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Split Regular Domination in Litact Graphs

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Abstract

Objectives: In the context of graph theory, a litact graph is a specific type of graph. This study introduces a new domination parameter, called split regular domination in litact graphs. **Methods**: When we talk about split regular domination in a litact graph during this investigation, we think about how to divide the litact graph into partitions that adhere to specific domination principles by taking a minimal split regular dominating set with all vertices of equal degree. We used a few common definitions and the ideas of several domination parameters in G to obtain the results. **Findings**: Numerous bounds on $\gamma_{sr}(m(G))$ were found in relation to the different parameters of G like vertices, edges, diameter, vertex covering number, maximum degree and so forth, and its relationship to other dominating parameters of G such as total domination, edge domination, connected domination and so on was also found. Furthermore, outcomes resembling those of Nordhaus-Gaddum were also obtained. **Novelty**: Graph G was used to find a litact graph. Subsequently, a few findings of a new domination parameter called split regular domination in a litact graph in terms of different parameters of G have been established.

Keywords: Graph; Litact Graph; Split Domination Number; Regular Domination Number; Split Regular Domination Number

1 Introduction

As usual, every graph taken into consideration here is undirected, simple, and finite. In many different contexts, numerous interactions and processes can be replicated using graphs, such as social, biological, information, physical systems and many more. Berge defined "coefficient of external stability" to speak of a graph's domination number. Ore referred to the same idea as "dominating set" then "dominating number" in 1962. E. J. Cockayne and S. T. Hedetniemi conducted a thorough and fascinating analysis of the findings at the time regarding dominating sets in graphs. Since then, the use of the symbol γ for a graph's domination number has grown significantly. Kulli and Janakiram presented split domination in G. Sampath Kumar who introduced the idea of regular domination number.

The study of domination numbers and associated subset problems, such as total, edge, connected, strong, split domination, and so on, is one of the fastest-growing fields in graph theory. Several conditions have been applied to dominating sets in order to study various sorts of domination parameters. Split regular domination in litact graphs has not been studied, despite a significant amount of study on domination parameters and related subjects in graphs.

The primary goal of this paper is to present split regular domination number of a litact graph, a new domination parameter, and develop some conclusions on it in terms of members of G such as vertices, edges and many more but not by itself. In a similar manner, we use this new domination parameter to find some boundaries with some other conventional domination parameters in G, including edge domination, total domination, split domination and many more.

For the current investigation, the authors referred the research articles, Renuka Lakshmi et. Al [1] about "Strong Split Litact Domination in Graphs", Mohanapriya A [2] regarding "Domination and its variants in split graphs -P versus NPC dichotomy", Sumathi R and Latha DL Yamini [3] about "The split domination in arithmetic graphs" and Amutha S [4] about "On α-Split Domination in Graphs".

2 Methodology

Using each of the definitions listed below, we were able to get the following results.

Definition 2.1. If all points in $V-S$ are connected to a point in S, then S is considered a **dominating set** in a graph $G(V,\ E)$. The **domination number** of G, represented by $\gamma(G)$, is the set's smallest cardinality.

Definition 2.2. Consider D is a subset of $V(G)$. The dominating set D of G is referred to as a **split dominating set** if the induced sub graph $\langle V-D\rangle$ is not connected. The **split domination number**$\gamma_s(G)$, is the set's minimal cardinality.

Definition 2.3. G is known as a **regular graph** if each point in it holds the equal degree. A dominating set D is called a **regular dominating set** of G if induced sub graph $\langle V-D\rangle$ is regular. The **regular domination number** of G, represented by $\gamma_r(G)$, is the minimal cardinality of such a set.

Definition 2.4. If the induced sub graph $\langle V-D\rangle$ is not connected and induced sub graph $\langle D\rangle$ is regular, then a dominating set $D\subseteq V(G)$ is a **split regular dominating set** in a graph G. Within this collection, the **split regular domination number**$\gamma_{sr}(G)$ represents the lowest cardinality of vertices in D.

Definition 2.5. The point set of a **litact graph**$m(G)$ is made up of the G's edges and cut vertices. If edges and cut vertices are incident or adjacent in G, then the two vertices in $m(G)$ are connected.

3 Results and Discussion

Numerous studies on split and regular domination numbers in different graphs have been conducted. However, by integrating the two distinct characteristics of split and regular dominations, a new domination parameter, split regular domination, has been established in the current work. Using this new parameter in litact graphs, which is the novelty work, some results have been obtained. Results on other graphs were found by other studies, but not on the litact graph. Several findings on split regular domination in a litact graph are presented in this paper, both in terms of other parameters and the domination parameters of a graph G, not onto itself.

This topic has many advantages and can be used to a variety of fields, including operations research, computer applications, networks, and more.

The subsequent findings are applicable to any graph where split regular domination exists; in the present instance, a non-trivial graph G with $p>3$ vertices is considered.

The subsequent theorem describes upper bound to $\gamma_{sr}-m(L_p)$, where L_p is a ladder graph.

Theorem 3.1. For any ladder graph $L_p,\ \gamma_{sr}(m(L_p))\geq 3$, where $p>3$.

Proof. Suppose $L_p(V,\ E)$ is a ladder graph with $2p$ points and $3p-2$ lines is a undirected planar graph such that $|V|=2p$ and $|E|=3p-2$. As per the definition of a litact graph, $m(L_p)$ has $3p-2$ vertices, which is $\left|V'(m(L_p))\right|=3p-2$, as the number of edges in L_p is $3p-2$ and the ladder graph has no cut vertices. Given a dominating set D in $m(L_p)$, D generates a split regular dominating set in $m(L_p)$ if $\langle V'-D\rangle$is disconnected and $\langle D\rangle$ is regular. Then, in this instance, it is evident that D contains at least three vertices. Therefore, $\gamma_{sr}(m(L_p))\geq 3$.

$\gamma_{sr}(m(G))$ and diameter of G are described by the following theorem.

Theorem 3.2. If G is a graph, then $\left\lceil\frac{diam(G)-1}{2}\right\rceil\leq\gamma_{sr}(m(G))$.

Proof. According to the definition of a litact graph, if E is the edge set and C is G's cut vertices, then $E\cup C=V(m(G))$. Assume edge set $E'\subseteq E(G)$ represents the largest distance between any two points such that $|E'|=diam(G)$. Let D forms a split regular dominating set in a litact graph $m(G)$. It follows that, $diam(G)\leq 2\gamma_{sr}(m(G))+1$.

$\therefore\ \frac{diam(G)-1}{2}\leq\gamma_{sr}(m(G))\Longrightarrow\left\lceil\frac{diam(G)-1}{2}\right\rceil\leq\gamma_{sr}(m(G))$.

The ensuing theorem connects $\gamma_{sr}(m(G))$ to $\gamma(G)$, $q(G)$ and $diam(G)$.

Theorem 3.3. For any (p,q) graph G, $\gamma(G)+diam(G) \leq \gamma_{sr}(m(G))+q$.

Proof. Assume that edge set $E_1 \subseteq E(G)$ has the longest path between any two seeming points in G, ensuring that $|E_1| = diam(G)$. Let X be the dominating set in G so that $|X| = \gamma(G)$. As usual q represents the number of lines in a (p,q) graph G so that $|E| = q$. Moreover, let D represent any dominating set that creates a split regular minimal set in a litact graph $m(G)$. This way, it is simple to determine that $|X \cup E_1| \leq |E \cup D|$ based on everything mentioned previously. Thus,

$\gamma(G)+diam(G) \leq \gamma_{sr}(m(G))+q$.

In terms of p, q and $\triangle(G)$, the succeeding result demonstrates a lower bound on $\gamma_{sr}(m(G))$.

Theorem 3.4. For each (p,q) graph G, $\gamma_{sr}(m(G)) < \left\lfloor \frac{p+q+\triangle(G)}{2} \right\rfloor$.

Proof. Let V, E and C be the point set, edge set and cut vertex set of G respectively so that $|V| = p$, $|E| = q$ and $|C| = c$. Moreover, $\triangle(G) = l$ can be achieved for at least one vertex $v \in V(G)$ of maximum degree. Then, a regular dominating set $D \in V(m(G))$ where $D \subseteq E(G) \cup C(G)$ so that $N(D) = V(m(G)) - D$ is disconnected. Therefore, D forms a split regular dominating set in $m(G)$ and $|D| = \gamma_{sr}(m(G))$. Thus,

$2|D| - l < |V(G) \cup E(G)|$

$\Longrightarrow 2\gamma_{sr}(m(G)) - \triangle(G) < p+q$

$\Longrightarrow \gamma_{sr}(m(G)) < \left\lfloor \frac{p+q+\triangle(G)}{2} \right\rfloor$.

The result that follows connects $\gamma_{sr}(m(G))$, $\gamma'(G)$ and $\delta(G)$.

Theorem 3.5. For any G, $\gamma_{sr}(m(G)) < \gamma'(G) + 2\delta(G) + 1$.

Proof. Let $G = (V,E)$ be the graph and $X = \{e_i; 1 \leq i \leq n\}$ is a smallest edge dominating set in G, so that $|X| = \gamma'(G)$. Moreover, assume that there is a vertex $v \in V$ of minimum degree $\delta(G)$ in G. Now, $E = \{e_j; 1 \leq j \leq m\}$ is the G's edge set and $C = \{c_k; 1 \leq k \leq l\}$ is the G's cut vertices, then by the definition of $m(G)$, $E \cup C \subseteq V(m(G))$. Thus, a regular dominating set $D \subseteq V(m(G))$ in $m(G)$ and $N(D) \in V(m(G)) - D$ is disconnected. Therefore, D forms the smallest split regular dominating set in $m(G)$ and $|D| < |X| + 2\delta(G) + 1 \Longrightarrow \gamma_{sr}(m(G)) < \gamma'(G) + 2\delta(G) + 1$.

This theorem-3.6 establishes a relationship between $\gamma_{sr}(m(G))$, $\gamma_t(G)$ and $\triangle(G)$.

Theorem 3.6. If G is a graph, then $\left\lfloor \frac{\gamma_{sr}(m(G))}{2} \right\rfloor \leq \gamma_t(G) + \triangle(G)$.

Proof. Consider G with the least dominating set X and the vertex and edge sets (V, E). If $V_1 = V - X$ and $V_2 \subseteq V_1$ such that $V_2 \subseteq N(X)$ in G, then the smallest dominating set $X \cup V_2$ forms a total dominating set in G. Thus, $|X \cup V_2| = \gamma_t(G)$. Suppose there is a vertex $v \in V$ has a maximum degree $\triangle(G)$ in G. Then, as usual C is the G's cut vertex set so that a minimal regular dominating set $D \subseteq E \cup C \subseteq V(m(G))$ in $m(G)$ and $N(D) \in V(m(G)) - D$ is disconnected. Thus, D constitute a $\gamma_{sr}-$set in $m(G)$. Consequently, based on the entire reasoning above, it is determined that

$\frac{|D|}{2} \leq |X \cup V_2| + \triangle(G)$

$\Longrightarrow \frac{\gamma_{sr}(m(G))}{2} \leq \gamma_t(G) + \triangle(G)$

$\Longrightarrow \left\lfloor \frac{\gamma_{sr}(m(G))}{2} \right\rfloor \leq \frac{\gamma_{sr}(m(G))}{2} \leq \gamma_t(G) + \triangle(G)$.

The following theorem relates $\gamma_{sr}(m(G))$ to $\gamma_s(G)$ and $\gamma_c(G)$.

Theorem 3.7. For each G, $\gamma_{sr}(m(G)) \leq \gamma_s(G) + \gamma_c(G)$.

Proof. In a graph G, consider a dominating set $X \subseteq V(G)$ and the sub graph $\langle V - X \rangle$ is disconnected. Then X forms a split dominating set in G and $|X| = \gamma_s(G)$. If a sub graph $\langle Y \rangle$ is connected, then a dominating set $Y \subseteq V(G)$ is considered a connected dominating set of G. In this case $|Y| = \gamma_c(G)$. Now, let D constitute a $\gamma_{sr}-$set in $m(G)$ so that $|D| = \gamma_{sr}(m(G))$. It is simple to confirm that $|D| \leq |X \cup Y|$ using the definitions of split domination, connected domination, and split regular domination. Thus,

$\gamma_{sr}(m(G)) \leq \gamma_s(G) + \gamma_c(G)$.

We develop the relationship between $\gamma_{sr}(m(G))$, $\alpha_0(G)$ and $\beta_1(G)$ in the succeeding result.

Theorem 3.8. For every G, $\gamma_{sr}(m(G)) < \alpha_0(G) + \beta_1(G)$.

Proof. A collection of points in G which covers each line of G is known as vertex cover. The lowest cardinality of G's vertex cover is $\alpha_0(G)$. If there are no two neighboring edges in the set, then $\beta_1(G)$ is the greatest cardinality of an independent set of lines. As required a minimum regular dominating set $D \subseteq E \cup C \subseteq V(m(G))$ in $m(G)$ and $N(D) \in V(m(G)) - D$ are not connected when C is G's cut vertex set. D therefore forms a $\gamma_{sr}-$set in $m(G)$. Given that the total of $\alpha_0(G)$ and $\beta_1(G)$ typically exceeds $|D|$, obtaining the desired outcome is straightforward. That is, $\gamma_{sr}(m(G)) < \alpha_0(G) + \beta_1(G)$.

An upper bound on $\gamma_{sr}(m(G))$ was obtained by the subsequent theorem.

Theorem 3.9. If $\triangle(G)$ is the greatest degree of G, then for each G,

$\gamma_{sr}(m(G)) \leq \left\lceil \frac{p.\triangle(G)}{\triangle(G)+1} \right\rceil$.

Proof. Given $\triangle(G)$ represents the largest degree between the points of G. Assume that $X \in V(m(G))$ be the smallest split regular dominating set in a litact graph $m(G)$. Since X is minimum, there is a vertex $v \in V(m(G)) - X$ so that u is next to v for each point u $\in X$. This suggests that $m(G)$'s split regular dominating set is determined if $V(m(G)) - X$ is not connected. Therefore, the fact that $|X| \leq \frac{p.\triangle(G)}{\triangle(G)+1}$ implies the outcome. Thus,

$\gamma_{sr}(m(G)) \leq \left\lceil \frac{p.\triangle(G)}{\triangle(G)+1} \right\rceil$

We derive $\gamma_{sr}(m(T))$'s lower bound in terms of T's cut vertices.

Theorem 3.10. T is any tree with a cut vertex set $C \geq 1$, then $\gamma_{sr}(m(T)) \geq |C| - 1$.

Proof. Suppose $V = \{v_i\}; 1 \leq i \leq n$ is set of points of a tree T and $C = \{v_j\} \subset \{v_i\}$ is T's cut vertex set. Additionally, let $J = \{e_j\}; 1 \leq j \leq l$ is set of all non-end edges in T such that $J \subset E(T)$ where $E(T) = \{e_1, e_2, \ldots, e_m\}$. The point set V' of $m(T)$ is represented by $V' = \{v_k'\} \subseteq E(T) \cup C(T) \subseteq V(m(T)); 1 \leq k \leq n-1$. Now, a regular dominating set $D \subseteq V(m(T))$ so that, $\langle V(m(T) - D\rangle$ is disconnected then D represents the smallest split regular dominating set in $m(T)$. Now, it is evident that $|D| \geq |C| - 1$ which follows the result.

$\gamma_{sr}(m(G))$, $\gamma_t(G)$ and $p(G)$ relation is explained by the succeeding theorem.

Theorem 3.11. For each $(p,\ q)$ graph G, $\gamma_{sr}(m(G)) \geq \left\lceil \frac{p}{2} \right\rceil - \gamma_t(G)$.

Proof. Given $V_1 = \{v_k\} \subseteq V(G); \forall 1 \leq k \leq n$ is the collection of every vertex in G that isn't an end. Assume that $V_2 \subseteq V_1$ is the minimal set of points in G. If, on the sub graph $\langle V_2\rangle$, $deg(v_i) \geq 1;\ \forall v_i \in V_2,\ 1 \leq i \leq m$, then V_2 constitutes a complete dominating set of G. Otherwise, attach the vertices $w_i \in N(v_i)$ if $deg(v_i) < 1$ in order to make $deg(v_i) \geq 2$ and ensure that $\langle V_2 \cup \{w_i\}\rangle$ does not contain any isolated vertex. It is evident that $V_2 \cup \{w_i\}$ constitutes G's minimal total dominating set. Let $E' = \{e_j'\}; 1 \leq j \leq k-1$ be the edge set that results from sandwiching a degree two vertex between each edge of $m(G)$ so that $E' \subset E(m(G))$ and in addition, $V(m(G)) = E(G) \cup C(G)$ where C is G's cut vertex set. Let a smallest regular dominating set $D = \{v_1, v_2, \ldots, v_{k-1}\} \subseteq V(m(G)) = E(G) \cup C(G)$ where E is the set of lines in G. From this discussion, D forms a minimal split regular dominating set in $m(G)$ if $\langle V(m(G) - D\rangle$ is disconnected and $|D| \cup |V_2 \cup \{w_1\}| \geq \left\lceil \frac{|V(G)|}{2} \right\rceil$. Thus,

$\gamma_{sr}(m(G)) + \gamma_t(G) \geq \left\lceil \frac{p}{2} \right\rceil$

$\Longrightarrow \gamma_{sr}(m(G)) \geq \left\lceil \frac{p}{2} \right\rceil - \gamma_t(G)$.

The results below can be used to support future findings.

Theorem A [4]. For any graph G, $\gamma_s(G) \leq \alpha_0(G)$.

Theorem B[5]. For any (p,q) graph G, $\gamma_{cot}(G) + 2 \leq \gamma_{sns}(G)$.

Theorem C [6]. For every graph G, $\gamma_{sm}(G) \leq \alpha_0(G) + \gamma(G) + diam(G)$.

Theorem D [7]. For any Gear graph of edges (G_n), $\gamma_s(G_n) \geq 3$, where $n \geq 3$.

Theorem E [8]. For any graph G, $\gamma_m^{-1}(G) \leq p - \beta_1(G)$.3

Theorem 3.12. For any graph G, $\gamma_{sm}(G) \leq \gamma_{sr}(m(G)) + q + \alpha_0(G)$.

Proof. Using Theorem-3.3, we obtain

$$\gamma(G) + diam(G) \leq \gamma_{sr}(m(G)) + q \tag{1}$$

From Theorem-C, we get $\gamma_{sm}(G) \leq \alpha_0(G) + \gamma(G) + diam(G)$

$$\Longrightarrow \gamma_{sm}(G) - \gamma(G) - diam(G) \leq \alpha_0(G) \tag{2}$$

Adding Equations (1) and (2) yields the following outcome.

$\gamma_{sm}(G) \leq \gamma_{sr}(m(G)) + q + \alpha_0(G)$.

Theorem 3.13. For any Gear graph of edges (G_n),

$2 + \gamma_{sr}(m(G)) < \gamma_s(G_n) + \gamma'(G) + 2\delta(G)$, where $n \geq 3$.

Proof. Theorem-3.5 and Theorem-D can be used to get this outcome.

Theorem 3.14. For any graph G,

$\left\lfloor \frac{\gamma_{sr}(m(G))}{2} \right\rfloor + \gamma_{cot}(G) + 2 \leq \gamma_t(G) + \triangle(G) + \gamma_{sns}(G)$

Proof. This outcome is derived from Theorem 3.6 and Theorem B.

Theorem 3.15. For any graph G, $\gamma_{sr}(m(G)) \leq \gamma_c(G) + \alpha_0(G)$.

Proof. From Theorem 3.7,

$$\gamma_{sr}(m(G)) - \gamma_s(G) \leq \gamma_c(G) \tag{3}$$

From Theorem A,

$$\gamma_s(G) \leq \alpha_0(G) \tag{4}$$

Equations (3) and (4) are added to obtain, $\gamma_{sr}(m(G)) \leq \gamma_c(G) + \alpha_0(G)$.

Theorem 3.16. For any graph G, $\gamma_{sr}(m(G)) + \gamma_m^{-1}(G) < p + \alpha_0(G)$.

Proof. This outcome can be established by applying Theorem 3.8 and Theorem E.

Next, Nordhaus-Gaddum type outcomes are established at the end.

Theorem 3.17. For any (p, q) connected graphs G and $\bar{G}$,

i) $\gamma_{sr}(m(G)) + \gamma_{sr}\left(m\left(\bar{G}\right)\right) < 2p$

ii) $\gamma_{sr}(m(G)) . \gamma_{sr}\left(m\left(\bar{G}\right)\right) < pq$.

Proof. It is obvious, the above-mentioned results hold for any (p, q) connected graphs G and $\bar{G}$.

4 Conclusion

Encryption relies heavily on graph features. A graph's dominating set is utilized to quickly crack the code. In a litact graph, the idea of a new domination parameter named split regular domination has been presented. To facilitate communication, the information retrieval system allows both the dominating set and the elements of split regular dominating sets to stand alone. It also has several uses in computer science, operations research, switching circuits, coding theory, and many other fields. In this instance, we explore and validate the findings on split regular domination in a graph's litact graph. Additionally, we establish a few relationships between this novel domination parameter and a few other standard parameters of a graph G. These findings are quite helpful for applying domination theory to actual circumstances. The results can be expanded upon, and the larger applications of the parameters studied by the readers in further work.

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