)| = n$ and $|E\,(G)| = n+1$. Let $S \subseteq V(G)$. The subset S is considered in such a way choose any $3p-7$ vertices of degree one and $\left\{x_{\frac{p}{3}}, x_{\frac{2p}{3}}\right\}$ vertex. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd\,(G) = 3p-4$.

Theorem:3.7 The $OP\,(\Gamma\,(Z_n))$, $n \neq 2p$ and $n \neq 3p\;\; n > 4$ under addition modulo $nidadd(OP\,(\Gamma\,(Z_n))) = n-2$.

Proof: Let graph G=$OP(\Gamma\,(Z_n))$ and $n \neq 2p$ and $n \neq 3p\; n > 4$ where p is a prime number

The vertex set of G is $V\,(G) = \{0,1,2,\ldots,n-1\} = \{x_0,\, x_1,\, x_2,\ldots,x_{n-1}\}$.

The edge set of G is $E\,(G) = \{x_0x_i\;/1 \leq i \leq n-1\}$.

$|V\,(G)| = n$ and $|E\,(G)| = n-1$. Let $S \subseteq V(G)$. The subset S is considered in such a way choose any $n-2$ vertices of degree one. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd\,(G) = n-2$.

Theorem: 3.8 The $OP\,(\Gamma\,(K_4))$ has distance average degree independent dimension to be 3

Proof: Let graph $G = OP(\Gamma\,(K_4))$ under composition.

The vertex set of G is $V\,(G) = \{e,a,b,ab\} = \{x_0,\, x_1,\, x_2,\, x_3\}$

The edge set of G is $E\,(G) = \{x_0x_i : 1 \leq i \leq 3\}$

$|V\,(G)| = 4$ and $|E\,(G)| = 3$. Let $S \subseteq V(G)$. The subset S is considered in such a way choose three vertices of degree one. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd\,(G) = 3$.

Theorem: 3.9 The $OP\,(\Gamma\,(Q_8))$ has distance average degree independent dimension to be 6

Proof: Let graph $G = OP(\Gamma\,(Q_8))$ under composition.

The vertex set of G is $V\,(G) = \{-1,1,i,-i,j,-j,k,-k\} = \{x_0,\, x_1,\, x_2,\, x_3, x_4,\, x_5,\, x_6,\, x_7\}$

The edge set of G is $E\,(G) = \{x_0x_i : 1 \leq i \leq 8\}$.

$|V\,(G)| = 8$ and $|E\,(G)| = 7$. Let $S \subseteq V(G)$. The subset S is considered in such a way choose six vertices of degree one. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e., sum of the distance between the vertex x_i to the set S and greatest integer of average of degree x_i and to the vertices in set S is distinct. Therefore, set S is a distance average degree resolving set. Now set S contains no pair of adjacent vertices. Therefore, set S is also independent set. The minimum cardinality of set S is called as distance average degree independent dimension i.e. $idadd\,(G) = 6$.

5 Conclusion

In this paper, distance average degree independent resolving sets has been found and then its corresponding dimensions has been found for the algebraic graphs like identity graph and order prime graph of finite group. Further, in future we have planned to extend this result for the network related graphs, and then we will compare with the algebraic graphs results.

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