INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 18-01-2024

Accepted: 09-02-2024

Published: 20-03-2024

Citation: Lambat PM, Pund AM (2024) String Viscous Fluid Cosmological Model in the Framework of Sen-Dunn Theory of Gravitation. Indian Journal of Science and Technology 17(12): 1189-1202. https://doi.org/10.17485/IJST/v17i12.134

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Funding: One of the authors P. M. Lambat File No. (08/758 (0001)/ 2020-EMR-I) is thankful to CSIR for providing financial assistance.

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

String Viscous Fluid Cosmological Model in the Framework of Sen-Dunn Theory of Gravitation

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Abstract

Objective: To present a new solution to the field equations for higher-dimensional Bianchi type-III string viscous fluid cosmological model in the context of the Sen-Dunn theory of gravity. **Methods:** To obtain definite solution of the field equations, we consider a power law relation between the scale factor and scalar field, and we take into account two distinct scale factors, $R(t) = (t^n e^t)^{\frac{1}{\beta}}$ and $R(t) = (sinh(at))^{\frac{1}{\beta}}$, in which n, β, a are positive constants. This produces a time-dependent deceleration parameter. We analyse the model with constant bulk viscosity $\xi = \xi_0 =$ constant to explain the current accelerated expansion of the universe. **Findings:** Spatial volume of the model increases with cosmic time, which shows that universe is spatially expanding. Also, the model possesses a point-type singularity. It is noted that these models depict the universe's accelerated expansion. **Novelty:** We obtained new solution to the field equations for higher dimensional Bianchi type-III generated by means of a cloud of strings with bulk viscous fluid in Sen-Dunn theory by using quadratic form.

Keywords: Bianchi type-III; Cosmic string; Bulk viscosity; Sen-Dunn Theory

1 Introduction

Einstein has proposed the general theory of relativity. It has been successful in producing a number of different models of the universe. But it has some flaws, such as singularity. So general theory of relativity does not provide an adequate explanation for the inertial properties of matter. Many scientists in this subject have proposed alternate theories that are modifications of the original theory in an effort to eliminate these flaws. One group of many alternative theories is scalar tensor theories of gravitation. These are considered as indispensable for describing gravitational interactions. Scalar field is implied by string theory, extended inflation, and numerous higher order theories. They are founded on the addition of a scalar field to the metric tensor field. Scalar tensor theories such as Brans-Dicke, Nordtvedt, Saez Ballester and Self creation are significant. One of the most well-known of them is the Senn-Dunn [1] theory. They have proposed a theory where both the scalar and tensor fields are intrinsically geometric. Therefore, it qualifies as a novel scalar tensor theory. The scalar field in this theory exhibits characteristics of the function $f = f(x_i)$ .x_i is the four dimensional Lyra geometry's coordinates, and the metric tensor g_{ij} of this geometry serves as the tensor field's identifier. The models in this theory have been investigated by Basumatary and Dewri[(2)], Dewri and Basumatary[(3)].

The study of spatially homogeneous and anisotropic cosmological models is important for comprehending the early phases of the evolution of the universe. These are the exact traits of Bianchi type space-times. Therefore, we see that these Bianchi type models have generated a lot of interest recently. Our cosmos is comparatively very small at the beginning of the expansion. This knowledge we get by the investigation of higher dimensional space-time. So, at the early era of universe higher dimensional cosmological models play an important role. Daimary and Baruah[(4)]. In the usual four dimensional space-times gravitational force cannot be unified with the other natural forces. But at the very early stage of the evolution these higher dimensional space times are very useful to unify gravitational force with other natural forces. Because with the evolution of time our usual standard dimensions get expanded, but the extra dimension give way to the plancanian dimension by shrinking, and with current available experimental facilities it is beyond the detection. Many researchers are attracted because of this fact, and they have investigated the cosmological problems with the higher dimensional space times. This motivated us to investigate the cosmological model in five dimensional space-time. The models of Bianchi type-III space-time have been examined by Ramprasad et al[(5)], Basumatary and Dewri[(6)].

From observable and theoretical data, it is almost certain that the universe has been expanding with acceleration since the big bang. According to the literature and theories that are now available, the cosmos has gained acceleration while losing deceleration. Even though the precise reason for the universe's expansion is still unknown, it has motivated all cosmologists and physics experts to determine the accelerated expansion. In the recent years, numerous authors have proposed various cosmological models using string theory to explain the accelerated expansion of the universe. Singh and Baro[(7)].

String cosmological models have a significant impact on the creation of structures in the early universe's evolution. Through the phase transition, the spontaneous breaking of the symmetry which occurs early in the universe is accomplished. It results in cosmic strings. Cosmic strings are topological imperfections that resemble lines. In the universe, they have a stable random network. The foundation of the galaxy clusters are these enormous cosmic strings. Academics naturally focused their efforts on string models. These strings are connected to the gravitational field and have stress energy. They provide thorough accounts of numerous cosmic string features. Letelier[(8)] and Stachel[(9)] provide the general relativistic formalisms of cosmic strings.

Bulk viscosity is important in cosmology. It also contributes to the phenomenon of the universe's accelerated expansion or inflationary phase. Sethi et al.[(10)] have examined viscosity's influence on the evolution of cosmos, for preventing singularity at the initial big bang. Several researchers have investigated cosmological models that incorporate cosmic strings within the viscous fluid. Hatkar et al.[(11)], Mishra and Dua[(12)], Santhi et al.[(13)], Bhoyar et al.[(14)] , Hegazy[(15)], Trivedi and Bhabor[(16)], Dixit et al.[(17)] and Mete and Deshmukh[(18)]. The behaviour of viscous fluid in the string cosmological model in the Lyra manifold, which takes into account the constant coefficient of bulk viscosity, has recently been explored by Mollah and Singh[(19)].

Motivated by these recent investigations, we studied a five-dimensional Bianchi type III space-time containing cosmic strings with the bulk viscous fluid with quadratic equation of state in the framework of Sen Dunn theory of gravitation. Interestingly, it is observed that both quadratic equations of state and bulk viscosity play a great job throughout the evolution of model universe and so our model can be thought as realistic universe. The contents of this paper are presented as follows: Section 2 contains field equations and their solutions, while in Section 3 we have formulated the model using different scale factors. The Section 3.1 and 3.2 give the dynamical and physical properties of the model through graphical representations and the conclusion is given in Section 4.

2 Field equations and the solution

We consider five-dimensional Bianchi type-III metric as

$$ds^2 = X^2\left(dx^2 + e^{-2bx}dy^2 + dz^2\right) + Y^2dm^2 - dt^2 \tag{1}$$

where scale factors $X(t)$ & $Y(t)$ are the functions of cosmic time t only, $b \neq 0$ is a constant, m is fifth coordinate which is space like, and the spatial curvature is taken as zero.

The field equation for Sen-Dunn theory which consider combined scalar and tensor fields (where $C = 1, G = 1$ in natural units) is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = \omega\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) - \varphi^{-2}T_{ij} \tag{2}$$

where $\omega = \frac{3}{2}$ and R_{ij} R, g_{ij}, φ and T_{ij} are the Ricci tensor, Ricci scalar, metric tensor, scalar field and energy momentum tensor respectively.

Energy-momentum tensor T_{ij} and total pressure $\overline{p}$which includes the proper pressure p for bulk viscous fluid are given as

$$T_{ij} = (\rho + \overline{p})v_i v_j + \overline{p} g_{ij} - \lambda x_i x_j, \tag{3}$$

with

$$\overline{p} = p - \xi \theta, \tag{4}$$

where, λ denotes string tension density, ρ denotes the energy density for a cloud of strings along with particles i.e. $\rho = \lambda + \rho_p$, ρ_p $\theta = v^j_{;j}\rho_p$represents the particle energy density, θrepresents expansion scalar factor, ξ represents the bulk viscosity coefficient, v^j represents the five velocity vector of fluid flow and x^j represents space like unit vector that represents direction of strings

where,

$$v^i = (0,0,0,0,1) \tag{5}$$

$$x^i = \left(0,0,0,\frac{1}{Y},0\right) \tag{6}$$

The direction of string x^i and velocity vector v^i satisfy following conditions in co-moving coordinates,

$$v^i v_i = -x^i x_i = -1, \quad v^i x_i = 0 \tag{7}$$

The equation of state is given as $p = p(\rho)$ in quadratic form

$$p = c\rho^2 + \rho \tag{8}$$

For the line element (**1**), following important and essential physical parameters are

Spatial volume

$$V = R^4 = X^3 Y \tag{9}$$

Expansion scalar

$$\theta = v^i_{;i} = 3\frac{X}{X} + \frac{Y}{Y} \tag{10}$$

Hubble parameter

$$4H = \theta = 3\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} \tag{11}$$

Declaration parameter

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{12}$$

Here the overhead dots represent derivatives with respect to time t.

Using Equations (1), (2), (3), (4), (5), (6) and (7), the field equations become

$$2\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}^2}{X^2} + 2\frac{\dot{X}\dot{Y}}{XY} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \varphi^{-2}\bar{P} \tag{13}$$

$$2\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}^2}{X^2} + 2\frac{\dot{X}\dot{Y}}{XY} - \frac{b^2}{X^2} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \varphi^{-2}\bar{P} \tag{14}$$

$$3\frac{\ddot{X}}{X} + 3\frac{\dot{X}^2}{X^2} - \frac{b^2}{X^2} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \varphi^{-2}\bar{p} + \lambda\varphi^{-2} \tag{15}$$

$$3\frac{\dot{X}^2}{X^2} + 3\frac{\dot{X}\dot{Y}}{XY} - \frac{b^2}{X^2} = -\frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \varphi^{-2}\rho \tag{16}$$

3 Solution of the field equations and the model

We consider $\xi = \xi_0$ (constant), as it is the simplest parametrization for the bulk viscosity. As there are four equations (Equations (13), (14), (15) and (16)) having six variables X, Y, φ, $\overline{p}$, λ, ρ which are unknown, so we need two extra conditions to get exact solution for the above equations.

Since, the field equations are highly non-linear therefore we first assume that -

(i) the power-law relation between scale factor R and scalar field φ as $\varphi \propto R^{\alpha}$, where α is any integer which implies that

$$\varphi = \varphi_0 R^{\alpha} = \varphi_0 V^{\frac{\alpha}{4}} \tag{17}$$

Where φ_0 is the constant of proportionality.

(ii) Several authors have used the law of variation for Hubble parameter. Bermann developed a specific equation of variation that produces a constant deceleration parameter.

The early decelerated phase of the universe is shown by the positive value of the deceleration parameter, whereas the accelerated phase is indicated by the negative value. Latest discoveries in modern cosmology confirm that the universe is currently undergoing an accelerated expansion phase. A smooth transition from the universe's initial slowing to its current accelerated expansion must be represented by the deceleration parameter. Thus, rather than being a constant, the deceleration parameter should be time-variable. We used the following format for the average scale factors from the above discussion. We get the time dependent deceleration parameter from this and arrived at the corresponding cosmological models.

Model 1. Here we consider the ansatz (mathematical approach) for the average scale factor as

$$R(t) = (t^n e^t)^{\frac{1}{\beta}} \tag{18}$$

The particular expression of Equation (18) is provided by

$$R(t) = \sqrt{t^n} e^t \tag{19}$$

We assume that, shear scalar is proportional to the expansion scalar in view of anisotropy of the space- time. So, we get the relation between X and Y metric coefficient i.e.

$$X = Y^k \tag{20}$$

where $k > 1$ is a constant. For the sake of simplicity, we have taken the integration constant as one.

Using Equations (11), (19) and (20) we get the expressions for metric functions as follows

$$Y = l_1 \left[t^n e^t\right]^{\frac{2}{3k+1}} \tag{21}$$

$$X = l_2 \left[t^n e^t\right]^{\frac{2k}{3k+1}} \tag{22}$$

where $l_1 = k_1^{\frac{-1}{3k+1}}, l_2 = l_1^k$ and k_1 is an integrating constant.

Hence, using Equations (21) and (22) the model takes the form,

$$ds^2 = l_2^2 \left[t^n e^t\right]^{\frac{4k}{3k+1}} \left[dx^2 + e^{-2bx} dy^2 + dz^2\right] + l_1^2 \left[t^n e^t\right]^{\frac{4}{3k+1}} dm^2 - dt^2. \tag{23}$$

3.1 Physical Aspects of the model 1

For the model (Equation (23)), the expressions for the spatial volume V, the Hubble parameter H, the expansion scalar θ, the deceleration parameter q, mean anisotropy parameter and shear scalar are given by,

$$V = R^4(t) = t^{2n} e^{2t} \tag{24}$$

$$H = \frac{\dot{R}}{R} = \frac{1}{2}\left[\frac{n}{t} + 1\right] \tag{25}$$

$$\theta = 4H = 2\left[\frac{n}{t} + 1\right] \tag{26}$$

$$q = -1 + \frac{2n}{(n+t)^2} \tag{27}$$

$$\Delta = \frac{3(k-1)^2}{(3k+1)^2} \tag{28}$$

$$\sigma^2 = \frac{9(k-1)^2}{8(1+3k)^2}\left(\frac{n}{t} + 1\right)^2 \tag{29}$$

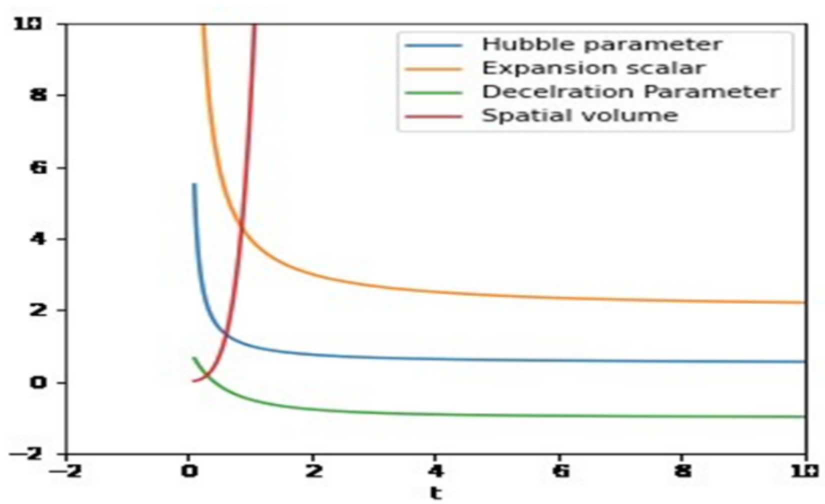

Fig 1. Variations of Hubble parameter, Expansion scalar, Deceleration parameter and Spatial Volume Vs time, whenever$n = 1$

Equations (21) and (22) show that the spatial scale factors are zero at the starting epoch, $t = 0$, and as a result, the model possesses a point-type singularity. According to Equations (24) and (26), the cosmos began to evolve with a zero volume at time $t = 0$ and the expansion scalar is infinite. This is consistent with the big bang theory.

Plots of spatial volume and Hubble's parameter against time are provided in Figure 1 for easier comprehension.

The expansion scalar (θ) starts from infinity at $t = 0$ and approaches to zero for high values of cosmic time t, according to the plot of the expansion scalar vs time shown in Figure 1. This demonstrates how the universe is expanding with the passage of time.

In early times, the deceleration parameter is $q > 0$, while at later times, it becomes $q < 0$. So, it is evident that this model varies from an early deceleration phase to the current accelerated phase.

From Equation (28), mean anisotropy parameter = constant $\neq 0$ for $k \neq 1$ and for $\Delta = 0$ for $k = 1$. So, we can conclude at late time the universe is anisotropic when $k \neq 1$, but it is isotropic for $k = 1$ throughout evolution.

Also, from Equation (29) the shear scalar decreases as cosmic time increases. Shear scalar σ is infinite at initial epoch and it approaches to zero at $t = \infty$, for $k \neq 1$, explaining a shearing model Universe throughout its evolution.

The jerk, snap, and lerk parameters are given below

$$j(t) = \frac{n^3 + (3t-6)n^2 + \left(3t^2 - 6t + 8\right)n + t^3}{(t+n)^3} \tag{30}$$

$$s(t) = \frac{1}{(t+n)^4}\left(n^4 + (4t-12)n^3 + \left(6t^2 - 24t + 44\right)n^2 + \left(4t^3 - 12t^2 + 32t - 48\right)n + t^4\right) \tag{31}$$

$$l(t) = \frac{1}{(t+n)^5}(n^5 + (5t-20)n^4 + \left(10t^2 - 60t + 140\right)n^3 + \left(10t^3 - 60t^2 + 220t - 400\right)n^2 + (5t^4 - 20t^3 + 80t^2 - 240t + 384)n + t^5 \tag{32}$$

From Equation (17), we get

$$\varphi = \varphi_0 t^{\frac{\alpha n}{2}} e^{\frac{t\alpha}{2}} \tag{33}$$

Equations (4), (13) and (16) give us the energy density ρ as

$$\rho = \frac{1}{\sqrt{c}}\left\{ \varphi_0 [t^n e^t]^{\frac{\alpha}{2}} \left[-\left(1+\frac{n}{t}\right)^2 u_1 - u_2\left(1+\frac{2n}{t}+\frac{n(n-1)}{t^2}\right) + b^2 l_0 (t^n e^t)^{-\frac{4k}{3k+1}} \right] + 2\xi\left(1+\frac{n}{t}\right) \right\}^{\frac{1}{2}} \tag{34}$$

From Equations (14) and (15), the string tension density λ is obtained as

$$\lambda = \varphi_0^2 [t^n e^t]^{\alpha} \left\{ u_3\left(\frac{n}{t}+1\right)^2 + u_4\left(1+\frac{2n}{t}+\frac{n(n-1)}{t^2}\right) \right\} \tag{35}$$

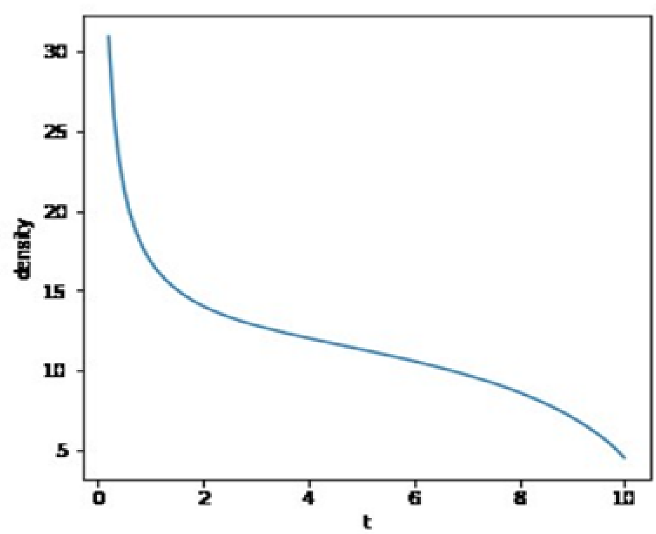

Fig 2. Variations of density Vs time

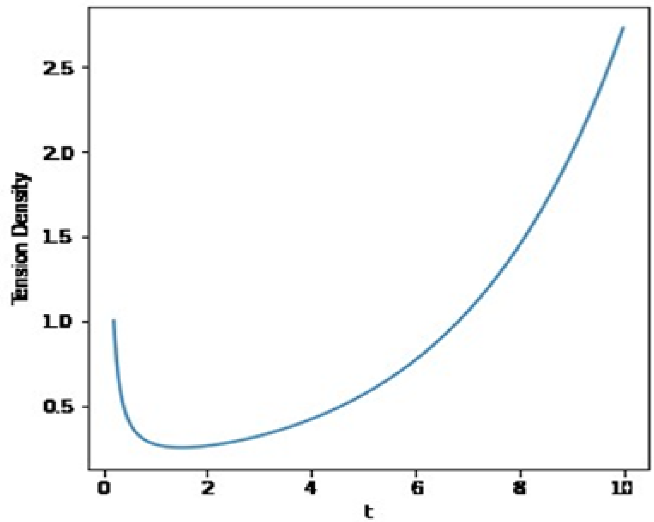

Fig 3. Variations of tension density Vs time

We can see from Equation (34), that the energy density of the fluid is a diminishing function of time. Figure 2 depicts the fluid's energy density vs time, whenever $n = 1$, phi=0.2, c=0.25, alpha $= 0.3,\ u1 = u2 = 0.5,\ \xi = 15,\ b = 0.5,\ l_0 = 0.1$. This demonstrates that the energy density ρ is decreasing function of time, but it remains positive. It satisfies the energy condition $\rho \geq 0$. When cosmic time $t \to 0$ then energy density $\rho \to \infty$ and as $t \to \infty$, the energy density decreases. So, we can see that initially the model has singularity. Initially tension density has some finite value then it rapidly increases as time increases i.e., string is survived for this model. The variation of tension density λ against cosmic time t is graphically presented in Figure 3 (Whenever $u3 = u4 = 0.5$).

Thus, using the Equations (34) and (35) in the relation, $\rho = \lambda + \rho_p$ the particle's energy density is as follows.

$$\rho_p = \frac{1}{\sqrt{c}} \left\{ \varphi_0 [t^n e^t]^{\frac{\alpha}{2}} \left[-\left(1+\frac{n}{t}\right)^2 u_1 - u_2 \left(1 + \frac{2n}{t} + \frac{n(n-1)}{t^2}\right) + b^2 l_0 (t^n e^t)^{-\frac{4k}{3k+1}} \right] + 2\xi \left(1+\frac{n}{t}\right) - \varphi_0^2 [t^n e^t]^{\alpha} \left\{ u_3 \left(1+\frac{n}{t}\right)^2 + u_4 \left(1 + \frac{2n}{t} + \frac{n(n-1)}{t^2}\right) \right\} \right\}^{\frac{1}{2}} \tag{36}$$

In Figure 4, we can see that $\rho_p \geq 0$ for all time t and it decreases with increase in time.

From Equations (4) and (8) the total pressure $\overline{p}$ and the proper pressure p are obtained as

$$\overline{p} = \frac{1}{\sqrt{c}} \left\{ \varphi_0 [t^n e^t]^{\frac{\alpha}{2}} \left[-\left(1+\frac{n}{t}\right)^2 u_1 - u_2 \left(1 + \frac{2n}{t} + \frac{n(n-1)}{t^2}\right) + b^2 l_0 (t^n e^t)^{-\frac{4k}{3k+1}} \right] + 2\xi \left(1+\frac{n}{t}\right) \right\}^{\frac{1}{2}} \tag{37}$$

$$+ \varphi_0 [t^n e^t]^{\frac{\alpha}{2}} \left\{ -u_1 \left(1+\frac{n}{t}\right)^2 - u_2 \left(1 + \frac{2n}{t} + \frac{n(n-1)}{t^2}\right) + b^2 l_0 [t^n e^t]^{-\frac{4k}{3k+1}} \right\}$$

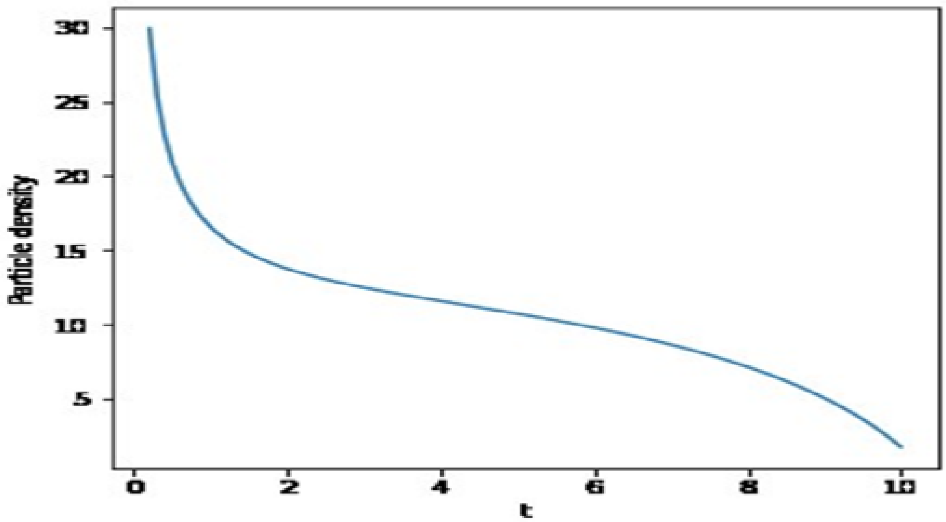

Fig 4. Variations of Particle density Vs time

$$p=\frac{1}{\sqrt{c}}\left\{\varphi_0[t^n e^t]^{\frac{\alpha}{2}}\left[-\left(1+\frac{n}{t}\right)^2 u_1-u_2\left(1+\frac{2n}{t}+\frac{n(n-1)}{t^2}\right)+b^2 l_0(t^n e^t)^{-\frac{4k}{3k+1}}\right]+2\xi\left(1+\frac{n}{t}\right)\right\}^{\frac{1}{2}}$$
$$+\varphi_0[t^n e^t]^{\frac{\alpha}{2}}\left\{-u_1\left(1+\frac{n}{t}\right)^2-u_2\left(1+\frac{2n}{t}+\frac{n(n-1)}{t^2}\right)+b^2 l_0[t^n e^t]^{-\frac{4k}{3k+1}}\right\}+2\xi\left(1+\frac{n}{t}\right) \quad (38)$$

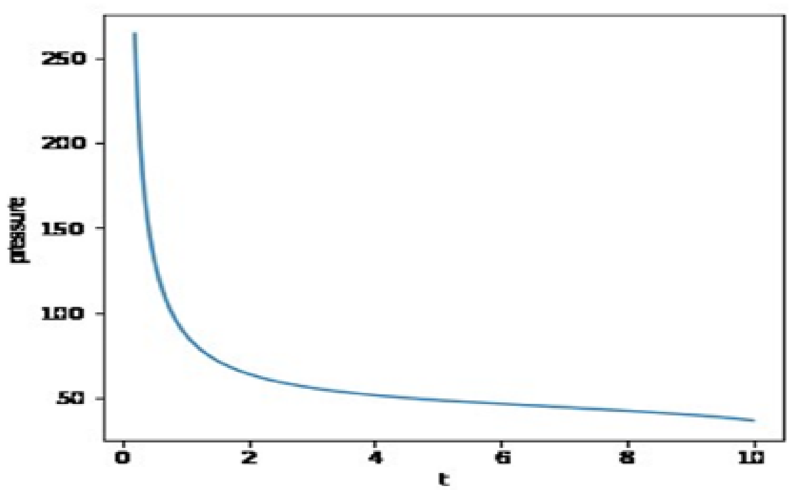

Fig 5. Variations of pressure Vs time

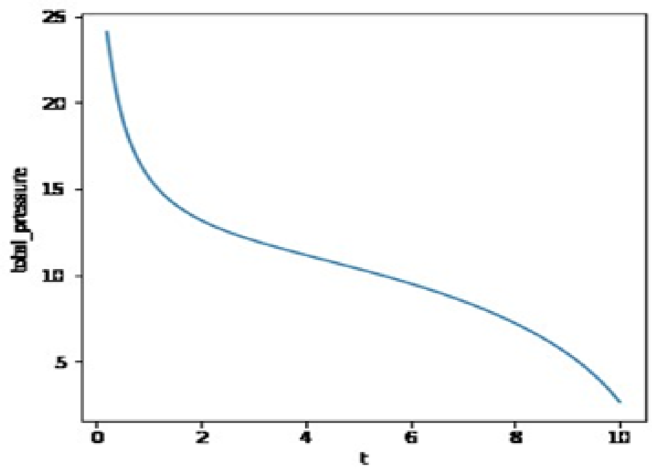

Fig 6. Variations of Total pressure Vs time. In the above figure pressure p is shown, which also diverges when $t = 0$ and becomes zero as $t \to \infty$

Model 2. Here we take another ansatz for average scale factor which is given by

$$R(t) = [\sinh(at)]^{\frac{1}{\beta}} \tag{39}$$

This gives a deceleration parameter which is time dependent. We select $\beta = 2$ as in model1 for the purpose of simplicity.

Now, using Equations (11), (20) and (39) we get the expressions as follows

$$X = l_2[\sinh(at)]^{\frac{2k}{3k+1}} \tag{40}$$

$$Y = l_1[\sinh(at)]^{\frac{2}{3k+1}} \tag{41}$$

Hence, the model takes the form as

$$ds^2 = l_2^2[\sinh(at)]^{\frac{4k}{3k+1}}\left[dx^2+e^{-2bx}dy^2+dz^2\right]+l_1^2[\sinh(at)]^{\frac{4}{3k+1}}dm^2. \tag{42}$$

3.2 Physical Aspects of the Model 2

The expressions of the physical parameters for the model 42 (Equation (42)) are as follows;

Spatial volume

$$V = \sinh^2(at) \tag{43}$$

The average Hubble parameter

$$H = \frac{1}{2}acoth(at) \tag{44}$$

The scalar expansion

$$\theta = 4H = 2acoth(at) \tag{45}$$

The deceleration parameter

$$q = -1 + 2\sec h^2(at) \tag{46}$$

Mean anisotropy parameter

$$\Delta = \frac{3(k-1)^2}{(3k+1)^2} \tag{47}$$

Shear scalar

$$\sigma^2 = \frac{9(k-1)^2 a}{8(3k+1)^2}(\coth(at))^2 \tag{48}$$

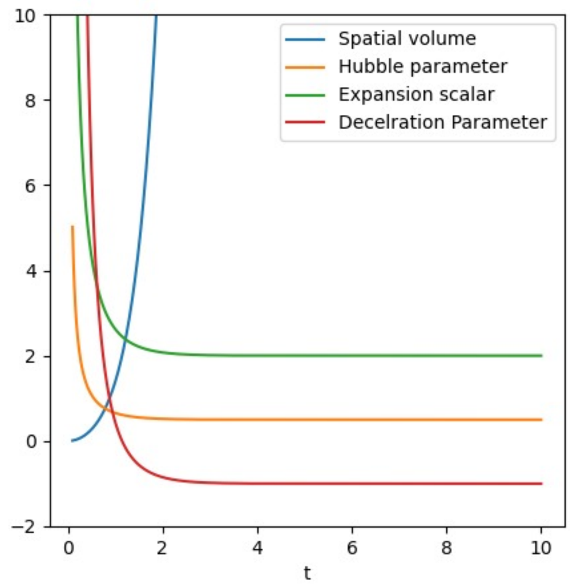

Fig 7. Variations of Spatial Volume, Hubble parameter, Expansion Scalar and Deceleration parameter Vs time whenever$a = 1,\ k = 0.2$

The spatial scale factors are zero at the starting epoch i.e. when $t = 0$, as shown from Equations (40) and (41), and as a result the model possesses a point type singularity. As, spatial volume of the model increases with cosmic time, it shows that universe is spatially expanding.

At a later time, both the Hubble parameter H and the expansion scalar θ become constant. They show the uniform expansion of the universe.

The variation of deceleration parameter q versus cosmic time t is depicted in the above figure. This demonstrates that the transition from the early decelerating phase $(q > 0)$ to the current accelerating phase $(q < 0)$ of the model is occurring smoothly. The present universe is accelerating is confirmed by the recent observations of SNe Ia and it also shows that the deceleration parameter value has the range $-1 \leq q < 0$ which is equal to $q = -0.73$ at $t \approx 13.7$ Gyr.

The anisotropy parameter behaves the same as model 1. i.e. the model is anisotropic in nature throughout the universe.

The jerk, snap, and lerk parameters are given below

Jerk parameter

$$j(t) = \frac{-2sinh^2(at) + 3cosh(at)^2}{cosh(at)^2} \tag{49}$$

Snap-parameter

$$s(t) = \frac{-4(sinh(at))^4 - 20(cosh(at))^2(sinh(at))^2 - 15(cosh(at))^4}{(cosh(at))^4} \tag{50}$$

Lerk parameter

$$l(t) = \frac{76(sinh(at))^4 - 180(cosh(at))^2(sinh(at))^2 + 105(cosh(at))^4}{(cosh(at))^4} \tag{51}$$

From Equation (17) we obtain,

$$\varphi = \varphi_0[\sinh(at)]^{\frac{\alpha}{2}} \tag{52}$$

By using the Equations (13), (16), (40) and (41) the energy density of the fluid is obtained as,

$$\rho = \frac{1}{\sqrt{c}}\left\{\begin{array}{l} \varphi_0[\sinh(at)]^{\alpha}\left[\frac{-4ka^2(5k+4)}{(3k+1)^2}\coth^2(at) - \frac{4ka^2+2a^2\operatorname{cosech}^2(at)}{(3k+1)} + b^2\left[\operatorname{cosech}^2(at)\right]^{\frac{4k}{3k+1}}\right] + \\ 2\xi a\coth(at) \end{array}\right\}^{\frac{1}{2}} \tag{53}$$

The string tension density λ is obtained using Equations (14) and (15) as

$$\lambda = \varphi_0^2[\sinh(at)]^{\alpha}\left\{u_5a^2\coth^2(at) + u_6k - u_7a^2\coth^2(at) + a^2\operatorname{cosech}^2(at)\right\}. \tag{54}$$

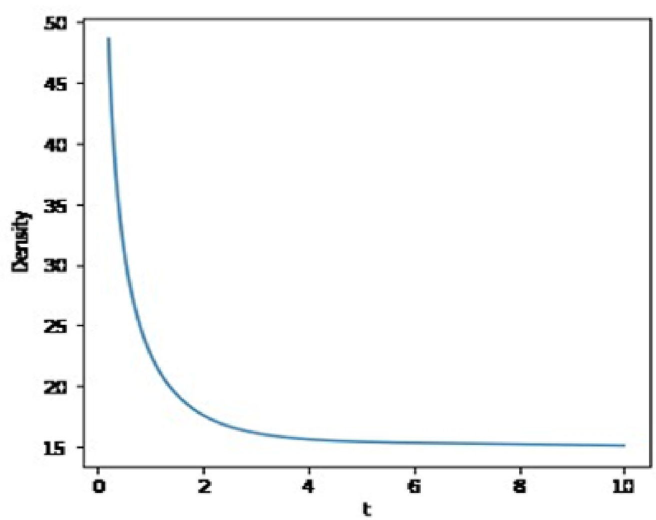

Fig 8. Variations of density Vs time

We can see from the above graphs that the viscous fluid has a positive energy density and tension density. According to Equation (54) for all values of cosmic time t, the string tension density is positive & $\lambda \geq 0$. Initially at $t \to 0$, λ is very large (attains the peak value) and just after that it becomes a decreasing function of cosmic time t and finally tends to a very small positive quantity i.e., string is present in the universe. In the above graph, the fluctuation of string tension density (λ) against cosmic time (t) is depicted.

We see from Equation (53) that the fluid's energy density $\rho(t)$ is a decreasing function of time. The curve of the fluid's energy density with time is shown in the above figure. We see that ρ is a positive decreasing function of time and as $t \to \infty$ it approaches to zero.

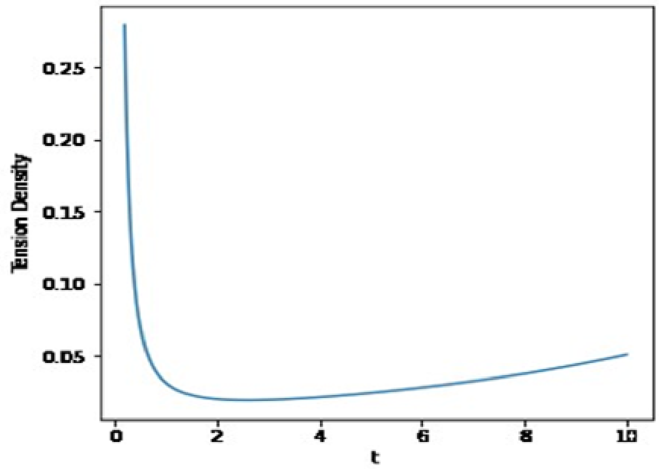

Fig 9. Variations of tension density Vs time, whenever u5=u6=u7=0.5

Thus, using Equations (53) and (54) in relation with $\rho = \lambda + \rho_p$, the energy density of particle is as follows

$$\rho_p = \frac{1}{\sqrt{c}} \left\{ \begin{array}{c} \varphi_0 [sinh(at)]^\alpha \left[\frac{-4ka^2(5k+4)}{(3k+1)^2} coth^2(at) - \frac{4ka^2 + 2a^2 cosech^2(at)}{(3k+1)} + b^2 \left[cosech^2(at)\right]^{\frac{4k}{3k+1}} \right] + \\ 2\xi acoth(at) \\ -\varphi_0 [sinh(at)]^\alpha \left\{ u_5 a^2 coth^2(at) + u_6 k - u_7 a^2 coth^2(at) + a^2 cosech^2(at) \right\}. \end{array} \right\}^{\frac{1}{2}} \tag{55}$$

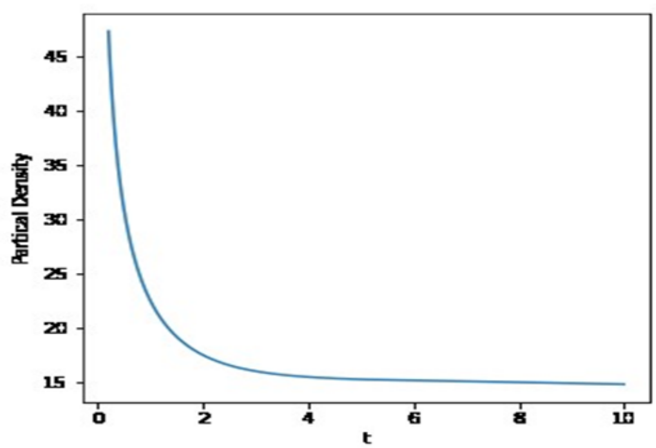

Fig 10. Variations of Particle Density Vs time

For all time $t, \rho_p \geq 0$ and with increasing of time ρ_p decreases. The above figure represents the variation of particle density ρ_p against cosmic time t. We see from the nature of graph and from Equation (55) that the particle density ρ_p is a decreasing function of cosmic time t. Particle density is initially very high when $t \to 0$ and initial singularity occurs when $t = 0$. In this model, as time goes on, the value of ρ_p decreases until it eventually reaches a constant value. This demonstrates that our universe will always have a finite number of particles. This may correspond to the period of decoupling radiation and the era of matter dominance. Our universe has a point-type initial singularity. The particle density and the string tension density both tend to infinity from the beginning i.e., $t \to 0$. This suggests that our universe began with a big bang, and that as time goes on, both ρ_p and λ decrease with the expansion of cosmos.

From Equations (4) and (8) we can obtain the total pressure $\overline{p}$ and the proper pressure p_{as}

$$p=\frac{1}{\sqrt{c}}\left\{\begin{array}{c}\varphi_0[sinh(at)]^{\alpha}\left[\frac{-4ka^2(5k+4)}{(3k+1)^2}coth^2(at)-\frac{4ka^2+2a^2cosech^2(at)}{(3k+1)}+b^2\left[cosech^2(at)\right]^{\frac{4k}{3k+1}}\right]+\\ 2\xi acoth(at)\end{array}\right\}^{\frac{1}{2}} \tag{56}$$
$$-\varphi_0[sinh(at)]^{\alpha}\left[\frac{-4ka^2(5k+4)}{(3k+1)^2}coth^2(at)-\frac{4ka^2+2a^2cosech^2(at)}{(3k+1)}+b^2\left[cosech^2(at)\right]^{\frac{4k}{3k+1}}\right]+2\xi acoth(at)$$

$$\overline{p}=\frac{1}{\sqrt{c}}\left\{\begin{array}{c}\varphi_0[sinh(at)]^{\alpha}\left[\frac{-4ka^2(5k+4)}{(3k+1)^2}coth^2(at)-\frac{4ka^2+2a^2cosech^2(at)}{(3k+1)}+b^2\left[cosech^2(at)\right]^{\frac{4k}{3k+1}}\right]+\\ 2\xi acoth(at)\end{array}\right\} \tag{57}$$
$$+\varphi_0[sinh(at)]^{\alpha}\left[\frac{-4ka^2(5k+4)}{(3k+1)^2}coth^2(at)-\frac{4ka^2+2a^2cosech^2(at)}{(3k+1)}+b^2\left[cosech^2(at)\right]^{\frac{4k}{3k+1}}\right]$$

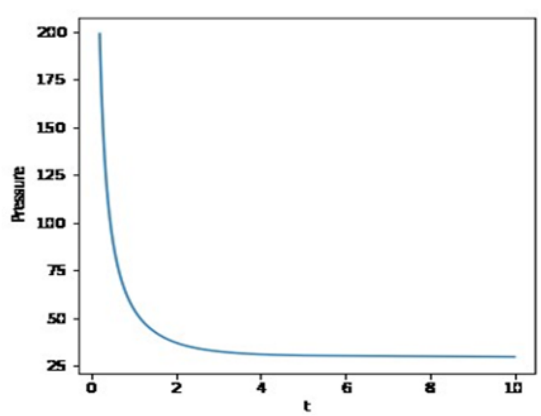

Fig 11. Variations of Pressure Vs time

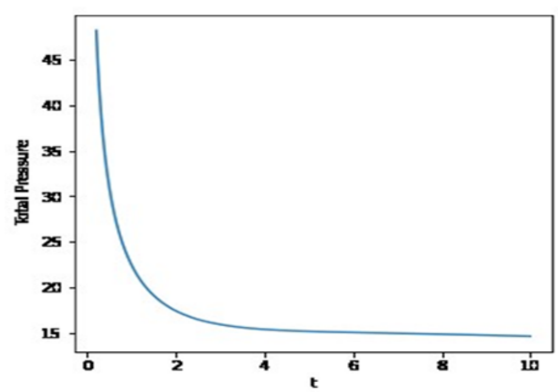

Fig 12. Variations of Total Pressure Vs time

From the above graphs we see pressure and total pressure, both are decreasing with increasing cosmic time t.

4 Conclusion

In the context of Sen-Dunn gravitational theory, we examined five-dimensional Bianchi type-III string with bulk viscous fluid cosmological models in this research. We have taken into account two separate scale factors in order to obtain deterministic solutions, which result in a time-dependent deceleration parameter. These simulations have the intriguing feature of depicting a seamless change from the decelerating phase to the accelerating situation of the current phase. Each model's cosmological parameters are analysed, and their applicability to the contemporary cosmology is examined. In both the cases, the model represents an exponentially expanding and accelerating Universe that starts with volume 0 and stops with infinite volume. The model has an initial singularity. It has been noted that the deceleration, and energy density behaviours of models 1 and 2 are comparable. It also agrees with the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ which in turn imply that our derived models are physically realistic as the present day observational data. Although the tension density and particle density are comparable, the tension density survive in this model. The model is anisotropic one and shearing throughout its evolution for $k \neq 1$ but approaches to small isotropy whenever $k = 1$. These properties resemble those of Naidu et al.[20]. Thus, we see that both the models show the accelerated expansion of the cosmos. This study is likely to be useful for the analysis of different kinds of Bianchi models in various space-times. Through this study, we hope to present a better knowledge of the cosmological evolution of the present universe with the help of higher dimensional Bianchi type-III universe in Sen-Dunn theory.

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