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RESEARCH ARTICLE

Poisson Input and Exponential Service Time Finite Capacity Interdependent Queueing Model with Breakdown and Controllable Arrival Rates

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Abstract

Objectives: This study aims at (i) introducing the finite capacity of the interdependent queueing model with breakdown and controllable arrival rates, (ii) calculating the average number of clients in the system, and identifying the expected waiting period of the clients in the system, (iii) dealing with the model descriptions, steady-state equations, and characteristics, which are expressed in terms of $P_{(1,1)}(0)$, and (iv) analyzing the probabilities of the queueing system and its characteristics with numerical verification of the obtained results. **Methods:** While providing the input, the arrival rates through faster and slower arrival rates are controlled using the Poisson process. Also, the service provides an exponential distribution. The server provides the service on an FCFS basis. In this article, two types of models are used: $(1,i)$ and $(2,i)$ which are the system's conditions, where i represents the number of units present in the queue in which their probability is $P_{1,i}$ and $P_{2,i}$. All probabilities are distributed based on the speed of advent using this concept. Then, the steady-state probabilities are computed using a recursive approach. **Findings:** This paper discovers the number of clients using the system on average (L_s) and the expected number of clients in the system (W_s) using the probability of the steady-state calculation. Little's formula is used to derive the expected waiting period of the clients in the system. **Novelty:** There are articles connected to the finite capacity of failed service in functioning and malfunctioning, but this takes the initiative to provide a link in connection with the rates of the controllable arrivals and interdependency in the arrival and service processes.

Mathematics Subject allocation: 60K25, 68M20, 90B22.

Keywords: M/M/1/K Queue Model; Finite Capacity; Breakdown; Controllable Arrival rates; FCFS Queue Discipline

1 Introduction

This paper connects the M/M/1/K interrelation representation of queue breakdown and controllable arrival rates. The active and breakdown times of a server are considered here. During the active time, the machine gets the work that has to be done. In other words, it provides service to the client. During the breakdown time, the machine does not provide any service until it is repaired. William J. Gray discussed the active service and repair process[(1)]. All probabilities are distributed based on the speed of arrivals using this concept. This interdependent model is applied here to simulate real-world scenarios in the context of novel queueing models. This framework incorporates and controls both arrival and service aspects. Numerous investigations are carried out on queueing systems to clarify the concept of breakdown service and establish a connection between the breakdown services. It helps in controlling the arrival rates.

Queueing models are more convenient for examining real-life circumstances. It emanates from situations like digital voice communication, computer transmission techniques, and neuro-physiological issues. These models supply stochastic behavior of the system, such as waiting times, the count of clients in the system, and so on. These articulations help researchers take time in selecting the circumstances and hold a suitable estimation to precise the stream. Generally, it is pretended that the advent and the avail procedures are connected. During the long and uncertain breakdown time, it is commonly considered in the majority of the prior studies that the server entirely stops its service. Deepa and Kalidass discussed the M/M/1/N queue with working breakdowns and vacations[(2)]. The idea of controlling the speed of arrival times in certain queueing models was discovered by A. Srinivasan and M. Thiagarajan in their study on controllable arrival rates in a variety of queueing models[(3)]. A large number of workloads that deal with interrelated queueing representations are described in the literature. Rahim KH and Thiagarajan M (2023) conducted comprehensive research by developing the concept of arrival rates within specific queueing models and exploring the modifiable arrival rates within various queueing models[(4)]. Shanthi S, Subramanian MG, and Sekar G (2022) explored the system of queueing[(5)]. The existing literature on breakdown queueing theory involves examinations of both the conduct of the client within the queue and the behaviors, and characteristics of the system's servers. These studies present an explanation of queueing systems, including Controlled arrivals, breakdown and repair, Working Breakdown, two-customer and a server subject to breakdown, bulk service queue with server breakdown and multiple vacations, M/M/1 queueing model with working vacation, and two types of server breakdown[(6–18)]. Gross D introduced steady-state equations and demonstrated them in certain models[(19)].

The present study addresses the controllable arrivals by dividing the faster and slower rates within the breakdown service queueing system. This establishes a link between the breakdown service and the controlled arrivals by allowing the probabilities to be partitioned based on the speed of arrivals. This interconnected model can be effectively employed to simulate real-world scenarios such as the development of new queueing models and featuring controls for both service and arrival processes.

2 Methodology

2.1 Model Description

The following assumptions are predicted based on the queueing model:

The arrival procedure - $\{Y_1(t)\}$ and the service process - $\{Y_2(t)\}$ of the system follow a bivariate Poisson process and correlated, as given below:

$$P[Y_1(t)=y_1,\, Y_2(t)=y_2]=\frac{e^{-(\lambda_i+\mu-\varepsilon)t}\sum_{j=0}^{min(y_1,y_2)}(\varepsilon t)^j\,[(\lambda_i-\varepsilon)t]^{y_1-j}\,[(\mu-\varepsilon)t]^{y_2-j}}{j!\,(y_1-j)!\,(y_2-j)!} \tag{2.1}$$

where $j=0,1,2\ldots,min(y_1,y_2); y_1\ ,\ y_2=0,1\ldots,\lambda_i>0,\ i=0,1\ ;\ \mu>0,\ 0\le\varepsilon<min\ (\lambda_i,\ \mu),\ i=0,1.$

The parameters λ_0, λ_1, μ, ε, f, and r indicate that

λ_0 = The mean faster arrival rate.

λ_1 = The mean slower arrival rate.

μ = The mean service rate.

ε = The mean dependence rate (the covariance of $\{Y_1(t)\&Y_2(t)\}$).

f = Breakdown rate.

r = Repair rate.

The Markov chain with state space is

$i=1,\ 2,\ \ldots,t-1,t\ ,t+1,\ldots,T-1,T,T+1\ldots,K$ (where K is the capacity of the system).

The movement of the system indicated by the arrival rate describes that:

- When the system size rises from low to T, the arrival rate, which was λ_0 until $T-1$, decreases to λ_1 and remains the same for consecutive upward movement of the system size.
- When the system size decreases to t, the arrival rate, which was λ_1 until $t+1$, increases to λ_0 and remains the same for subsequent downward movement to 0 and upward movement up to $T-1$. The same procedure is repeated again and again.

In this article, two types of models are used. They are, the system's conditions which are denoted by $(1,i)$ and $(2,i)$ in which i represents the number of units present in the queue, where $(1,i)$ is i clients in the system while in active service $(i \geq 1)$ and its probability is $P_{1,i}$.$(2,i)$ is i clients in the system while in the repair process $(i \geq 2)$ and its probability is $P_{2,i}$.

The steady-state probabilities are expressed as follows:

Let $P_{1,i}(0)$ describes the steady-state probability that there are queued i clients when active service and the system is at a faster arrival rate.

Let $P_{1,i}(1)$ describes the steady-state probability that there are queued i clients when active service and the system is at a slower arrival rate.

Let $P_{2,i}(0)$ describes the steady-state probability that there are queued i clients when repair service and the system is at a faster arrival rate.

Let $P_{2,i}(1)$ describes the steady-state probability that there are queued i clients when repair service and the system is at a slower arrival rate.

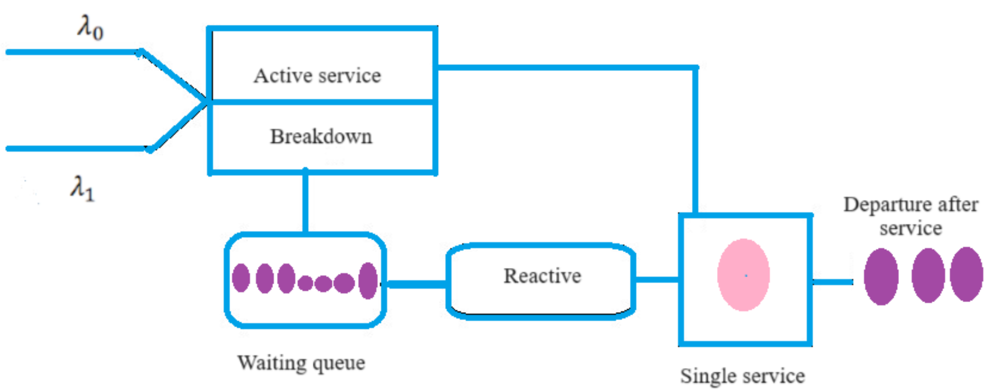

Fig 1. M/M/1/K Model with breakdown and controllable arrival rates

3 Results and Discussion

3.1 Steady-state Equations

The steady state is characterized by the probability of the client count in a queuing system being time-independent, signifying the steady state of the system. Shortle JF, Thompson JM, Thompson JM, Harris CM. (2018) Fundamentals of Queueing Theory explained the steady-state equations[19]. Deepa and Kalidass (2018) discussed the breakdown concept, steady-state equations, and their explanations[2]. It seems that only $P_{1,i}(0)$& $P_{2,i}(0)$ exists when $i=1,\ 2,\ \ldots, t-1, t$. Both $P_{1,i}(0)$, $P_{2,i}(0)$, $P_{1,i}(1)$ & $P_{2,i}(1)$ when $i=t+1,\ldots,T-1$. $P_{1,i}(1)$ and $P_{2,i}(1)$ exists when $i=T,\ldots,K$. Additionally $P_{1,i}(0)=P_{2,i}(0)=P_{1,i}(1)=P_{2,i}(1)=0$ if $i>K$. The steady-state equations are specified as

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,1}(0)= (\mu-\varepsilon)P_{1,2}(0)+rP_{2,1}(0) \tag{3.1}$$

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,i}(0)= (\lambda_0-\varepsilon)P_{1,i-1}(0)+(\mu-\varepsilon)P_{1,i+1}(0)+rP_{2,i}(0); \quad (i=2,3,\ldots,t-1) \tag{3.2}$$

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,t}(0)= (\lambda_0-\varepsilon)P_{1,t-1}(0)+(\mu-\varepsilon)P_{1,t+1}(0)+(\mu-\varepsilon)P_{1,t+1}(1)+rP_{2,t}(0) \tag{3.3}$$

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,i}(0)= (\lambda_0-\varepsilon)P_{1,i-1}(0)+(\mu-\varepsilon)P_{1,i+1}(0)+rP_{2,i}(0); \quad (i=t+1,t+2,\ldots,T-2) \tag{3.4}$$

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,T-1}(0)= (\lambda_0-\varepsilon)P_{1,T-2}(0)+rP_{2,T-1}(0) \tag{3.5}$$

$$(\lambda_1+\mu+f-2\varepsilon)P_{1,t+1}(1)= (\mu-\varepsilon)P_{1,t+2}(1)+rP_{2,t+1}(1) \tag{3.6}$$

$$(\lambda_1+\mu+f-2\varepsilon)P_{1,i}(1)= (\lambda_1-\varepsilon)P_{1,i-1}(1)+(\mu-\varepsilon)P_{1,i+1}(1)+rP_{2,i}(1); \quad (i=t+2,t+3,\ldots,T-1) \tag{3.7}$$

$$(\lambda_1+\mu+f-2\varepsilon)P_{1,T}(1)= (\lambda_0-\varepsilon)P_{1,T-1}(0)+(\lambda_1-\varepsilon)P_{1,T-1}(1)+(\mu-\varepsilon)P_{1,T+1}(1)+rP_{2,T}(1) \tag{3.8}$$

$$(\lambda_1+\mu+f-2\varepsilon)P_{1,i}(1)= (\lambda_1-\varepsilon)P_{1,i-1}(1)+(\mu-\varepsilon)P_{1,i+1}(1)+rP_{2,i}(1); \quad (i=T+1,T+2,\ldots,K) \tag{3.9}$$

$$(\lambda_0-\varepsilon+r)P_{2,1}(0)=fP_{1,1}(0) \tag{3.10}$$

$$(\lambda_0-\varepsilon+r)P_{2,i}(0)=fP_{1,i}(0)+(\lambda_0-\varepsilon)P_{2,i-1}(0);(i=2,3,\ldots,T-1) \tag{3.11}$$

$$(\lambda_1-\varepsilon+r)P_{2,t+1}(1)=0 \tag{3.12}$$

$$(\lambda_1-\varepsilon+r)P_{2,i}(1)=fP_{1,i}(1)+(\lambda_1-\varepsilon)P_{2,i-1}(1);(i=t+2,t+3,\ldots,T-1) \tag{3.13}$$

$$(\lambda_1-\varepsilon+r)P_{2,T}(1)=fP_{1,T}(1)+(\lambda_1-\varepsilon)P_{2,T-1}(1)+(\lambda_0-\varepsilon)P_{2,T-1}(0) \tag{3.14}$$

$$(\lambda_1-\varepsilon+r)P_{2,i}(1)=fP_{1,i}(1)+(\lambda_1-\varepsilon)P_{2,i-1}(1);(i=T+1,T+2\ldots,K) \tag{3.15}$$

3.2 Computation of steady-state solutions

The steady-state equations are expressed as follows :

Let $A=\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}$; $B=\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}$; $C=\frac{f}{\mu-\varepsilon}$; $D=\frac{r}{\mu-\varepsilon}$; $E=\frac{A}{A+D}$;
$F=\frac{B}{B+D}$; $G=\frac{C}{B+D}$; $H=\frac{A}{B+D}$; $J=\frac{C}{A+D}$

From Equation (3.10) we have

$$\left(\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)+\left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,1}(0)=\left(\frac{f}{\mu-\varepsilon}\right)P_{1,1}(0)$$

$$(A+D)P_{2,1}(0) = CP_{1,1}(0)$$

$$P_{2,1}(0) = \left(\frac{C}{A+D}\right)P_{1,1}(0)$$

$$P_{2,1}(0) = JP_{1,1}(0) \tag{3.2.1}$$

From Equation (3.1)

$$(\lambda_0+\mu+f-2\varepsilon)P_{1,1}(0) = (\mu-\varepsilon)P_{1,2}(0)+rP_{2,1}(0)$$

$$(\mu-\varepsilon)P_{1,2}(0) = (\lambda_0+\mu+f-2\varepsilon)P_{1,1}(0)-rP_{2,1}(0)$$

$$P_{1,2}(0) = \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}+\frac{\mu-\varepsilon}{\mu-\varepsilon}+\frac{f}{\mu-\varepsilon}\right)P_{1,1}(0)-\left(\frac{r}{\mu-\varepsilon}\right)P_{2,1}(0)$$

$$P_{1,2}(0) = (A+1+C)P_{1,1}(0)-DP_{2,1}(0)$$

From Equation (3.2)

$$P_{1,i+1}(0) = \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}+\frac{\mu-\varepsilon}{\mu-\varepsilon}+\frac{f}{\mu-\varepsilon}\right)P_{1,i}(0)-\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{1,i-1}(0)-\left(\frac{r}{\mu-\varepsilon}\right)P_{2,i}(0)$$

$$P_{1,i+1}(0) = (A+1+C)P_{1,i}(0)-AP_{1,i-1}(0)-DP_{2,i}(0)$$

$$P_{1,i}(0) = (A+1+C)P_{1,i-1}(0)-AP_{1,i-2}(0)-DP_{2,i-1}(0)$$

Recursively using the Equation (3.1) and Equation (3.2) we obtain,

$$P_{1,i}(0) = (A+1+C)P_{1,i-1}(0)-AP_{1,i-2}(0)-DP_{2,i-1}(0);(i=2,3,\ldots,t) \tag{3.2.2}$$

Using Equation (3.3) we get,

$$P_{1,t+1}(0) = \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}+\frac{\mu-\varepsilon}{\mu-\varepsilon}+\frac{f}{\mu-\varepsilon}\right)P_{1,t}(0)-\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{1,t-1}(0)-P_{1,t+1}(1)$$

$$-\left(\frac{r}{\mu-\varepsilon}\right)P_{2,t}(0)$$

$$P_{1,t+1}(0) = (A+1+C)P_{1,t}(0)-AP_{1,t-1}(0)-P_{1,t+1}(1)-DP_{2,t}(0)$$

From Equation (3.4)

$$P_{1,i+1}(0) = \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}+\frac{\mu-\varepsilon}{\mu-\varepsilon}+\frac{f}{\mu-\varepsilon}\right)P_{1,i}(0)-\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{1,i-1}(0)-\left(\frac{r}{\mu-\varepsilon}\right)P_{2,i}(0)$$

$$P_{1,i+1}(0) = (A+1+C)P_{1,i}(0) - AP_{1,i-1}(0) - DP_{2,i}(0)$$

$$P_{1,i}(0) = (A+1+C)P_{1,i-1}(0) - AP_{1,i-2}(0) - DP_{2,i-1}(0)$$

Recursively using Equations (3.3) and (3.4) we get,

$$P_{1,i}(0) = (A+1+C)P_{1,i-1}(0) - AP_{1,i-2}(0) - P_{1,t+1}(1) - DP_{2,i-1}(0); \quad (i = t+1, t+2, \ldots, T-1) \tag{3.2.3}$$

Using Equations (3.5), (3.6) and (3.7) we obtain,

$$P_{1,i}(1) = (B+1+C)P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1); \quad (i = t+1, t+2, \ldots, T) \tag{3.2.4}$$

From Equation (3.8),

$$P_{1,T+1}(1) = \left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon} + \frac{\mu-\varepsilon}{\mu-\varepsilon} + \frac{f}{\mu-\varepsilon}\right)P_{1,T}(1) - \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{1,T-1}(0) - \left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)P_{1,T-1}(1) - \left(\frac{r}{\mu-\varepsilon}\right)P_{2,T}(1)$$

$$P_{1,T+1}(1) = (B+1+C)P_{1,T}(1) - AP_{1,T-1}(0) - BP_{1,T-1}(1) - DP_{2,T}(1) \tag{3.2.5}$$

From Equation (3.9) we get,

$$P_{1,i+1}(1) = \left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon} + \frac{\mu-\varepsilon}{\mu-\varepsilon} + \frac{f}{\mu-\varepsilon}\right)P_{1,i}(1) - \left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)P_{1,i-1}(1) - \left(\frac{r}{\mu-\varepsilon}\right)P_{2,i}(1)$$

$$P_{1,i}(1) = (B+1+C)P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1); (i = T+2, T+3 \ldots, K) \tag{3.2.6}$$

From Equation (3.11) we get,

$$\left(\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right) + \left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,i}(0) = \left(\frac{f}{\mu-\varepsilon}\right)P_{1,i}(0) + \left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{2,i-1}(0)$$

$$(A+D)P_{2,i}(0) = CP_{1,i}(0) + AP_{2,i-1}(0)$$

$$P_{2,i}(0) = \left(\frac{C}{A+D}\right)P_{1,i}(0) + \left(\frac{A}{A+D}\right)P_{2,i-1}(0)$$

$$P_{2,i}(0) = JP_{1,i}(0) + EP_{2,i-1}(0); (i = 2, 3, \ldots, T-1) \tag{3.2.7}$$

From Equation (3.12) we get,

$$\left(\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right) + \left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,t+1}(1) = 0$$

$$(B+D)P_{2,t+1}(1) = 0$$

$$P_{2,t+1}(1) = \frac{1}{B+D} \tag{3.2.8}$$

From Equation (3.13)

$$\left(\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)+\left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,i}(1) = \left(\frac{f}{\mu-\varepsilon}\right)P_{1,i}(1)+\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)P_{2,i-1}(1)$$

$$(B+D)P_{2,i}(1) = CP_{1,i}(1)+BP_{2,i-1}(1)$$

$$P_{2,i}(1) = \left(\frac{C}{B+D}\right)P_{1,i}(1)+\left(\frac{B}{B+D}\right)P_{2,i-1}(1)$$

$$P_{2,i}(1) = GP_{1,i}(1)+FP_{2,i-1}(1);\ (i = t+2, t+3, \ldots, T-1) \tag{3.2.9}$$

From Equation (3.14) we have

$$\left(\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)+\left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,T}(1) = \left(\frac{f}{\mu-\varepsilon}\right)P_{1,T}(1)+\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)P_{2,T-1}(1)+\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right)P_{2,T-1}(0)$$

$$(B+D)P_{2,T}(1) = CP_{1,T}(1)+BP_{2,T-1}(1)+AP_{2,T-1}(0)$$

$$P_{2,T}(1) = \left(\frac{C}{B+D}\right)P_{1,T}(1)+\left(\frac{B}{B+D}\right)P_{2,T-1}(1)+\left(\frac{A}{B+D}\right)P_{2,T-1}(0)$$

$$P_{2,T}(1) = GP_{1,T}(1)+FP_{2,T-1}(1)+HP_{2,T-1}(0) \tag{3.2.10}$$

From Equation (3.15)

$$\left(\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)+\left(\frac{r}{\mu-\varepsilon}\right)\right)P_{2,i}(1) = \left(\frac{f}{\mu-\varepsilon}\right)P_{1,i}(1)+\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)P_{2,i-1}(1)$$

$$(B+D)P_{2,i}(1) = CP_{1,i}(1)+BP_{2,i-1}(1)$$

$$P_{2,i}(1) = \left(\frac{C}{B+D}\right)P_{1,i}(1)+\left(\frac{B}{B+D}\right)P_{2,i-1}(1)$$

$$P_{2,i}(1) = GP_{1,i}(1)+FP_{2,i-1}(1);(i = T+1, T+2, \ldots, K) \tag{3.2.11}$$

3.3 Characteristics of the Model

In this framework, the expected results and the analytical discoveries are derived for the system. The probability in the system (λ_0 & λ_1) are specified as follows:

Probability of faster arrival

Here, the probability of faster arrival $P(0)$ is calculated.

$$P(0) = \sum_{i=1}^{K} P_{1,i}(0) + \sum_{i=1}^{K} P_{2,i}(0) \tag{3.3.1}$$

$P(0)$ exists only when $i = 1, 2, 3, \ldots, T-1$

$P(0) = \sum_{i=2}^{t} P_{1,i}(0) + \sum_{i=t+1}^{T-1} P_{1,i}(0) + P_{2,1}(0) + \sum_{i=2}^{T-1} P_{2,i}(0)$

$P(0) = Z_1 P_{1,1}(0);$

where,

$$\begin{aligned} Z_1 \; &= \sum_{i=2}^{t} \{(A+1+C) P_{1,i-1}(0) - AP_{1,i-2}(0) - DP_{2,i-1}(0)\} \\ &+ \sum_{i=t+1}^{T-1} \{(A+1+C) P_{1,i-1}(0) - AP_{1,i-2}(0) - DP_{2,i-1}(0)\} + J \\ &+ \sum_{i=2}^{T-1} \{JP_{1,i}(0) + EP_{2,i-1}(0)\} \end{aligned}$$

Probability of slower arrival

Here, the probability of slower arrival $P(1)$ is calculated.

$$P(1) = \sum_{i=1}^{K} P_{1,i}(1) + \sum_{i=1}^{K} P_{2,i}(1) \tag{3.3.2}$$

$P(1)$ exists only when $i = t+1, t+2, \ldots, K$

$P(1) = \sum_{i=t+1}^{T} P_{1,i}(1) + P_{1,T+1}(1) + \sum_{i=T+2}^{K} P_{1,i}(1) + P_{2,t+1}(1) + \sum_{i=t+2}^{T-1} P_{2,i}(1)$
$+ P_{2,T}(1) + \sum_{i=T+1}^{K} P_{2,i}(1)$

$P(1) = Z_2 P_{1,1}(0);$

where,

$$\begin{aligned} Z_2 = &\sum_{i=t+1}^{T} \{(B+1+C) P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1)\} \\ &+ \{(B+1+C) P_{1,T}(1) - AP_{1,T-1}(0) - BP_{1,T-1}(1) - DP_{2,T}(1)\} \\ &+ \sum_{i=T+2}^{K} \{(B+1+C) P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1)\} \\ &+ \left\{\frac{1}{B+D}\right\} + \sum_{i=t+2}^{T-1} \{GP_{1,i}(1) + FP_{2,i-1}(1)\} \\ &+ \{GP_{1,T}(1) + FP_{2,T-1}(1) + HP_{2,T-1}(0)\} + \sum_{i=T+1}^{K} \{GP_{1,i}(1) + FP_{2,i-1}(1)\} \end{aligned}$$

Normalizing form

At present, the normalizing conditions are used to calculate the probability of the faster and slower arrival rate. The probability $P_{1,1}(0)$ is determined from the normalizing form

$$P(0) + P(1) = 1$$

$$[Z_1 + Z_2] P_{1,1}(0) = 1; \; P_{1,1}(0) = [Z_1 + Z_2]^{-1}$$

The mean count of clients

The sum of the mean count of clients in the faster rates of arrival (L_{S_0}) and the mean count of clients with slower rates of arrival (L_{S_1}), gains the value of the mean count of clients in the structure which is L_s. Therefore, $L_S = L_{S_0} + L_{S_1}$,

where

$$L_{S_0} = \sum_{i=2}^{t} iP_{1,i}(0) + \sum_{i=t+1}^{T-1} iP_{1,i}(0) + P_{2,1}(0) + \sum_{i=2}^{T-1} iP_{2,i}(0) = L_1 P_{1,1}(0) \tag{3.3.3}$$

Here,

$$\begin{aligned} L_1 = &\sum_{i=2}^{t} i \{(A+1+C) P_{1,i-1}(0) - AP_{1,i-2}(0) - DP_{2,i-1}(0)\} \\ &+ \sum_{i=t+1}^{T-1} i \{(A+1+C) P_{1,i-1}(0) - AP_{1,i-2}(0) - DP_{2,i-1}(0)\} \\ &+ J + \sum_{i=2}^{T-1} i \{JP_{1,i}(0) + EP_{2,i-1}(0)\} \end{aligned}$$

$$L_{s_1} = \sum_{i=t+1}^{T} iP_{1,i}(1) + iP_{1,T+1}(1) + \sum_{i=T+2}^{K} iP_{1,i}(1) + iP_{2,t+1}(1) + \sum_{i=t+2}^{T-1} iP_{2,i}(1) + iP_{2,T}(1) + \sum_{i=T+1}^{K} iP_{2,i}(1) = L_2 P_{1,1}(0) \quad (3.3.4)$$

Here

$$\begin{aligned} L_2 = &\sum_{i=t+1}^{T} i\,\{(B+1+C)P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1)\} \\ &+ i\,\{(B+1+C)P_{1,T}(1) - AP_{1,T-1}(0) - BP_{1,T-1}(1) - DP_{2,T}(1)\} \\ &+ \sum_{i=T+2}^{K} i\,\{(B+1+C)P_{1,i-1}(1) - BP_{1,i-2}(1) - DP_{2,i-1}(1)\} + i\,\left\{\tfrac{1}{B+D}\right\} \\ &+ \sum_{i=t+2}^{T-1} i\,\{GP_{1,i}(1) + FP_{2,i-1}(1)\} + i\,\{GP_{1,T}(1) + FP_{2,T-1}(1) + HP_{2,T-1}(0)\} \\ &+ \sum_{i=T+1}^{K} i\,\{GP_{1,i}(1) + FP_{2,i-1}(1)\} \end{aligned}$$

Waiting Time

The Little's formula is used here to calculate the expected waiting period of the clients in the system which is represented as W_s.

$$W_S = \frac{L_S}{\bar{\lambda}},\ where\ \bar{\lambda} = \lambda_0(P_{1,i}(0) + P_{2,i}(0)) + \lambda_1(P_{1,i}(1) + P_{2,i}(1)) \quad (3.3.5)$$

3.4 Numerical Experiments

The numerical examples, which strongly support the theoretical results, are considered here. For various values of $r, f, \lambda_0, \lambda_1, \mu, and\ \varepsilon$, the values of $P_{1,1}(0), P(0), P(1), L_s, W_s$ are calculated. Consider $t = 3,\ T = 8,\ K = 12,\ r = 2,\ f = 5$. The behavior of L_s and W_s by the faster and slower arrivals is graphically represented for better understanding.

Table 1. M/M/1/K with breakdown and controllable arrival rates

Parameters		$P_{1,1}(0)$	Probability of faster arrival $P(0)$	Probability of slower arrival $P(1)$	Average count of clients L_s	Expected clients waiting time W_s
Faster rate of arrival (λ_0) Varying (λ_0) and keeping other parameters $\lambda_1 = 3,\ \mu = 12,\ \varepsilon = 1$ as fixed values.	10	0.0011	0.0188	0.9812	21.2564	6.7873
	7	0.0032	0.0555	0.9445	20.8040	6.4570
	5	0.0083	0.1101	0.8899	20.1289	6.2510
	4	0.0143	0.1570	0.8430	19.5320	6.1870
	8	0.0022	0.0396	0.9604	20.9999	6.5666
Slower rate of arrival (λ_1) Varying (λ_1) and keeping other parameters $\lambda_0 = 11,\ \mu = 15, \varepsilon = 0$ as fixed.	9	0.0007	0.0104	0.9896	21.7934	2.4159
	7	0.0011	0.0149	0.9851	22.1890	3.1431
	4	0.0019	0.0258	0.9742	22.1621	5.3013
	2	0.0029	0.0384	0.9616	20.3208	8.6620
	6	0.0013	0.0178	0.9822	22.3143	3.6646
Mean service rate (μ) Varying (μ) and keeping other parameters$\lambda_0 =$ $8,\ \lambda_1 = 6,\ \varepsilon = 0.5$ as fixed values.	16	0.0042	0.0501	0.9499	22.0964	3.6222
	14	0.0021	0.0300	0.9700	22.1684	3.6582
	20	0.0100	0.0926	0.9074	21.6244	3.4961
	10	0.0002	0.0043	0.9957	21.6379	3.6012
	18	0.0069	0.0719	0.9281	21.9004	3.5647
Mean dependence rate (ε) Varying (ε) and keeping other parameters $\lambda_0 = 5, \lambda_1 = 2, \mu = 9$ as static values.	0.5	0.0023	0.0489	0.9511	20.6866	9.6359
	0	0.0019	0.0385	0.9615	21.4444	10.1361
	0.75	0.0027	0.0559	0.9441	20.1360	9.2885
	1	0.0031	0.0646	0.9354	19.4444	8.8631
	0.25	0.0021	0.0432	0.9568	21.1164	9.9152

Due to the congestion in the queue, the rate of arrivals is changed from faster to slower. Therefore, the oscillation in the queue length happens. The numerical experiments demonstrate that (i) when a faster mean arrival rate (λ_0) increases, (L_s) &

(W_s) also increase, while the other variables are unaltered. (ii) when a slower mean arrival rate (λ_1) increases, (L_s) increases & (W_s) decreases, while the other variables are unaltered. (iii) when the mean service rate (μ) increases, (L_s) & (W_s) decrease, the other variables are unaltered. (iv) when the mean dependence rate (ε) becomes greater, (L_s) & (W_s) decrease and the other variables are fixed in the same manner.

Table 2. M/M/1/K with breakdown without controllable arrival rates

Parameters		**Average count of clients L_s**	**Expected clients waiting time W_s**
Arrival rate (λ) Varying $(\lambda_0 = \lambda_1 = \lambda)$ and keeping other parameters $\mu = 20,\ \varepsilon = 0$ as fixed values.	7	0.5384	0.0769
	13	1.8571	0.1428
	14	2.3333	0.1666
	17	5.6666	0.3333
	19	19	1.0000
Mean service rate (μ) Varying (μ) and keeping other parameters $\lambda = 7, \varepsilon = 0$ as fixed.	9	3.5	0.5
	12	1.4	0.2
	14	1	0.1428
	17	0.7	0.1
	20	0.5384	0.0769

Table 2 demonstrates that the average count of clients (L_s) increases with the rise in the arrival rate, which follows the real condition. (i) When a mean arrival rate (λ) increases, (L_s) & (W_s) also increase, while the other variables are unaltered; (ii) When the mean service rate (μ) increases, (L_s) & (W_s) decrease, the other variables are unaltered.

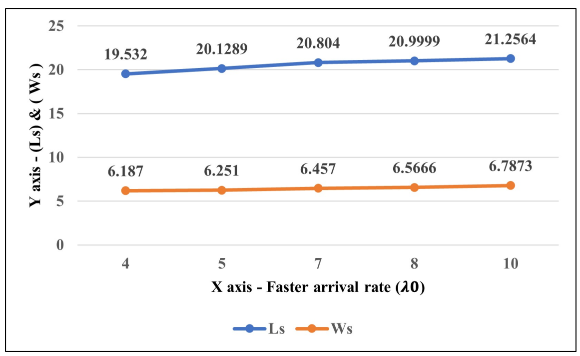

Fig 2. Variation of (L_S) & (W_S) when they lie in the faster arrival rates (λ_0) , and the unaltered of the other variables

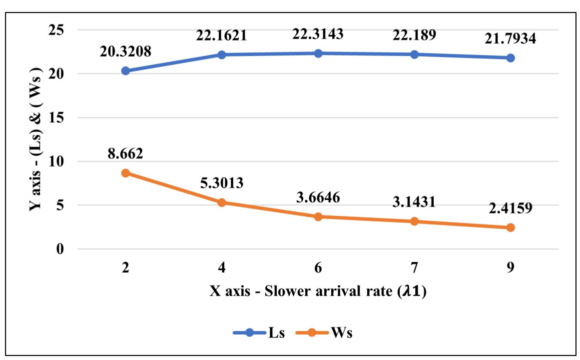

Fig 3. Variation of (L_S) & (W_S) when they lie in the slower arrival rates (λ_1) , and the unaltered of the other variables

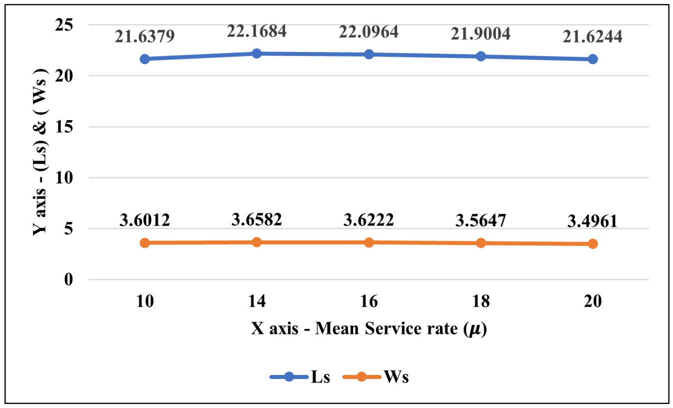

Fig 4. Variation of (L_S) & (W_S) when they lie in the mean service rate (μ) , and the unaltered of the other variables

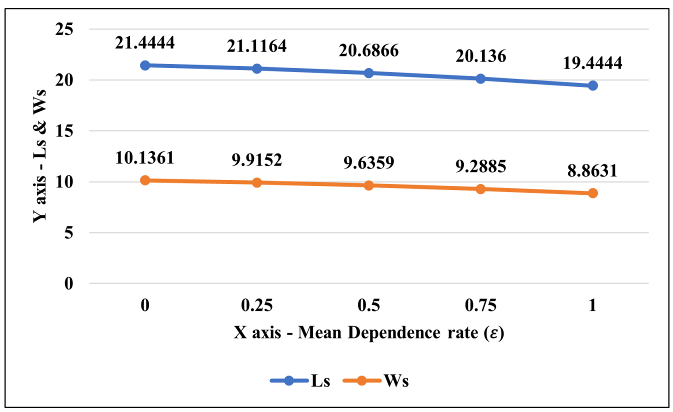

Fig 5. Variation of (L_S) & (W_S) when they lie in the mean dependence rate (ε) , and the unaltered of the other variables

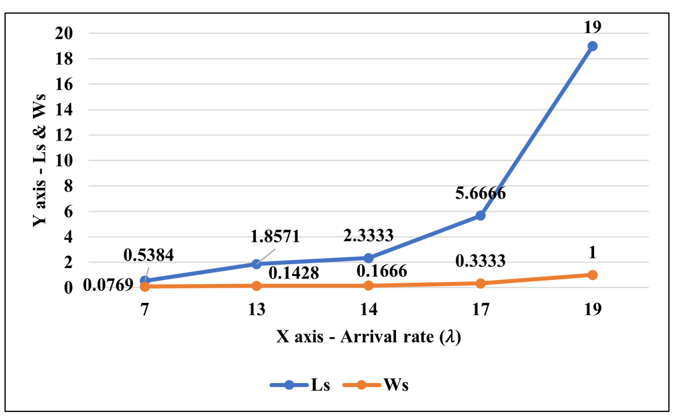

Fig 6. (L_S) & (W_S) by the varying arrival rate $(\lambda_0 = \lambda_1 = \lambda)$, and other variables are unaltered

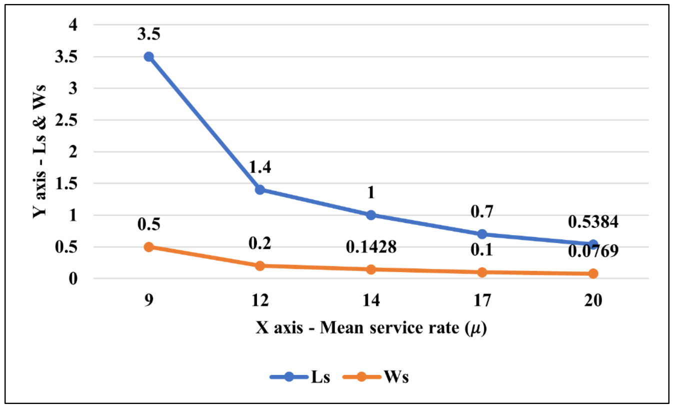

Fig 7. (L_S) & (W_S) **by varying mean service rate** (μ)**, and other variables are unaltered , when** $(\lambda_0 = \lambda_1 = \lambda)$.

4 Conclusion

This study proves that a new approach can be used to control the arrival rates that include both faster and slower rates in a breakdown service. Moreover, interdependence in the arrival and service processes is being gained. The explicit expression for $P_{1,1}(0)$ is obtained by using a recursive approach. The average number of clients in the system, and identifying the expected waiting period of the clients in the system are studied using a steady-state probability. The probabilities of the queueing system and its characteristics with numerical verification of the obtained results and graphs are carried out. These models can be applied to analyze practical situations in banks, hospital emergency rooms with limited beds, computer systems with limited buffers, and telephone centres with a finite number of waiting rooms. In these circumstances, of breakdown services, this novel study presents a unique method for controlling varying arrival rates, which includes both faster and slower rates. It also explores how arrival and service performance are interrelated, providing an exhaustive structure for investigation and improvement. Traits and associated Probabilities are established according to varying rates of faster and slower arrivals. Thus, the furnishing valuable perspectives for forthcoming research pursuits are exposed. As it is a promising avenue for prospective investigations, this model demonstrates the capability to undertake mathematical modeling of real-world situations. This specific model functions as a fundamental framework, designed to access and promote equivalent queuing models of this kind. The current model comprehensively includes the preceding iterations as specific instances. This article serves as a connection between the breakdown services and the controllable arrival rates.

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