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Some Standard Seidel Energy Results of the Minimum Maximal Dominating Graphs

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Abstract

Objectives: Let $F\left(\gamma',\delta'\right)$ be a finite and connected graph with β points and d edges. In this research, introduced the graph's minimum maximal dominating seidel energy ($SE_X(F)$) and the properties of the latent roots of the given parameters are discussed. **Method**: In this research, the seidel energy of several graphs and its properties are investigated. Examined its minimum maximal limits and computed a few conventional seidel energy outcomes for the minimum maximal dominating graphs. **Finding**: Using the minimum maximal dominating seidel energy of graphs, significant outcomes were achieved for complete graphs, complete bipartite graphs, and star graphs. The properties of the class of graphs were computed. The established upper and lower bound is $\sqrt{\beta^2-\beta+\mu_\theta} \leq SE_X(F) \leq \sqrt{\beta(\beta^2-\beta+\mu_\theta)}$. **Novelty:** The seidel energy of the proposed research findings is used in various graphs based on the research. The fundamental characteristics of a graph, such as its energy upper and lower bounds, have been determined, and this knowledge has found notable chemical applications in the conjugated molecular orbital theory. Recommendations for future energy-related research are presented and examined.

Keywords: Connected graph; Dominating set; Latent roots; Minimum maximal; Seidel energy

1 Introduction

Consider a finite, connected graph $F\left(\gamma',\delta'\right)$ with β points and d edges. Let B $=(b_{ij})$ be the adjacency matrix of F. The various authors implemented their work in different dominations of graphs that were motivated by this[(1–3)]. So, we introduced the concept of a graph's minimum maximal dominating seidel energy. The Wiener index of some topological indices in every connected graph, as well as the vertex degree-based[(4)] molecular structure and the F-coindexindices of the physical characteristics and biological activities minimum maximal dominations, can be solved[(5,6)]. Some standard seidel energy of domination in comprising transition metals of the bonds between various elements reveals typical open-shell properties[7–9]. Such authors investigated their research in domination set of class of graph concepts such as metric dimension, domination set, edge-identification chain and ring can be developed in the concept of dominating sets in the paper[10,11]. This research investigates the prevailing of acyclic graphs and binomial graphs metric dimension of Corona product graphs[12–14]. The symmetric difference network, minimal spanning tree algorithm, labelled vertices, and congruent dominating graph can be used to compute the seidel energy[15,16]. This method of seidel energy can solve the fundamental knowledge of fuzzy fractional colouring in the minimum maximal dominating manner[17,18]. The latent roots of F are denoted as $\tau_1,\ \tau_2,\tau_3,\ldots,\tau_\beta$, arranged in non-increasing order. In 1978, Gutman developed the notion of graph energy[19] and is defined by $E(F)=\sum_{i=1}^{\beta}|\tau_i|$, where τ_i are the characteristic roots of F. Some of the authors studied the minimum maximal domination energy of a graph and minimum dominating seidel energy of a graph, further we extend this parameter to minimum maximal dominating seidel energy of a graph. Consider a finite, connected graph $F=\left(\gamma',\delta'\right)$ with β points and d edges. A set $X\in\gamma'$ of F is said to be a dominating set if each point in γ' but not in X is adjacent to some member of X. Let $\gamma'=\{\alpha_1,\alpha_2,\ldots,\alpha_\beta\}$ and $\delta'=\{y_1,y_2,\ldots,y_d\}$. And F is the $\beta\times\beta$ seidel matrix obvious by $S(F)=(s_{ij})$,

$$s_{ij}=\begin{cases}-1 & if\ \alpha_i\alpha_j\in\delta'\\ 1 & if\ \alpha_i\alpha_j\notin\delta'\\ 0 & if\ \alpha_i=\alpha_j\end{cases}$$

The polynomial formula for $S(F)$ is represented as $f_\beta(F,\tau)=det(\tau_i-\ S(F))$. $SE(F)=\sum_{i=1}^{\beta}|\tau_i|$. Any seidel set with minimum cardinality is called a dominating seidel set of F.

This research gap is a novel feature selection strategy to be adopted to make the decision to raise a graph's minimal maximal dominating seidel energy value. This research has focused on various types of graphs, which help us to study the physical and chemical properties of the chemical structures.

2 Methodology

The dominating energy of a minimum maximal seidel graph F is defined in this section.

- **The dominating energy of a minimum maximal seidel graph**

Let $F=\left(\gamma',\delta'\right)$ be a finite and connected graph. Let $\gamma'=\{\alpha_1,\alpha_2,\ldots,\alpha_\beta\}$ and $\delta'=\{y_1,y_2,\ldots,y_d\}$. If a dominating set X is said to be maximal dominating set of F then $\gamma' difference\ X$ is not a dominating set of F. The maximal domination number $\mu_\theta(F)$ of F is the minimum cardinality of a maximal dominating set in F. The term minimum maximal dominating set refers to any maximal dominating set that has minimum cardinality. The dominating seidel matrix of minimum maximal graph F of $\beta\times\beta$ is $S_X(F)=(s_{ij})$

$$s_{ij}=\begin{cases}-1 & if\ \alpha_i\alpha_j\in\delta'\\ 1 & if\ \alpha_i\alpha_j\notin\delta'\\ 1 & if\ \alpha_i=\alpha_j\ and\ \alpha_i\in X\\ 0 & otherwise\end{cases}$$

We write $f_\beta(F,\tau)=det(\tau I-S_X(F))$ to represent the characteristic equation of $S_X(F)$.Since $S_X(F)$ is a real and symmetric, its latent roots are real numbers and the labelled in non-increasing order: $\tau_1\geq\tau_2\geq\tau_3\geq,\ldots,\geq\tau_\beta$. Given $SE_X(F)=\sum_{i=1}^{\beta}|\tau_i|$, the minimum maximal dominating seidel energy of F is evident. A minimum maximal dominating seidel set is any maximal dominating seidel set that has the minimum cardinality. Note that the trace $(SE_X(F))$ =Domination Number= μ_θ.

3 Results and Discussion

3.1 Some standard seidel energy results of the minimum maximal dominating graphs:

In this research focus, we comparatively improved our results from the minimum maximal domination energy to the minimum maximal dominating seidel energy of some classes of graphs. Findings from some standard Seidel energy results for the minimum maximal domination, of the following graphs: complete graphs, complete bipartite graphs, and star graphs are computed. The unique features of this study comparatively to the existing works of the seidel energy is increased.

Theorem 3.2. Let K_a be a complete graph and if $a \geq 2$, then the minimum maximal dominating seidel energy of K_a,$SE_X(K_a) \leq 3a-4$.

Proof: Let the complete graph K_a has the point set $\gamma' = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_a\}$. Then $\mu_\theta = a$. Hence, the complete graph's minimum maximal dominating seidel set is$X = \{\alpha_1, \alpha_2, \alpha_3, \ldots.., \alpha_a\}$. The minimum maximal dominating seidel matrix is

$$S_X(K_a) = \begin{bmatrix} 1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & 1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & 1 & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \cdots & 1 & -1 \\ -1 & -1 & -1 & \cdots & -1 & 1 \end{bmatrix}$$

$$f_a(K_a, \tau) = \begin{bmatrix} \tau-1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & \tau-1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & \tau-1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & \tau-1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & \tau-1 \end{bmatrix}$$

The respective characteristic equation is $(\tau-2)^{a-1}(\tau+(a-2)) = 0$.

The minimum maximal dominating seidel latent roots are $\tau = 2\,[(a-1)\ \textit{times}]$ and $\tau = -(a-2)$.

The complete graph's minimum maximal dominating seidel energy is

$SE_X(K_a) = |2|(a-1) + |-(a-2)| \leq 3a-4$.

Theorem 3.3 . Let $K_{a,b}$ be a complete bipartite graph and if $b \geq a$, then the minimum maximal dominating seidel energy of $K_{a,b}$, $SE_X(K_{a,b}) \leq -1 + a + 2\left(\sqrt{b+a^2}\right)$.

Proof: If $a \leq b$, then the complete bipartite graph of the $K_{a,b}$ *with point set*.

$\gamma' = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_a, \delta_1, \delta_2, \delta_3, \ldots, \delta_b\}$. The complete bipartite graph's minimum maximal dominating seidel set is $X = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_a, \delta_1\}$.

$$S_X(K_{a,b}) = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & -1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & 1 & \cdots & 1 & -1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & 1 & \cdots & 1 & -1 & -1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & .. & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & -1 & -1 & -1 & \cdots & -1 \\ -1 & -1 & -1 & \cdots & -1 & 1 & 1 & 1 & \cdots & 1 \\ -1 & -1 & -1 & \cdots & -1 & 1 & 0 & 1 & \cdots & 1 \\ -1 & -1 & -1 & \cdots & -1 & 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & -1 & 1 & 1 & 1 & \cdots & 0 \end{bmatrix}$$

$$f_\beta(K_{a,b}, \tau) = \begin{bmatrix} \tau-1 & -1 & -1 & \cdots & -1 & 1 & 1 & 1 & \cdots & 1 \\ -1 & \tau-1 & -1 & \cdots & -1 & 1 & 1 & 1 & \cdots & 1 \\ -1 & -1 & \tau-1 & \cdots & -1 & 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & .. & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & \tau-1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 & \tau-1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & 1 & \cdots & 1 & -1 & \tau & -1 & \cdots & -1 \\ 1 & 1 & 1 & \cdots & 1 & -1 & -1 & \tau & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & -1 & -1 & -1 & \cdots & \tau \end{bmatrix}$$

The respective characteristic equation is $\tau^{b-1}(\tau+1)^{a-1}\left(\tau^2-2a\tau-b\right)=0$.

The minimum maximal dominating seidel latent roots are

$\tau=0\quad[(b-1)\ times], \tau=-1\ [(a-1)\ times]$ and $\tau=\pm\sqrt{b+a^2}+a\ [one\ time\ each]$.

The complete bipartite graph's minimum maximal dominating seidel energy is

$$SE_X\left(K_{a,b}\right)=|0|\,(b-1)+|-1|\,(a-1)+\left|\sqrt{b+a^2}\ +a\right|+\left|-\sqrt{b+a^2}\ +a\right|$$

$SE_X\left(K_{a,b}\right)\leq -1+a+2\sqrt{b+a^2}$.

Theorem 3.4 . Let $K_{1,a-1}$ be a star graph and if $a\geq 2$, then the minimum maximal dominating seidel energy $K_{1,a-1}$, $SE_X\left(K_{1,a-1}\right)\leq a-1+\sqrt{a^2-2a+9}$.

Proof: Let $K_{1,a-1}$ be a star with $\gamma'=\{\alpha_0,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ldots,\ \alpha_a\}$, α_0 is the centre and the minimum maximal dominating seidel set is $X=\{\alpha_0,\ \alpha_1\}$.

$$S_X\left(K_{1,a-1}\right)=\begin{bmatrix}1 & -1 & -1 & \cdots & -1\\ -1 & 1 & 1 & \cdots & 1\\ -1 & 1 & 0 & \cdots & 1\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -1 & 1 & 1 & \cdots & 0\end{bmatrix}$$

$$f_a\left(K_{1,-1+a},\tau\right)=\begin{bmatrix}\tau-1 & 1 & 1 & \cdots & 1\\ 1 & -1+\tau & -1 & \cdots & -1\\ 1 & -1 & \tau & \cdots & -1\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & -1 & 1 & \cdots & \tau\end{bmatrix}$$

The respective characteristic equation is $\tau(\tau+1)^{a-3}\left(\tau^2-(a-1)\,\tau-2\right)=0$.

The minimum maximal dominating seidel latent roots are

$\tau=0,\ \tau=-1\,[(-3+a)\ times]\ and\ \tau=\frac{(-1+a)\pm\sqrt{a^2-2a+9}}{2}$.

The star graph's minimum maximal dominating seidel energy is

$$SE_X(K_{1,a-1})=|0|+|-1|\,(-3+a)+\frac{(-1+a)+\sqrt{a^2-2a+9}}{2}+\frac{(-1+a)-\sqrt{a^2-2a+9}}{2}$$

$SE_X\left(K_{1,a-1}\right)\leq(a-3)+\sqrt{a^2-2a+9}$.

Next, we have discussed the properties of the latent roots of the minimum maximal dominating seidel graph.

Theorem 3.5. Let $F\left(\gamma',\delta'\right)$ be a finite and connected graph and $\gamma'=\left\{\alpha_1,\ \alpha_2,\ \alpha_3,\ldots,\ \alpha_\beta\right\}$. Then $\sum_{i=1}^{\beta}\tau_i=|X|$ and $\sum_{i=1}^{\beta}\tau_i^2\leq|X|+\beta^2-\beta$.

Proof :

Let the latent roots of the minimum maximal dominating seidel matrix $S_X\left(F\right)$ are $\tau_1,\tau_2,\ldots,\tau_\beta$. We know that the sum of the latent roots of $S_X\left(F\right)$ = The trace of $S_X\left(F\right)$= $\sum_{i=1}^{\beta}\tau_i=\sum_{i=1}^{\beta}b_{ii}=|X|=\mu_\theta$.

The sum of the squares of the latent roots of $S_X\left(F\right)=\ \sum_{i=1}^{\beta}\tau_i^2=\sum_{i=1}^{\beta}b_{ij}\sum_{j=1}^{\beta}b_{ji}=\sum_{i=1}^{\beta}\left(b_{ii}\right)^2+\sum_{i\neq j}b_{ij}b_{ji}=\sum_{i=1}^{\beta}\left(b_{ii}\right)^2+2\sum_{j>i}\left(b_{ij}\right)^2$

$$=2\left[(-1)^2d+\left(\frac{\beta^2-\beta}{2}-d\right)(1)^2\right]+|X|\leq\ |X|+\beta^2\leq\beta^2-\beta+\mu_{\theta.}$$

The sum of the squares of the latent roots of $S_X\left(F\right)\leq\beta^2-\beta+\mu_{\theta.}$

- **Bounds for the dominating energy of minimum maximal seidel graph**

The upper and lower bounds on the minimum maximal dominating seidel energy of a finite graphs are defined in this section.

Theorem 3.6. Let F be a finite and connected graph with β points and d edges then $\sqrt{\beta^2-\beta+\mu_\theta} \leq SE_X(F) \leq \sqrt{\beta(\beta^2-\beta+\mu_\theta)}$.

Proof: By the inequality of Cauchy-Schwartz,

$$\left(\sum_{s=1}^{\beta}\partial_s\varepsilon_s\right)^2 \leq \left(\sum_{s=1}^{\beta}\partial_s^2\right)\left(\sum_{s=1}^{\beta}\varepsilon_s^2\right)$$

By Chosen $\partial_s = 1$ & $\varepsilon_s = |\tau_s|$,

$$(SE_X(F))^2 = \left(\sum_{s=1}^{\beta}(\tau_s|\right)^2 \leq \left(\sum_{s=1}^{\beta}1\right)\left(\sum_{s=1}^{\beta}\tau_s^2\right)$$

From theorem 3.5, then we get $(SE_X(F))^2 \leq \beta\left(\beta^2-\beta+|X|\right) \leq \beta(\beta^2-\beta+\mu_\theta)$

Consequently, hold is applicable to the upper bound; nevertheless, since the lower bound

$$\left(\sum_{s=1}^{\beta}(\tau_s|\right)^2 \geq \left(\sum_{s=1}^{\beta}\tau_s^2\right)$$

Then, $(SE_X(F))^2 \geq \sum_{s=1}^{\beta}\tau_s{}^2 = \beta^2-\beta+(X| = \beta^2-\beta+\mu_\theta$.

$SE_X(F) \geq \sqrt{(\beta^2-\beta+\mu_\theta)}$.

Therefore, $\sqrt{\beta^2-\beta+\mu_\theta} \leq SE_X(F) \leq \sqrt{\beta(\beta^2-\beta+\mu_\theta)}$.

Theorem 3.7 . Let F be a finite and connected graph with β points and d edges. If X is the minimum maximal dominating seidel set and

$D = det\, S_X(F)$, then $SE_X(F) \geq \sqrt{\beta^2-\beta+\mu_\theta+(\beta^2-\beta)D^{\frac{2}{\beta}}}$.

Proof:

Since, $(SE_X(F))^2 = \left(\sum_{s=1}^{\beta}|\tau_s|\right)^2 = \left(\sum_{s=1}^{\beta}|\tau_s|\right)\left(\sum_{s=1}^{\beta}|\tau_s|\right) = \sum_{s=1}^{\beta}|\tau_s|^2 + 2\sum_{s\neq j}|\tau_s||\tau_j|$,

Since the arithmetic mean is not smaller than the geometric mean, we have

$$\frac{1}{\beta(\beta-1)}\sum_{s\neq j}|\tau_s||\tau_j| \geq \left(\prod_{s\neq j}|\tau_s||\tau_j|\right)^{\frac{1}{\beta(\beta-1)}}$$

$$(SE_X(F))^2 \geq \sum_{s=1}^{\beta}|\tau_s|^2 + \beta(\beta-1)\left(\prod_{s\neq j}|\tau_s||\tau_j|\right)^{\frac{1}{\beta(\beta-1)}} \geq \sum_{s=1}^{\beta}|\tau_s|^2 + (\beta^2-\beta)\left(\prod_{s\neq j}|\tau_s|^{2(\beta-1)}\right)^{\frac{1}{\beta(\beta-1)}}$$

From theorem 3.5 we get,

$$= \sum_{s=1}^{\beta}|\tau_s|^2 + (\beta^2-\beta)\left|\prod_{s\neq j}\tau_s\right|^{\frac{2}{\beta}} = \beta^2-\beta+\mu_\theta+(\beta^2-\beta)D^{\frac{2}{\beta}}.$$

$Therefore,\ SE_X(F) \geq \sqrt{\beta^2-\beta+\mu_\theta+(\beta^2-\beta)D^{\frac{2}{\beta}}}$.

4 Conclusion

This research establishing the Seidel energy of minimum maximal dominating graphs and explores Seidel energy of some standard graphs. This research computed the properties of class of graph and provides the upper and lower bounds of minimum maximal dominating graphs. In future research, discussing with how to use seidel energy in various types of chemical bonds. In future, the studied parameters can be extended to the different types of dominating seidel energy for some class of graphs.

Open questions:

1. How to construct the seidel energy (any types of seidel energy) for an infinite graph in graphs?
2. How to construct an optimal seidel energy (any types of seidel energy) of the graph in graphs?

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