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RESEARCH ARTICLE

M/M/1 Interdependent Queuing Model with Vacation and Controllable Arrival Rates

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Abstract

Objectives: Rather than working nonstop in the service area, servers take vacations when they have no clients. To determine the probability and features of the queuing system, this study introduces controllable arrival rates and interdependency in the system's service and arrival processes. It also performs a numerical verification of the results. **Methods:** A recursive method is employed to solve the steady-state probability equations, yielding explicit iterative formulas under the assumption that a single server provides services to all clients. Here, customer arrivals are controlled as either faster or slower, with Poisson assumed by default. **Findings:** For this model, steady-state solutions and characteristics are derived and explored, and some numerical analysis is carried out using MATLAB. All the probabilities are expressed in terms of $P_{0,0}(0)$, which indicates the system when empty. The movement of the average number of customers in the system and the expected waiting time, L_s and W_s respectively, of the customers in the system is investigated through a graph. L_s and W_s decrease when dependence service rate, and faster arrival rate increase. Additionally, L_s increases and W_s decreases when the slower arrival rate increases. **Novelty:** Although there have been studies on vacation in queuing theory, this new approach aims to bridge the gap between vacation and interdependency in the arrival and service process, as well as controllable arrival rates. When vacations with predictable arrival rates are utilised advantageously for the benefit of both the server and the client, waiting times may be minimised and the most practical, economical service can be provided.

Keywords: Markovian Queuing System; Vacation; Loss and Delay; Finite Capacity; Interdependent Arrival and Service Rates; Varying Arrival Rates; Bivariate Poisson Process

1 Introduction

The model of queuing process that the present research, assumes a vacationing server with a regulated arrival rate of indefinite customers. Between vacation and controlled arrival rates, this article fills the gap. Previously, the M/M/1/∞ interdependent queuing model with controllable arrival rates was covered by Srinivasa Rao et al.[1] but that study does not include vacation rate. The research gap in vacation rate is now included in this article and compared to the previous model. Additional information on controlled arrival rates includes the work of the other authors[2,3] who have also studied the controllable arrival rates and not vacation.

In the 1970s, the Vacation model made its debut. Doshi[4] produced a fantastic overview study on vacation. Additional information about the vacation model is studied from the other articles[5–11]. All these authors have studied the vacation model but have not included the controllable arrival rates. Now the present research article studies vacation with two different arrival types—one slower and the other faster. Here, the vacation period is assumed to be when there are no services to be rendered i.e.) when there are no customers in the queue.

In citation[12,13] the same researchers have studied the system with a definite number of customers in the arrival system and about the loss of customers due to delay in providing service making the customer lose his patience and quit the queue, consequences of loss to the business. Now the present research studies about indefinite customer arrival (M/M/1) which is very practical in the modern computer-assisted system and to avoid losing customers due to delayed service. The novelty of the present article is very much suitable in the computer communication system to provide faster service by avoiding delay. This model can be used in a computer communication system to avoid buffering due to excess load. By controlling the arrival rates and giving a vacation rate the system runs smoothly thus providing faster service.

2 Description of the Model

The arrival process and the service process are $\{X_1(t)\}$, $\{X_2(t)\}$ respectively are correlated and follow a bivariate Poisson process given by

$$P[X_1(t) = x_1 \ , \ X_2(t) = x_2] = \frac{e^{-(\lambda_i+\mu-\varepsilon)t} \sum_{j=0}^{min(x_1,x_2)} (\varepsilon t)^j [(\lambda_i-\varepsilon)t]^{x_1-j} [(\mu-\varepsilon)t]^{x_2-j}}{j!\,(x_1-j)!\,(x_2-j)!} \tag{1}$$

Where $x_1, x_2 \geq 0; \lambda_{01},\ \lambda_{02}, \lambda_{11},\ \mu > 0,\ 0 \leq \varepsilon < min(\lambda_{ij}, \mu)$.

i) Here, we consider a single server queuing system with parameter

$\lambda_0,\ \lambda_1$ - Mean faster rate and slower rate of arrivals respectively,

μ - Mean service rate,

ε - Mean dependence rate,

v - Vacation rate.

ii) When the system size increases to R from below the arrival rate which was λ_0 until $R-1$, decreases to λ_1 and remains same for subsequent upward movement of the system size.

iii) When the system size decreases to r from above, the arrival rate which was λ_1 until $r+1$, increases to λ_0 and remains same for subsequent downward movement to 0 and upward movement up to $R-1$. This process is repeated.

iv) The states for the model are as follows:

(a) (0, n) is the state where there are n customers in the queue and the server is in vacation, $n \geq 0$. Its probability is $P_{0,n}$

(b) (1, n) is the state where there are n customers in the system during active service, $n \geq 1$. Its probability is $P_{1,n}$.

3 Steady State Equations

We observe $P_{0,n}(0)$ and $P_{1,n}(0)$ exists when $n = 0,1,2,\ldots,r-1,r$; $P_{0,n}(0),\ P_{1,n}(0),\ P_{0,n}(1),\ P_{1,n}(1)$ exists when $n = r+1,\ r+2,\ldots,R-1$ and $P_{0,n}(1)$ *and* $P_{1,n}(1)$ exists only when $n = R, R+1,\ldots$.

$$(\lambda_0-\varepsilon)P_{0,0}(0) = (\mu-\varepsilon)P_{1,1}(0) \tag{2}$$

$$(\lambda_0+v-\varepsilon)P_{0,n}(0) = (\lambda_0-\varepsilon)P_{0,n-1}(0) \quad (1 \leq n \leq R-1) \tag{3}$$

$$(\lambda_1+v-\varepsilon)P_{0,r+1}(1) = 0 \tag{4}$$

$$(\lambda_1+v-\varepsilon)P_{0,n}(1) = (\lambda_1-\varepsilon)P_{0,n-1}(1) \quad (r+1 \leq n \leq R-1) \tag{5}$$

$$(\lambda_1+v-\varepsilon)P_{0,R}(1)=(\lambda_1-\varepsilon)P_{0,R-1}(1)+(\lambda_0-\varepsilon)P_{0,R-1}(0) \tag{6}$$

$$(\lambda_1+v-\varepsilon)P_{0,n}(1)=(\lambda_1-\varepsilon)P_{0,n-1}(1)\quad (n\geq R+1) \tag{7}$$

$$(\lambda_0+\mu-2\varepsilon)P_{1,1}(0)=(\mu-\varepsilon)P_{1,2}(0)+vP_{0,1}(0) \tag{8}$$

$$(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)=(\lambda_0-\varepsilon)P_{1,n-1}(0)+(\mu-\varepsilon)P_{1,n+1}(0)+vP_{0,n}(0) \quad (2\leq n\leq r-1) \tag{9}$$

$$(\lambda_0+\mu-2\varepsilon)P_{1,r}(0)=(\lambda_0-\varepsilon)P_{1,r-1}(0)+(\mu-\varepsilon)P_{1,r+1}(0)+(\mu-\varepsilon)P_{1,r+1}(1)+vP_{0,r}(0) \tag{10}$$

$$(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)=(\lambda_0-\varepsilon)P_{1,n-1}(0)+(\mu-\varepsilon)P_{1,n+1}(0)+vP_{0,n}(0) \quad (r+1\leq n\leq R-2) \tag{11}$$

$$(\lambda_0+\mu-2\varepsilon)P_{1,R-1}(0)=(\lambda_0-\varepsilon)P_{1,R-2}(0)+vP_{0,R-1}(0) \tag{12}$$

$$(\lambda_1+\mu-2\varepsilon)P_{1,r+1}(1)=(\mu-\varepsilon)P_{1,r+2}(1)+vP_{0,r+1}(1) \tag{13}$$

$$(\lambda_1+\mu-2\varepsilon)P_{1,n}(1)=(\mu-\varepsilon)P_{1,n+1}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+vP_{0,n}(1) \quad (r+2\leq n\leq R-1) \tag{14}$$

$$(\lambda_1+\mu-2\varepsilon)P_{1,R}(1)=(\mu-\varepsilon)P_{1,R+1}(1)+(\lambda_1-\varepsilon)P_{1,R-1}(1)+(\lambda_0-\varepsilon)P_{1,R-1}(0)+vP_{0,R}(1) \tag{15}$$

$$(\lambda_1+\mu-2\varepsilon)P_{1,n}(1)=(\mu-\varepsilon)P_{1,n+1}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+vP_{0,n}(1) \quad (n\geq R+1) \tag{16}$$

Let

$$A=\frac{\lambda_0-\varepsilon}{\mu-\varepsilon},\ B=\frac{\lambda_1-\varepsilon}{\mu-\varepsilon},\ C=\frac{v}{\mu-\varepsilon},\ D=\frac{A}{A+C},\ E=\frac{B}{B+C}$$

From Equation (2) we get

$$P_{1,1}(0)=AP_{0,0}(0) \tag{17}$$

From Equation (3) we get

$$P_{0,n}(0)=D^nP_{0,0}(0)\quad (1\leq n\leq R-1) \tag{18}$$

From Equations (4) and (5) we get

$$P_{0,n}(1) = 0 \quad (r+1 \le n \le R-1) \tag{19}$$

From Equation (6) we get

$$P_{0,R}(1) = \frac{A}{B+C} D^{R-1} P_{0,0}(0) \; = \; J\, P_{0,0}(0) \tag{20}$$

From Equation (7) we get

$$P_{0,n}(1) = E^{n-R} J P_{0,0}(0) \quad (n \ge R+1) \tag{21}$$

From Equations (8) and (9) we get

$$P_{1,n}(0) = \left\{ \left[A + A^2 + \cdots + A^n\right] - \left[1 + A + \ldots A^{n-2}\right] CD - \left[1 + A + \ldots A^{n-3}\right] CD^2 - \cdots - CD^{n-1} \right\} P_{0,0}(0) \quad (2 \le n \le r) \tag{22}$$

From Equations (10) and (11) we get,

$$\begin{aligned} &P_{1,n}(0) = \left\{ \left([A + \cdots + A^n] - \left[1 + A + \cdots + A^{n-2}\right] CD - \left[1 + A + \cdots + A^{n-3}\right] CD^2 - \cdots - CD^{n-1} \right) \right. \\ &\left. P_{0,0}(0) - \left(1 + A + \cdots + A^{n-r-1}\right) P_{1,r+1}(1) \right\} (r+1 \le n \le R-1) \end{aligned} \tag{23}$$

From Equation (12) we get

$$P_{1,r+1}(1) = F P_{0,0}(0) \tag{24}$$

Where,

$$F = \frac{\left\{ (A + A^2 + \cdots + A^R) - (1 + A + \cdots + A^{R-2}) CD - \ldots CD^{R-1} \right\}}{1 + A + \cdots + A^{R-r-1}}$$

From Equations (13) and (14) we get

$$P_{1,n}(1) = \left(1 + B + \ldots . B^{n-r-1}\right) F P_{0,0}(0) \, ; (r+2 \le n \le R) \tag{25}$$

From Equation (15) we get

$$P_{1,R+1}(1) = \left\{ F \begin{bmatrix} [1 + B + \cdots + B^{R-r}] + A\{A + A2 + \cdots + A^{R-1} \\ -(1 + A + \cdots + A^{R-3}) CD - \cdots - CD^{R-2}\} - (1 + A + \cdots + A^{R-r-2}) F \\ -CJ \end{bmatrix} \right\} P_{0,0}(0) \tag{26}$$

From Equation (16) we get

$$\begin{aligned} &P_{1,n}(1) = \{F \left(1 + B + \cdots + B^{n-r-1}\right) + \left(1 + B + \cdots + B^{n-R+1}\right) \\ &\left[A \left[\left(A + A^2 + \cdots + A^{R-1}\right) - \left(1 + A + \cdots + A^{R-3}\right) CD - \cdots - CD^{R-2} - \left(1 + A + \cdots + A^{R-r-2}\right) F \right] - CJ \right] \\ &- \left(1 + B + \cdots + B^{n-R-2}\right) CEJ - \left(1 + B + \cdots + B^{n-R-3}\right) CE^2 J - \cdots - CE^{n-R-1} J \} P_{0,0}(0) \quad (n > R+1) \end{aligned} \tag{27}$$

4 Characteristics of the Model

$$P(0) = \sum_{n=0}^{\infty} P_{1,n}(0)$$

P(0) exists only when $1 \le n \le R-1$, we get

$$P(0) = \sum_{n=1}^{R-1} P_{0,n}(0) + \sum_{n=1}^{r} P_{1,n}(0) + \sum_{n=r+1}^{R-1} P_{1,n}(0) \tag{28}$$

Now,

$$P(1) = \sum_{n=0}^{\infty} P_{1,n}(1)$$

P(1) exists only when $n \geq r+1$

$$P(1) = \sum_{n=r+1}^{\infty} P_{0,n}(1) + \sum_{n=r+1}^{R} P_{1,n}(1) + \sum_{n=R+1}^{\infty} P_{1,n}(1) \tag{29}$$

The system is empty can be calculated from the normalizing condition

$$P(0) + P(1) = 1 \tag{30}$$

The average number of customers in the system

$$L_s = L_{s_0} + L_{s_1} \tag{31}$$

Where

$$L_{s_0} = \sum_{n=1}^{R-1} nP_{0,n}(0) + P_{1,1}(0) + \sum_{n=2}^{r} nP_{1,n}(0) + \sum_{n=r+1}^{R-1} nP_{1,n}(0)$$

and

$$L_{s_1} = \sum_{n=r+1}^{\infty} nP_{0,n}(1) + \sum_{n=r+1}^{R} nP_{1,n}(1) + \sum_{n=R+1}^{\infty} nP_{1,n}(1)$$

Now by using Little's formula, the average waiting time of the customers in the system

$$W_s = \frac{L_s}{\bar{\lambda}} \tag{32}$$

Where

$$\bar{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$$

5 Results and Discussion

For various values of $\lambda_0, \lambda_1, \mu, \varepsilon, v$ the values of $P_{0,0}(0)$, $P(0)$, $P(1)$, L_s, W_s are computed.

Let us assume $r = 4$ and $R = 7$.

Table 1. When there is only one server with vacation, $v = 20$

λ_0	λ_1	μ	ε	$P_{0,0}(0)$	**P(0)**	**P(1)**	L_s	W_s
3	2	4	0	0.6288	0.8403	0.1597	6.6011	2.3241
4	2	4	0	0.2470	0.5925	0.4075	8.0615	2.5311
5	2	4	0	0.0980	0.3203	0.6797	8.8332	2.9834
6	1	5	0.5	0.1245	0.4114	0.5886	8.1063	2.6516
6	3	5	0.5	0.1033	0.3413	0.6587	8.9399	2.2216
6	5	5	0.5	0.0819	0.2707	0.7293	9.7808	1.8557
6	4	5	1	0.0860	0.2812	0.7188	9.3418	2.0476
6	4	8	1	0.6341	0.8727	0.1273	7.4648	1.2993
6	4	9	1	0.8233	0.9302	0.0698	7.4644	1.2737
4	3	4	0	0.2280	0.5469	0.4531	8.4414	2.3799
4	3	4	0.3	0.2324	0.5507	0.4493	8.3518	2.3521
4	3	4	1	0.2446	0.5625	0.4375	8.1086	2.2761

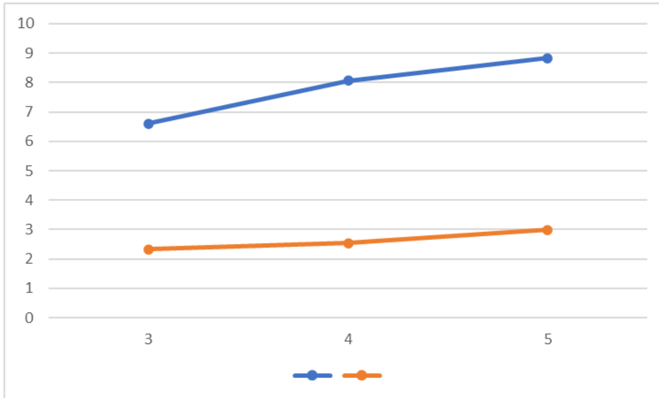

Fig 1. L_s **and** W_s **by varying** λ_0 **and keeping other parameters fixed**

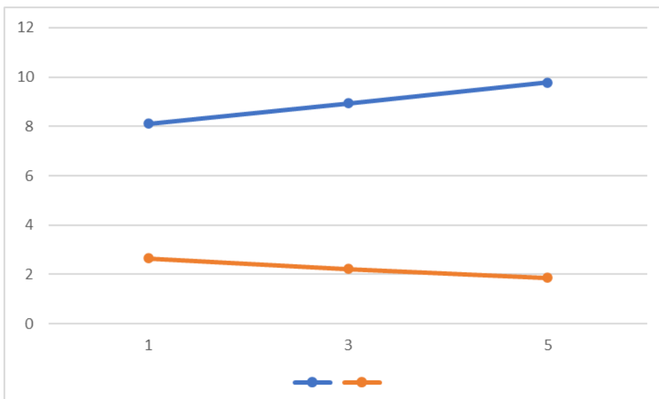

Fig 2. L_s **and** W_s **by varying** λ_1 **and keeping other parameters fixed**

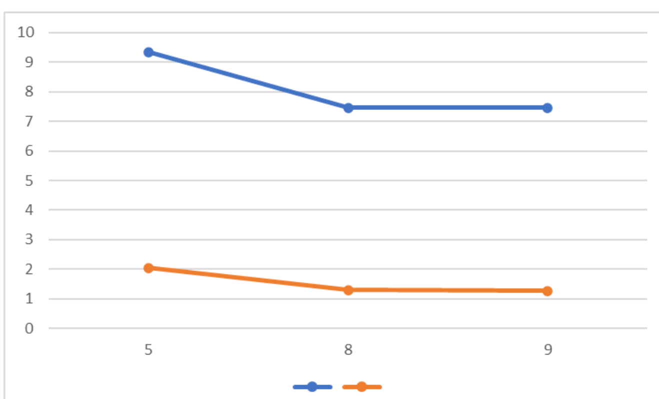

Fig 3. L_s **and** W_s **by varying** μ **and keeping other parameters fixed**

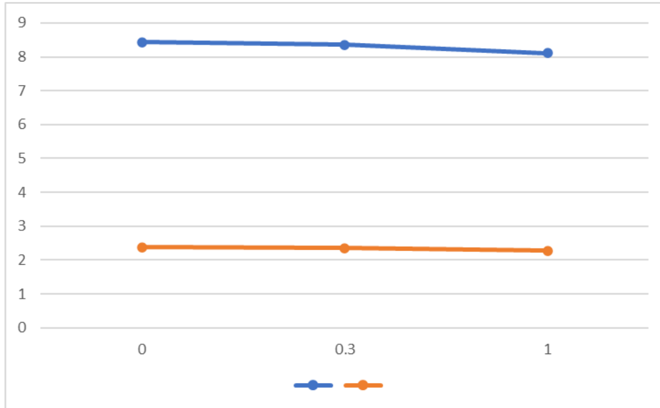

Fig 4. L_s **and** W_s **by varying** ε **and keeping other parameters fixed**

Table 1 is created by assuming that there would be just one server and that vacation time would be taken at a rate of 20. Figures 1, 2, 3 and 4 show how L_s and W_s move when one parameter is altered while the remaining is left constant. We are given a generalised picture of the assumed values of r, R, and v via this table and graphs. Assuming any values will also provide the same.

Table 2. When there is only one server without vacation

λ_0	λ_1	μ	ε	$P_{0,0}(0)$	P(0)	P(1)	L_s	W_s
6	2	4	0	0.0433	0.9367	0.0633	3.6941	0.6428
10	2	4	0	0.0025	0.3216	0.6784	5.4611	1.1942
11	2	4	0	0.0025	0.2826	0.7174	5.5601	1.2239
10	3	5	0.5	0.0058	0.4113	0.5887	5.2255	0.8889
10	5	5	0.5	0.0054	0.3876	0.6124	5.3277	0.7679
10	8	5	0.5	0.0050	0.3538	0.6462	5.4735	0.6286
8	5	2	1	8.2601e-06	0.0486	0.9514	6.4644	1.2563
8	5	3	1	3.5421e-04	0.1557	0.8443	6.0550	1.1076
8	5	4	1	0.0031	0.3115	0.6885	5.5730	0.9391
13	6	3	0	1.2243e-04	0.1183	0.8817	6.1469	0.9002
13	6	3	0.3	7.9908e-05	0.1048	0.8952	6.1900	0.9193
13	6	3	1	2.2316e-05	0.0731	0.9269	6.3001	0.9675

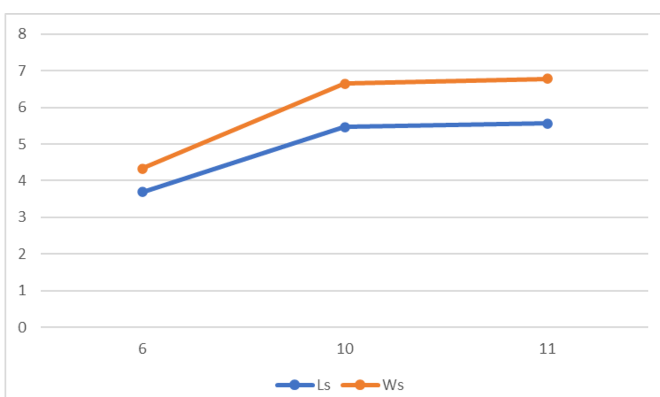

Fig 5. L_s **and** W_s **by varying** λ_0 **and keeping other parameters fixed**

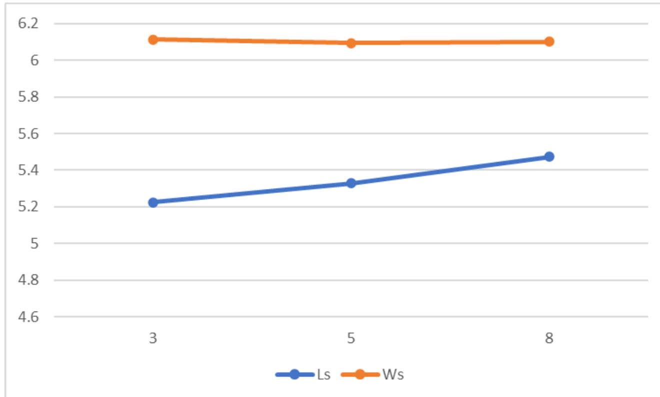

Fig 6. L_s **and** W_s **by varying** λ_1 **and keeping other parameters fixed**

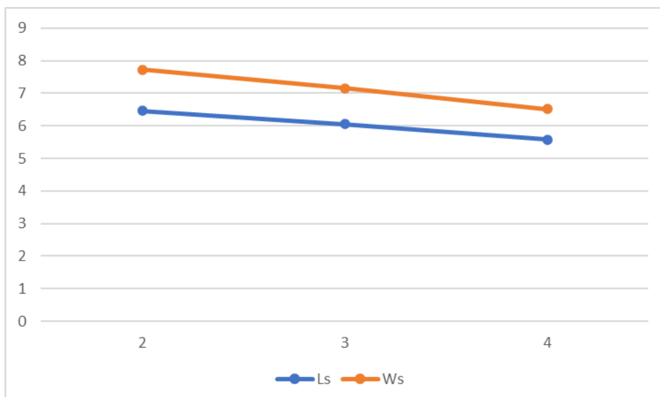

Fig 7. L_s **and** W_s **by varying** μ**and keeping other parameters fixed**

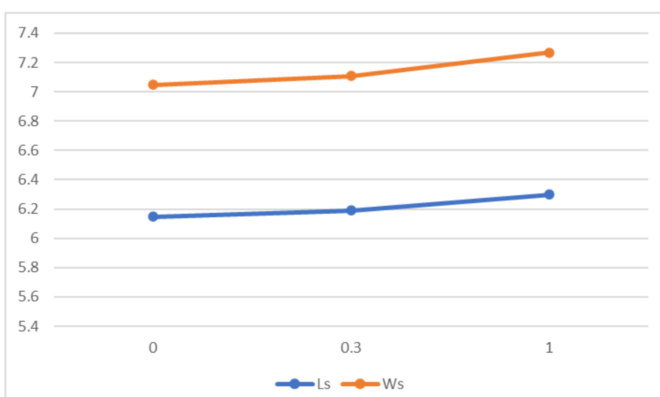

Fig 8. L_s **and** W_s **by varying** ε**and keeping other parameters fixed**

Assuming that there would be a single server with no vacation rate ($v = 0$), Table 2 is generated. We might consult citation [1] research article. Figures 5, 6, 7 and 8 show how L_s and W_s move when one parameter is altered while the remaining is left constant. These graphs correspond with Figures 1, 2, 3 and 4.

Table 3. When there is only one server without vacation and controllable arrival rates

		L_s	W_s
Varying $\lambda_0 = \lambda_1 = \lambda$ and keeping $\mu = 20,\ \varepsilon = 0$ fixed	6	0.4286	0.0714
	9	0.8182	0.0909
	11	1.2222	0.1111
	15	3.1000	0.2000
Varying μ and keeping $\lambda_0 = \lambda_1 = \lambda = 10,\ \varepsilon = 0$ fixed	12	5.1000	0.5000
	14	2.5000	0.2500
	15	2.1000	0.2000
	19	1.1111	0.1111

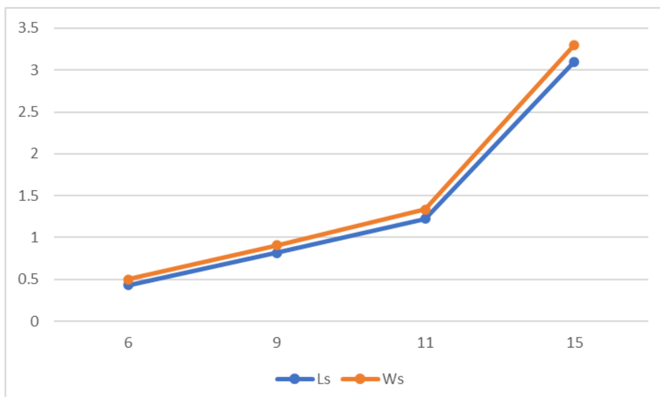

Fig 9. L_s **and** W_s **by varying** $\lambda_0 = \lambda_1 = \lambda$ **and keeping other parameters fixed**

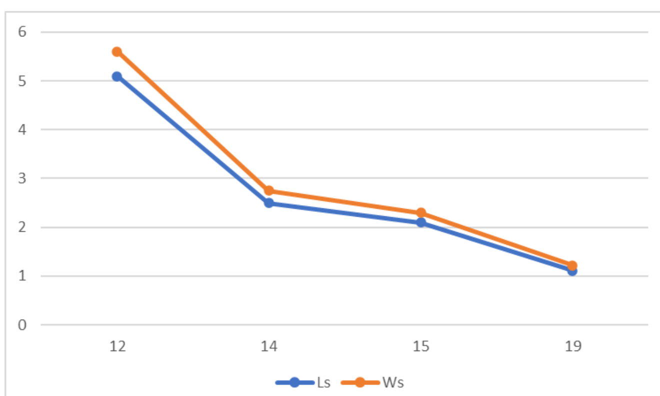

Fig 10. L_s **and** W_s **by varying** μ **and keeping other parameters fixed**

Table 3 is constructed assuming that there is a single server with no vacation ($v = 0$) and no adjustable arrival rates ($\lambda_0 = \lambda_1 = \lambda$). We might refer to the conventional model. Figures 9 and 10show how L_s and W_s move when one parameter is altered while the remaining is constant. These graphs correspond with Figures 1, 2, 3 and 4.

6 Conclusion

The movement of L_s and W_s is observed in Table 1.

- When the mean dependence rate increases and the other parameters are kept fixed, both L_s and W_s decrease.
- When the service rate increases and the other parameters are kept fixed, both L_s and W_s decrease.
- When the faster arrival rate increases and the other parameters are kept fixed, both L_s *and* W_s increase.
- When the slower arrival rate increases and the other parameters are kept fixed, L_s increases W_s and decreases.

When there is no vacation ($v = 0$), Table 2 specifies the model. The results of Srinivasa Rao et al.[1] are consistent with this numerical result. The model with no vacation and adjustable arrival rates ($v = 0$ and $\lambda_0 = \lambda_1 = \lambda$) is defined in Table 3. The conventional model and this numerical finding are in agreement. This model simplifies to the M/M/1/∞ queuing model with vacation[5] when λ_0 approaches to λ_1 and $\varepsilon = 0$.

Therefore, a single server vacation queuing model with quicker and slower arrival rates has been examined in this article. Graphs are used to support this model's validity even further. Numerous stochastic service systems in the actual world may be used using this model. The future scope of the idea of this model can be extended to analyse other related models such as the delayed vacation model, multiple vacation model, time-dependent failures, second optional vacation policy etc.,

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