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RESEARCH ARTICLE

Anisotropy Bianchi Type-III Cosmological Model in Brans-Dicke Theory of Gravitation

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Abstract

Objectives: To derive the multi-fluid anisotropic Bianchi type-III cosmological model filled with radiation and matter in the presence of Brans-Dicke scalar-tensor theory. **Method:** In this paper, we have calculated Brans-Dicke field equations in anisotropic Bianchi type -III space times in the presence of matter and radiation which are minimally interacting fields. To obtain the determinate solution of the model, we considered i) the state of the equation $(\rho_m - 3p_m) = 0$, ii) the shear scalar σ is directly proportional to the expansion scalar θ i.e., $\sigma\alpha\theta$. Also, consider the deceleration parameter, $q = -\gamma t + \alpha - 1$, where $\gamma \geq 0, \alpha \geq 0$ with $\gamma = 0$ which yields models with constant deceleration parameter. **Findings:** This solution shows an anisotropic radiation model in Brans-Dicke's theory. Also, it shows accelerated expansion of the universe and transition of the universe from the deceleration phase to the accelerated phase. **Novelty:** The derived model's many parameters support recent cosmological findings, indicating the model's good discovery. Thus, within the framework of our established model, we are able to comprehend the cosmic development.

Keywords: Bianchi type-III; Brans-Dicke theory; Multifluid; Deceleration parameter; Λ CDM

1 Introduction

During the last few decades, there has been a surge in interest in building cosmological models based on many alternative theories of gravity. The Brans-Dicke scalar tensor theory is one of them. Importance of this theory-: 1) it has enormous cosmic consequences. 2) The scalar field is important in the cosmological model. 3) This theory produces a long-range scalar field that interacts similarly with all kinds of matter with the metric tensor and dimensionless coupling constant except electromagnetism. 4) It suggests a probable solution for the missing matter problem that arises in non-flat FRW cosmologies. Many authors have discussed several aspects of Brans-Dicke theory viz, Neelima et al.[1], Nimkar et al.[2] Tripathy et al.[3], Kapse[4], Sadeghi et al.[5], Aditya et al.[6], Singh et al.[7], Singla et al.[8], Kumar et al.[9], Sharif et al.[10], Rasouli et al.[11].

In this theory, a cosmological model based on two fluids is preferable to a single-fluid model with two sequences. Two fluids here denote matter and radiation. As the cosmos evolves, this type of two-fluid model helps to explain the transition from the era of radiation domination to the era of matter domination. We have assumed in this theory that throughout cosmic evolution, both fluids exist with one fluid dominating the other. Some authors have discussed this aspect viz; Solanke et al.[(12)], Pawar et al.[(13)], Santhi et al.[(14)], Trivedi et al.[(15)], Kumar et al.[(16)], Siddiqui et al.[(17)], Goswami et al.[(18)], Borikar[(19)], Raju et al.[(20)].

We have taken up the study of two fluid cosmological models in Bianchi type III space-time in the framework of Brans-Dicke[(21)] theory based on the above discussion and research. to obtain a definite solution of the field equation, we consider the state of the equation and shear scalar is directly proportional to the expansion scalar. We also looked at the linear deceleration parameter given by Akarsu and Dereli[(22)], which we reduce to Berman's law[(23)]. The exact field equations of Brans-Dicke's theory with two fluid sources have been derived under specific physical assumptions in the third section. We looked at a few cosmological, kinematic, and physical features in section 4. In section 5 we reviewed the findings and section 6 has the written conclusion.

2 Metric and field equation

The Bianchi type-III space-time is spatially homogenous and anisotropic, has been considered here,

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 dz^2 \tag{1}$$

Scale factors A, B, and C are the functions of cosmic time t only.

The field equations of the combined scalar and tensor fields, according to Brans-Dicke's[(21)] theory are

$$R_{ij} - \frac{1}{2} g_{ij} R + \omega \phi^{-2} (\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k}) + \phi^{-1} (\phi_{i;j} - g_{ij} \phi^{k}_{;k}) = -8\pi \phi^{-1} (T^{(m)}_{ij} + T_{ij}(r)) \tag{2}$$

In this equation R is the Ricci scalar, R_{ij} is the Ricci tensor, ω is the dimensionless constant and ϕ is the Brans-Dicke scalar field.

The energy-momentum tensor of two fluid source is,

$$T_{ij} = T_{ij}{}^{(m)} + T_{ij}{}^{(r)} \tag{3}$$

Here $T_{ij}{}^{(m)}$ is the energy momentum tensor of matter field and $T_{ij}{}^{(r)}$ is the energy-momentum tensor of radiation field and are given by (Coley and Dunn[(24)])

$$T_{ij}{}^{(m)} = (p_m + \rho_m) u_i{}^m u_j{}^m - p_m g_{ij} \tag{4}$$

$$T_{ij}{}^{(r)} = \left(\frac{4}{3}\right) \rho_r u_i{}^r u_j{}^r - \left(\frac{1}{3}\right) \rho_r g_{ij} \tag{5}$$

where ρ_m, ρ_r are the energy densities of matter and radiation and p_m is the pressure.

Also, the scalar field satisfies the following wave equation

$$\phi_{;k}{}^{k} = 8\pi (3 + 2\omega)^{-1} \left(T_{ij}{}^{(m)} + T_{ij}{}^{(r)}\right) \tag{6}$$

The energy conservation equation is

$$\left(T_{ij}{}^{(m)} + T_{ij}{}^{(r)}\right)_{;j} = 0 \tag{7}$$

With the use of Equation (3), BD field Equations (2) and (6) in a commoving co-ordinate for the metric (Equation (1)) can be given as,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{BC} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -8\pi \phi^{-1} \left(p_m + \frac{\rho_r}{3}\right) \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}C}{AC}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)=-8\pi\phi^{-1}\left(p_m+\frac{\rho_r}{3}\right) \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=-8\pi\phi^{-1}\left(p_m+\frac{\rho_r}{3}\right) \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{B\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{A^2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=-8\pi\phi^{-1}\left(\rho_m+\rho_r\right) \tag{11}$$

$$\frac{A}{A}-\frac{B}{B}=0 \tag{12}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=8\pi(3+2\omega)^{-1}\left(\rho_m-3p_m\right) \tag{13}$$

Where an overhead dot denotes differentiation with respect to t.

From conservation Equation (7), we get;

$$\dot{\rho}_m+\dot{\rho}_r+(\rho_m+\rho_r)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\left(p_m+\frac{\rho_r}{3}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{14}$$

We have, the conservation equations split into two parts,

$$\dot{\rho}_m+(\rho_m+p_m)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{15}$$

$$\dot{\rho}_r+\left(\rho_r+\frac{\rho_r}{3}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{16}$$

We get from Equation (12),

$$A=k_1B \tag{17}$$

In which k_1 is a constant of integration which can be set equal to the set unity. As a result of the (Equation (17)), the field Equations (8), (9), (10), (11), (12) and (13) become the following independent equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{B\dot{C}}{BC}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=-8\pi\phi^{-1}\left(p_m+\frac{\rho_r}{3}\right) \tag{18}$$

$$2\frac{\ddot{B}}{B}+\left(\frac{\dot{B}}{B}\right)^{2}-\frac{1}{B^2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{\phi}}{\phi}\frac{\dot{B}}{B}=-8\pi\phi^{-1}\left(p_m+\frac{\rho_r}{3}\right) \tag{19}$$

$$2\frac{B\dot{C}}{BC}+\left(\frac{\dot{B}}{B}\right)^{2}-\frac{1}{B^2}-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\dot{\phi}}{\phi}\left(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=-8\pi\phi^{-1}\left(p_m+\rho_r\right) \tag{20}$$

$$\ddot{\phi}+\dot{\phi}\left(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=8\pi(3+2\omega)^{-1}\left(\rho_m-3p_m\right) \tag{21}$$

The Spatial Volume (V) and the average scale factor $(a(t))$ for the metric (Equation (1)) is defined as,

$$a(t) = \left(B^2C\right)^{\frac{1}{3}}$$

$$V = a^3(t) = B^2C \tag{22}$$

The Hubble parameter is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{23}$$

where, $H_1 = H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the direction of x, y and z axes respectively.

The average anisotropic parameter is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \; where \; \Delta H_i = H_i - H \tag{24}$$

The expansion scalar θ and shear scalar σ are given by

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{1}{3}\left[\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right]^2 \tag{25}$$

$$And \; \theta = 2\frac{B}{B} + \frac{C}{C}, \; \sin ce \; \theta = 3H \tag{26}$$

3 Solutions of the field equations

There are six unknowns in all of the four independent Equations (18), (19), (20) and (21). Two more conditions are required to obtain a determinate solution. So, we consider,

i) the state of the equation

$$(\rho_m - 3p_m) = 0 \tag{27}$$

ii) the shear scalar σ is directly proportional to the expansion scalar θ

$$\sigma \alpha \theta$$

$$\sigma = k_2\theta \tag{28}$$

From Equations (25), (26) and (28),

$$B = k_3 C^{\frac{\sqrt{3}k_1 + 1}{1 - 2\sqrt{3}k_1}} \tag{29}$$

Now, consider deceleration parameter (Akarsu and Dereli[22]), which is as follows:

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -\gamma t + \alpha - 1 \tag{30}$$

where $\gamma \geq 0$ and $\alpha \geq 0$ are constant and $\gamma = 0$ reduces to the law of Berman[23] which yields models with constant deceleration parameter.

$$a(t) = k_6 (k_4 t + k_5)^{\frac{1}{\alpha}}, \quad for \; \gamma = 0 \quad and \; \alpha > 0 \tag{31}$$

From Equations (17), (22), (29) and (31), we obtain

$$A = k_8 \left(k_4 t + k_5\right) \frac{\sqrt{3}k_2 + 1}{\alpha} \tag{32}$$

$$B = k_8 \left(k_4 t + k_5\right) \frac{\sqrt{3}k_2 + 1}{\alpha} \tag{33}$$

$$C = k_7 \left(k_4 t + k_5\right) \frac{1 - 2\sqrt{3}k_2}{\alpha} \tag{34}$$

Therefore, the metric (Equation (1)) takes the form,

$$ds^2 = dt^2 - \left[k_8 \left(k_4 t + k_5\right)^{\frac{\sqrt{3}k_2 + 1}{\alpha}}\right]^2 dx^2 - \left[k_8 \left(k_4 t + k_5\right) \frac{\sqrt{3}k_2 + 1}{\alpha}\right]^2 e^{-2x} dy^2 - \left[k_7 \left(k_4 t + k_5\right) \frac{1 - 2\sqrt{3}k_2}{\alpha}\right]^2 dz^2 \tag{35}$$

From Equation (26), we get

$$\phi = k_{11} \left(k_4 t + k_5\right)^{\frac{\alpha - 3}{\alpha}} + \phi_0 \tag{36}$$

4 Cosmological parameters of the model

Using the Brans-Dicke scalar-tensor theory of gravitation, two fluids model of Bianchi type III space time is described by Equation (35). The following physical and kinematical aspects are important in this theory and are essential to cosmological discussions.

Spatial volume is,

$$V = k_9 \left(k_4 t + k_5\right)^{\frac{3}{\alpha}} \tag{37}$$

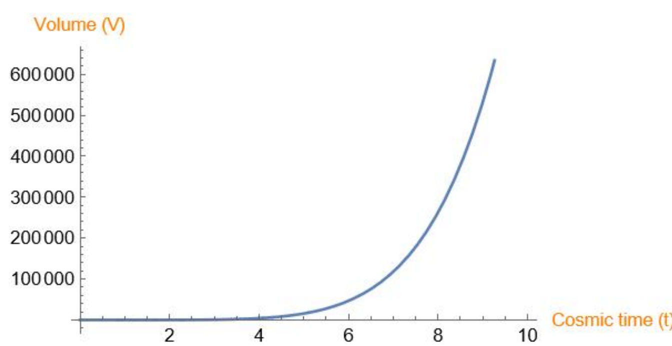

Fig 1. V vs*t*

Scale factor

$$a^3(t) = k_9 \left(k_4 t + k_5\right)^{\frac{1}{\alpha}} \tag{38}$$

Hubble parameter

$$H = \frac{k_4}{\alpha \left(k_4{}^{t + k_5}\right)} \tag{39}$$

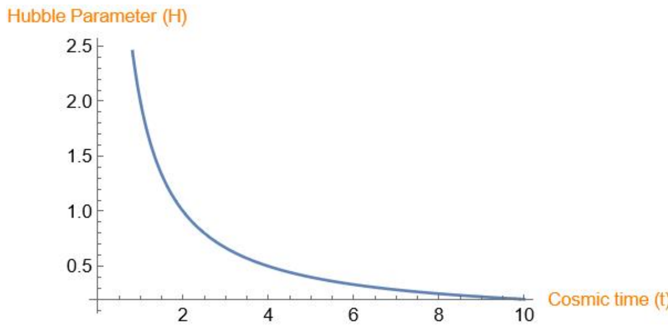

Fig 2. H **vs** t

Expansion Scalar

$$\theta = \frac{3k_4}{\alpha\,(k_4 t + k_5)} \tag{40}$$

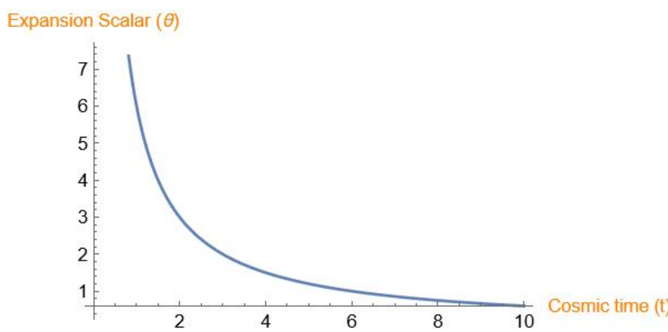

Fig 3. θ **vs** t

Shear scalar

$$\sigma^2 = \frac{9k_2 {k_4}^2}{\alpha^2\,(k_4 t + k_5)^2} \tag{41}$$

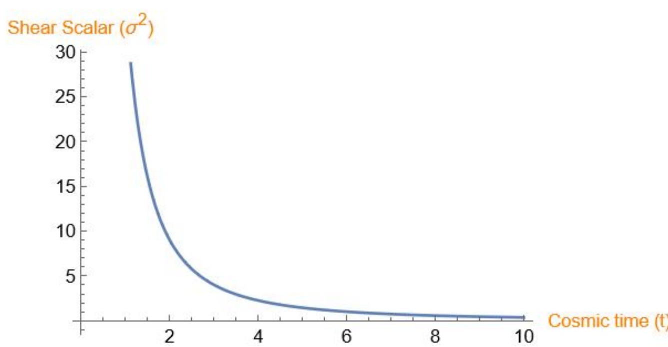

Fig 4. σ^2 **vs** t

From Equations (15) and (16) the energy density for matter and radiation and pressure are given by,

$$\rho_m = 3k_{12}(k_4 t + k_5)^{-\frac{4}{\alpha}} \tag{42}$$

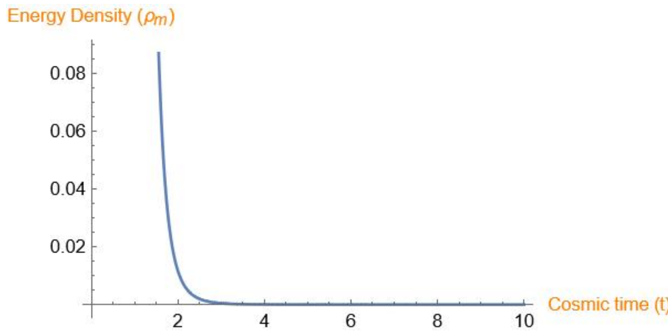

Fig 5. ρ_m **vs** t

$$\rho_r = k_{12}\left(k_4 t + k_5\right)^{-\frac{4}{\alpha}} \tag{43}$$

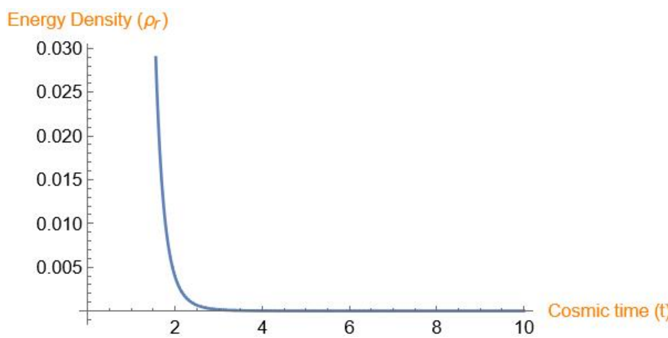

Fig 6. ρ_r **vs** t

$$p_m = k_{12}(k_4 t + k_5)^{-\frac{4}{\alpha}} \tag{44}$$

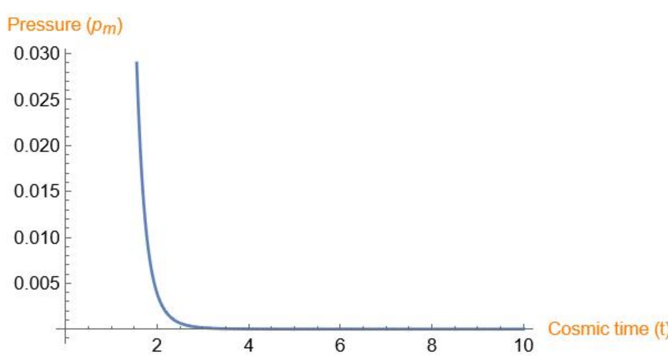

Fig 7. p_m **vs** t

$$q = -1 + \alpha \tag{45}$$

The jerk parameter (j) is a dimensionless and third derivative of the scale factor with respect to cosmic time (t). It is defined as,

$$j(t)=\frac{a}{aH^3} \tag{46}$$

Equation (45) can be written as; $j(t)=q+2q^2-\frac{\dot{q}}{H}=$ constant

$$j(t)=1+\alpha(2\alpha-3) \tag{47}$$

It has been believed by the Cosmologists that deceleration to acceleration transition of the universe occurs for models with negative value of deceleration parameter and positive value of jerk parameter. From Equation (47), it is observed that for any value α, the jerk parameter is positive except $\alpha=0$. If $\alpha=0$, then value of the jerk parameter is one. i.e., $j=1$. So that for $\alpha=0$, Flat Λ CDM model is obtained.

The look back time (t_L) to an object is the difference between the present age of the Universe (t_0) and the age of the Universe at the time (t) the photons were emitted (according to the object). Look back t_L time is given by,

$$t_L=t_0-t(z)=\int_R^{R_0}\frac{dR}{R} \tag{48}$$

Suffix ' 0 ' refers to present epoch. Here R_0 is the size of universe at the time the light from object is observed and R is the size of Universe at the time it was emitted. i.e., scale factor Cosmological red shift z is directly proportional to size of Universe i. e. scale factor $R(t)$ and is written as,

$$1+z=\frac{R(t_0)}{R(t)} \tag{49}$$

From Equations (48) and (49), we get

$$H_0(t_0-t(z))=\frac{1}{\alpha}\left[1-(1+z)^{-\alpha}\right] \tag{50}$$

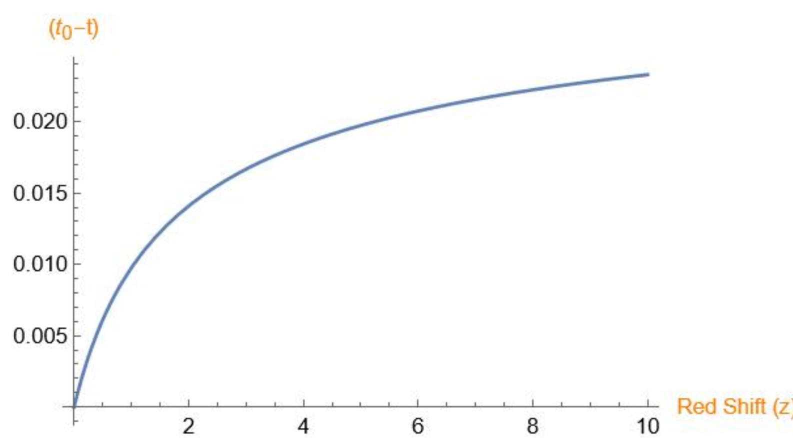

Fig 8. (t_0-t) **vs** z

The proper distance $(d(z))$ is defined as the distance between a cosmic source emitting light at any instant $t=t_1$ located at $r=r_1$ with red shift (z) and the observer receiving the light from the source emitted at $r=0$ and $t=t$. The proper distance $d(z)$ is given by,

$$d(z)=r_1R_0 \tag{51}$$

Where $r_1=\int_t^{t_0}\frac{dt}{R}$

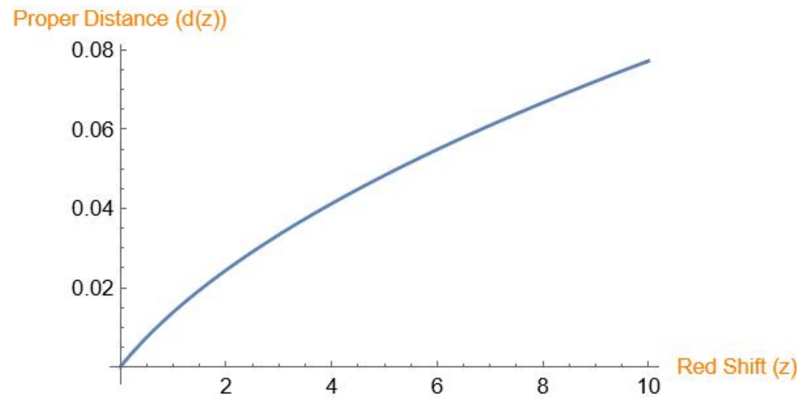

Fig 9. $d(z)$ **vs z**

From Equations (49) and (51), we get,

$$d(z) = \frac{(k_9)^{\frac{2}{\alpha}}}{H_0(\alpha-1)}\left[1-(1+z)^{1-\alpha}\right] \tag{52}$$

Luminosity distance is given by,

$$d_L = r_1(z)R_0(1+z) = d(z)(1+z) \tag{53}$$

Using Equation (52) in Equation (53), we get,

$$d_L = \frac{(k_9)^{\frac{2}{\alpha}}}{H_0(\alpha-1)}\left[1-(1+z)^{1-\alpha}\right](1+z) \tag{54}$$

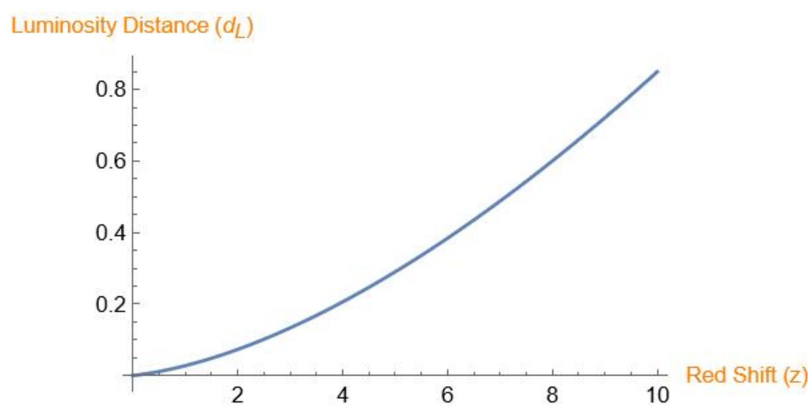

Fig 10. $d_L vs z$

The ratio of an object's physical transverse size to its angular size yields the angular diameter distance.
Angular diameter distance is given by,

$$d_\Lambda = d(z)(1+z)^{-1} \tag{55}$$

Using Equation (52) in Equation (55), we get

$$d_A = \frac{(k_9)^{\frac{2}{\alpha}}}{H_0(\alpha-1)}\left[1-(1+z)^{1-\alpha}\right](1+z)^{-1} \tag{56}$$

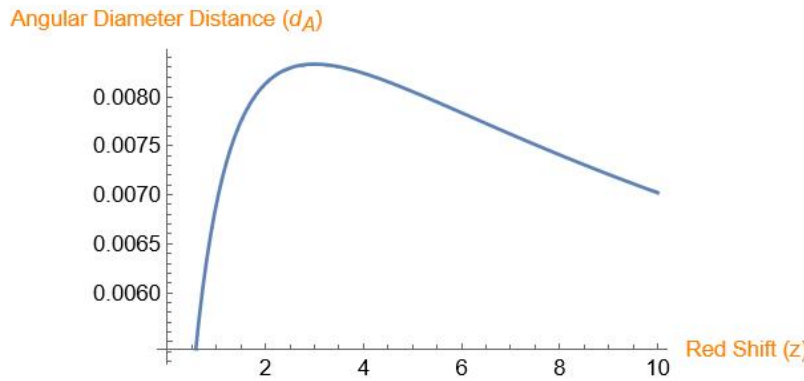

Fig 11. d_A **vs** z

The distance modulus $\mu(z)$ is given by,

$$\mu(z) = 5 log d_L + 25 \tag{57}$$

Using Equation (54) in Equation (57), we get

$$\mu(z) = 5log\left[\frac{(k_9)^{\frac{2}{\alpha}}}{H_0(\alpha-1)}\left[1-(1+z)^{1-\alpha}\right](1+z)\right]+25 \tag{58}$$

5 Result and Discussion

The multi-fluid anisotropic Bianchi type-III cosmological model filled with radiation and matter in the presence of Brans-Dicke scalar-tensor theory is represented by Equation (35). One noticed that our investigation related to cosmological model in the presence of constant deceleration parameter is a viable cosmological model. The physical and kinematical properties of the obtained model are also discussed below:

- Figure 1, shows spatial volume of the model increases with time and vanishes at $t = 0$. Hence, we have an expanding model having point-type singularity. It can be seen from Figures 2, 3 and 4 that H, θ and shear scalar decreases as time increases.
- It can be also observed from Figures 5 and 6 that energy density diverges at the initial epoch $t = 0$ and decreases as the universe expands as $t \to \infty$. Figure 7 shows that pressure is infinite during the early stage of the universe, and it decreases as time increases. We observed that our results of the transition are very similar to Raju et al.[20].
- The relation $\frac{\sigma^2}{\theta^2} = k_2 =$ constant, shows that the model given by Equation (35) does not approach to isotropy for $k_2 \neq 0$. Hence, our model is fully anisotropic and spatially homogeneous. It is in contrast with the result discussed by Nimkar et al.[2] and Pawar et al.[13] as their models does not approach to isotropy.
- A positive sign of the deceleration parameter q indicates the standard decelerating model, whereas the negative sign indicates the inflating model. Recent observation shows that the deceleration parameter of the range of $-1 < q < 0$ and the present-day universe is undergoing an accelerated expansion. From Equation (45), it is observed that the deceleration parameter q is in the range $(-1, 0)$ for $\alpha \in (0, 1)$ and, hence, the model represents an accelerated expansion of the universe. In this cosmological model, it is seen that the jerk parameter is positive for any value of α except at $\alpha = 0$. When $\alpha = 0$, then the value of jerk parameter (j) is one. Therefore, flat Λ CDM model is obtained. The transition of the universe which takes place from the deceleration phase to the accelerated phase is seen from the negative deceleration parameter and positive jerk parameter. Also, we can observe that our cosmological parameters satisfy the result of the cosmological model constructed by Basumatay[25].
- According to Figure 8, the look-back time grows as the redshift increases, indicating that light from previous cosmic epochs with higher red shift values travelled for a long time before reaching us. From Figure 9, we can see that the correct distance increases as z increases. Thus, light sources with higher redshifts are more distant according to well-defined

cosmological distances. As $z \to \infty, d(z) \to 0$ expands which starts with a big bang. Figure 10, shows that the luminosity distance increases continuously with redshift. Sources at higher redshifts appear brighter because greater spatial expansion causes light to be diluted at greater distances. Figure 11, shows the distance module, which determines the decrease of light from distant sources due to spatial expansion. We see an increase in $\mu(z)$ with z, indicating that higher- z sources appear fainter than low- z sources, as expected in the expanding universe model, which makes light travel long distances. The same result was found in a previous study that considered two fluid cosmological models in the $f(R,T)$ theory of gravitation (Solanke et al.[12]).

6 Conclusion

Using Bianchi space-time in the context of Brans-Dicke's theory of gravity, we developed two fluid cosmologic models. Using physically meaningful assumptions, we have derived an exact solution to very nonlinear field equations. In general, model (Equation (35)) is anisotropic. The scalar field decreases steadily and at $t \to \infty$ tends to be constant. The early universe has an infinitely large energy density and as it expands, this energy density decreases with cosmic time t. At large times $t \to \infty$, the energy density becomes null. As time increases to infinity, the scale factor and volume tend to infinity but the σ, θ and H tends to zero. Therefore, the rate of expansion decreases with increasing time. From Equation (45) we can see that $-1 < q < 0$ and the present is going through an accelerated expansion. The jerk parameter (j) has a value of one which is similar to the results discussed by Raju et al.[20]. Consequently, a flat Λ CDM model is produced. In addition, we have extracted expressions for the look back time redshift, the proper distance, the luminosity distance, and the distance modulus vs redshift, and discussed these expressions and their importance. The next step was to look at the changes in each of the cosmological parameters corresponding to those model parameters. The results of this analysis of the various parameters show that the model obtained represents a cosmological model that is expanding and accelerating, which is consistent with more recent models of cosmology. This study will provide some information on the structure of the universe which has astrophysical implications.

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