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RESEARCH ARTICLE

Separation Axioms of $\alpha b^{*}g\alpha-$ Closed Sets in Topological Spaces

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Abstract

Objectives: The notion of the article is to introduce new spaces called $cT_{\alpha b*g\alpha}$- space, $T^{*}_{\alpha b*g\alpha}$- space, $*g\alpha T***_{1/2}$- space by using $\alpha b*g\alpha$- closed set. **Methods:** The characteristics and properties of the new spaces are explained using theorems and propositions. To prove the converse part of the theorems, counter examples are utilized. **Findings:** As the reverse implications of the theorems which explains the properties of the new spaces, are proved to be not true using the examples we could find the weaker and stronger spaces. Several of their fundamental characterizations and their relationships with other corresponding kinds of spaces are discussed. **Novelty:** The inter relationship between the spaces are identified and investigated and relation among the spaces are graphically illustrated. Independence relation among the $cT_{\alpha b*g\alpha}$- space, $T^{*}_{\alpha b*g\alpha}$- space, $*g\alpha T***_{1/2}$- space and other existing spaces are identified. Several examples are discussed for the independence relation.

Keywords: $\alpha b^{*}g\alpha$ - closed set; $b^{*}g\alpha$ - open set; $cT_{\alpha b*ga}$ - spce; $T_{\alpha b*ga}$ - space; $*g\alpha T***_{1/2}$ - space

1 Introduction

R. Devi, H. Maki and K. Balachandran [1] introduced αT_b- space. M. Vigneshwaran and R. Devi [2] proposed the idea of $*g\alpha$- closed sets and studied the properties of αT**1/2-space and TC**- space. M. Vigneshwaran and S. Saranya [3] introduced the concepts of b*gα- closed sets and functions and defined c$T_{b*g\alpha}$- spaces and *bT***1/2- spaces based on b*gα- closed set in topological spaces. Suthi Keerthana K, M. Vigneshwaran and L.Vidyarani [4] introduced the concepts of αb*gα- closed sets in topological spaces.

In this paper, we introduced and study some new spaces by applying αb*gα- closed sets, namely c$T_{\alpha b*g\alpha}$- space, $T^{*}_{\alpha b*g\alpha}$- space, *gαT***1/2- space. Moreover we have studied the properties and characteristics of these spaces with the other existing sets.

2 Preliminaries

The basic definitions, which are used in the next section, are referred from the references[1–8].

Definition 2.1 : A Subset E of the topological space (S, τ) is αb*gα- closed set[4] if $\alpha cl(E) \subseteq V$ whenever E $\subseteq$ V and V is b*gα- open in (S, τ).

Notation: For a space (S, τ), C(S, τ)(resp SC(S, τ), αC(S, τ), GC(S, τ), GSC(S, τ), GPC(S, τ), SGC(S, τ), αGC(S, τ), GαC(S, τ), *GαC(S, τ), GSPC(S, τ), G*SPC(S, τ), G*C(S, τ), *$\alpha G\alpha$C(S, τ), S*GαC(S, τ), B*GαC(S, τ), P*GαC(S, τ), αB*GαC(S, τ)) denote the class of all closed (resp. semi- closed, α- closed, g- closed, gs- closed, gp-closed, sg- closed, αg- closed, gα- closed, * gα- closed, gsp- closed, g*sp- closed, g*- closed, *αgα- closed, s*gα- closed. b*gα- closed, αb*gα- closed) subsets of (S, τ). CS and OS denote Closed sets and Open sets in topological spaces.

3 Results

Definition 3.1 : A Space (S, τ) is called c$T_{\alpha b*g\alpha}$- space if every αb*gα- closed set is closed.

Theorem 3.2. If (S, τ) is c$T_{\alpha b*g\alpha}$- space, then any singleton of S is either CS or αb*gα- OS.
Proof: Let s $\in$ S & {s} is not CS of (S, τ). Then S/s is not open.
$\Rightarrow$ S is the only OS containing s/{s}.
$\because$ (S, τ) is c$T_{\alpha b*g\alpha}$- space, S/s is αb*gα- CS or{s} is αb*gα- OS in (S, τ).
The reverse implication of the theorem need not be true.

Example 3.3. Let S = {f, g, h}, τ = {S, ϕ, {f}, {h}, {f, h}}
αb*gα- OS of (S, τ) = {S, ϕ, {f}, {h}, {f, g}, {f, h}, {g, h}}
αB*GαC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, g}, {g, h}}.
{f} & {h} are αb*gα- OS, {g} is CS but (S, τ) is not c$T_{\alpha b*g\alpha}$- space.
$\because$ {f} is αb*gα- CS but not CS of (S, τ).

Theorem 3.4. Any T_b- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is gs-CS & (S, τ) is T_b- space, K is CS.
$\therefore$ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.

Example 3.5. Let S = {f, g, h}, τ = {S, ϕ, {g}, {f, h}}
GSC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, g}, {f, h}, {g, h}}
αB*GαC(S, τ) = {S, ϕ, {g}, {f, h}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not T_b- space.
$\because$ {f} is gs- CS but not CS in (S, τ).

Theorem 3.6. Any αT_b- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is αg-CS & (S, τ) is αT_b- space, K is CS.
$\therefore$ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.

Example 3.7. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
αGC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h}, {g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not αT_b- space.
$\because$ {f} is αg- CS but not CS in (S, τ).

Theorem 3.8. Any c$T_{\alpha b*g\alpha}$- space is αT**1/2-space.
Proof: Let K be *gα- CS.
As every *gα- CS is αb*gα- CS, K is αb*gα- CS. Since (S, τ) is c$T_{\alpha b*g\alpha}$- space, K is Closed.
$\therefore$ (S, τ) is αT**1/2-space.
The reverse implication of the theorem need not be true.

Example 3.9. Let S = {f, g, h}, τ = {S, ϕ, {f}, {f, h}}
*GαC(S, τ) = {S, ϕ, {g}, {g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {g}, {g, h}}.
(S, τ) is αT**1/2-space but not c$T_{\alpha b*g\alpha}$- space.
$\because$ {h} is αb*gα- CS but not CS in (S, τ).

Theorem 3.10. Any semi T1/2- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is sg-CS & (S, τ) is semi T1/2- space, K is Closed.
∴ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.11. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
SGC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h}, {f. g},{g, h}}
SC(S, τ) = {S, ϕ, {h}, {f, g}},
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not semi T1/2- space.
∵ {g} is sg- CS but not semi- CS in (S, τ).
Theorem 3.12. Any *αT**1/2- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is *αgα -CS & (S, τ) is *αT**1/2- space, K is CS.
∴ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.13. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
*$\alpha G\alpha$C(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h},{g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not *αT**1/2- space.
∵ {g} is *αgα -CS but not CS in (S, τ).
Theorem 3.14. Any *sT**1/2- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is s*gα -CS & (S, τ) is *sT**1/2- space, K is CS.
∴ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.15. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
S*$G\alpha$C(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h},{g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not *sT**1/2- space.
∵ {g} is s*gα -CS but not CS in (S, τ).
Theorem 3.16. Any c$T_{b*g\alpha}$- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is b*gα -CS & (S, τ) is c$T_{b*g\alpha}$- space, K is CS.
∴ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.17. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
B*$G\alpha$C(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h},{g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not c$T_{b*g\alpha}$- space.
∵ {f} is b*gα -CS but not CS in (S, τ).
Theorem 3.18. Any pgT_{α}- space is c$T_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
As every αb*gα- CS is p*gα -CS & (S, τ) is pgT_{α}- space, K is CS.
∴ (S, τ) is c$T_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.19. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
P*$G\alpha$C(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, h},{g, h}}
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}.
(S, τ) is c$T_{\alpha b*g\alpha}$- space but not pgT_{α}- space.
∵ {f} is p*gα -CS but not CS in (S, τ).
The above figure illustrates the relation among c$T_{\alpha b*g\alpha}$- space and other existing spaces
Definition 3.20: A Space (S, τ) is called $T^{*}_{\alpha b*g\alpha}$- space if every αb*gα- CS is α- CS.

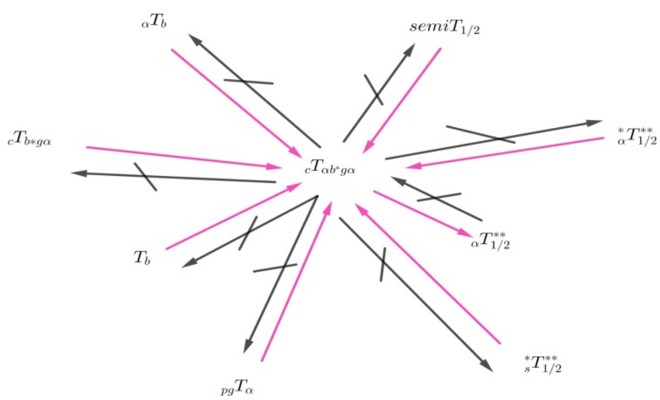

Fig 1. The relation among $cT_{\alpha b*g\alpha}$- space and other existing spaces

Theorem 3.21. If (S, τ) is $T^{*}_{\alpha b*g\alpha}$- space, then any singleton of S is either α- CS or αb*gα- OS.
Proof: Let s $\in$ S & {s} is not α- CS of (S, τ). Then S/s is not α- open.
$\Rightarrow$ S is the only α- OS containing s/{s}.
$\because$ (S, τ) is $T^{*}_{\alpha b*g\alpha}$ space, S/s is αb*gα- CS or{s} is αb*gα- OS in (S, τ).
The reverse implication of the theorem need not be true.
Example 3.22. Let S = {f, g, h}, τ = {S, ϕ, {f}, {h}, {f, h}}
αC(S, τ) = {S, ϕ, {g}, {f, g}, {g, h}}
αb*gα- OS of (S, τ) = {S, ϕ, {f}, {h}, {f, g}, {f, h}, {g, h}}
αB^{*}GαC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, g}, {g, h}}.
{f} & {h} are αb*gα- OS, {g} is α- CS but (S, τ) is not $T^{*}_{\alpha b*g\alpha}$- space.
$\because$ {f} is αb*gα- CS but not α- CS of (S, τ).
Theorem 3.23. Any c$T_{\alpha b*g\alpha}$- space is $T^{*}_{\alpha b*g\alpha}$- space.
Proof: Let K be αb*gα- CS.
Since (S, τ) is c$T_{\alpha b*g\alpha}$- space, K is Closed. But every CS is α- CS, K is α- closed.
$\therefore$ (S, τ) is $T^{*}_{\alpha b*g\alpha}$- space.
The reverse implication of the theorem need not be true.
Example 3.24. Let S = {f, g, h}, τ = {S, ϕ, {f}, {f, h}}
αC(S, τ) = {S, ϕ, {g}, {h}, {g, h}}
αB^{*}GαC(S, τ) = {S, ϕ, {h}, {g}, {g, h}}.
(S, τ) is $T^{*}_{\alpha b*g\alpha}$- space but not c$T_{\alpha b*g\alpha}$- space.
$\because$ {h} is αb*gα- CS but not CS in (S, τ).
Definition 3.25: A Space (S, τ) is called $*g\alpha T***_{1/2}$- space if every αb*gα- CS is *gα- CS.
Theorem 3.26. If (S, τ) is $*g\alpha T***_{1/2}$- space, then any singleton of S is either *gα- CS or αb*gα- OS.
Proof: Let s $\in$ S & {s} is not *gα- CS of (S, τ). Then S/s is not *gα- open.
$\Rightarrow$ S is the only *gα- OS containing s/{s}.
$\because$ (S, τ) is $*g\alpha T***_{1/2}$- space, S/s is αb*gα- CS or{s} is αb*gα- OS in (S, τ).
The reverse implication of the theorem need not be true.
Example 3.27. Let S = {f, g, h}, τ = {S, ϕ, {f}, {h}, {f, h}}
*gαC(S, τ) = {S, ϕ, {g}, {f, g}, {g, h}}
αb*gα- OS of (S, τ) = {S, ϕ, {f}, {h}, {f, g}, {f, h}, {g, h}}
αB^{*}GαC(S, τ) = {S, ϕ, {f}, {g}, {h}, {f, g}, {g, h}}.
{f} & {h} are αb*gα- OS, {g} is *gα- CS but (S, τ) is not $*g\alpha T***_{1/2}$- space.
$\because$ {f} is αb*gα- CS but not *gα- CS of (S, τ).

Theorem 3.28. Any $^*\alpha$T***1/2- space is $*g\alpha T***_{1/2}$- space.
Proof: Let K be αb*gα- CS.
But every αb*gα- CS is $^*\alpha$gα- CS, K is $^*\alpha$gα- CS.
Since (S, τ) is $^*\alpha$T***1/2- space, K is *gα- CS.
$\therefore$ (S, τ) is $*g\alpha T***_{1/2}$- space
The reverse implication of the theorem need not be true.
Example 3.29. Let S = {f, g, h}, τ = {S, ϕ, {g}, {f, h}}
*GαC(S, τ) = {S, ϕ, {g}, {f, h}}, αB^*GαC(S, τ) = {S, ϕ, {g}, {f, h}},
* αGαC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
(S, τ) is $*g\alpha T***_{1/2}$- space but not $^*\alpha$T***1/2- space.
$\because$ {f, g} is * αgα- CS but not *g α- CS in (S, τ).
Theorem 3.30. Any *sT***1/2- space is $*g\alpha T***_{1/2}$- space.
Proof: Let K be αb*gα- CS.
But every αb*gα- CS is s*gα- CS, K is s*gα- CS.
Since (S, τ) is *sT***1/2- space, K is *gα- CS.
$\therefore$ (S, τ) is $*g\alpha T***_{1/2}$- space
The reverse implication of the theorem need not be true.
Example 3.31. Let S = {f, g, h}, τ = {S, ϕ, {g}, {f, h}}
*GαC(S, τ) = {S, ϕ, {g}, {f, h}}, αB^*GαC(S, τ) = {S, ϕ, {g}, {f, h}},
S* GαC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
(S, τ) is $*g\alpha T***_{1/2}$- space but not *sT***1/2- space.
$\because$ {f, g} is s*gα- CS but not *g α- CS in (S, τ).
Theorem 3.32. Any *bT***1/2- space is $*g\alpha T***_{1/2}$- space.
Proof: Let K be αb*gα- CS.
But every αb*gα- CS is b*gα- CS, K is b*gα- CS.
Since (S, τ) is *bT***1/2- space, K is *gα- CS.
$\therefore$ (S, τ) is $*g\alpha T***_{1/2}$- space
The reverse implication of the theorem need not be true.
Example 3.33. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
*GαC(S, τ) = {S, ϕ, {h}, {f, g}}, αB^*GαC(S, τ) = {S, ϕ, {h}, {f, g}},
B* GαC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
(S, τ) is $*g\alpha T***_{1/2}$- space but not *bT***1/2- space.
$\because$ {f, h} is b*gα- CS but not *g α- CS in (S, τ).
Theorem 3.34. Any Tc**- space is $*g\alpha T***_{1/2}$- space.
Proof: Let K be αb*gα- CS.
But every αb*gα- CS is gs- CS, K is gs- CS.
Since (S, τ) is Tc**- space, K is *gα- CS.
$\therefore$ (S, τ) is $*g\alpha T***_{1/2}$- space
The reverse implication of the theorem need not be true.
Example 3.35. Let S = {f, g, h}, τ = {S, ϕ, {g}, {f, h}}
*GαC(S, τ) = {S, ϕ, {g}, {f, h}}, αB^*GαC(S, τ) = {S, ϕ, {g}, {f, h}},
GSC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
(S, τ) is $*g\alpha T***_{1/2}$- space but not Tc**- space.
$\because$ {g, h} is gs- CS but not *g α- CS in (S, τ).
Theorem 3.36. Any αTc**- space is $*g\alpha T***_{1/2}$- space.
Proof: Let K be αb*gα- CS.
But every αb*gα- CS is αg- CS, K is αg- CS.
Since (S, τ) is αTc**- space, K is *gα- CS.
$\therefore$ (S, τ) is $*g\alpha T***_{1/2}$- space
The reverse implication of the theorem need not be true.
Example 3.37. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}}
*GαC(S, τ) = {S, ϕ, {h}, {f, g}}, αB^*GαC(S, τ) = {S, ϕ, {h}, {f, g}},
αGC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.

Here (S, τ) is a $*g\alpha T***_{1/2}$- space but not αTc**- space.
∵ {f, h} is αg- CS but not *g α- CS in (S, τ).

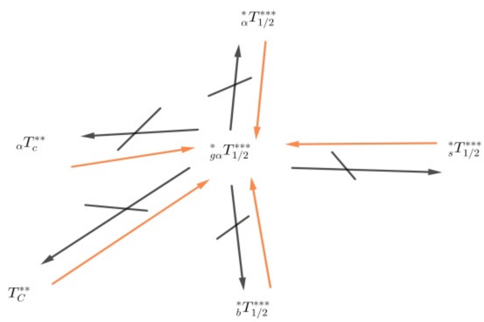

Fig 2. The relation among $*g\alpha T***_{1/2}$- space and other existing spaces

The above figure illustrates the relation among $*g\alpha T***_{1/2}$- space and other existing spaces
Theorem 3.38. Any $T^*_{\alpha b*g\alpha}$- space is independent of $*g\alpha T***_{1/2}$- space.
Proof: It can be seen by the following example.
Example 3.39. Let S = {f, g, h}, τ = {S, ϕ, {h}}, *GαC(S, τ) = {S, ϕ, {f, g}},
αB*GαC(S, τ) = {S, ϕ, {f}, {g}, {f, g}}, αC(S, τ) = {S, ϕ, {f},{g}, {f, g}}.
Here (S, τ) is a $T^*_{\alpha b*g\alpha}$- space but not $*g\alpha T***_{1/2}$- space,
∵ {g} is αb*gα- CS but not *g α- CS in (S, τ).
Example 3.40. Let S = {f, g, h}, τ = {S, ϕ, {f, g}}, *GαC(S, τ) = {S, ϕ, {h}, {f, h}, {g, h}},
αB*GαC(S, τ) = {S, ϕ, {h}, {g, h}, {f, h}}, αC(S, τ) = {S, ϕ, {h}}.
Here (S, τ) is a $*g\alpha T***_{1/2}$- space but not $T^*_{\alpha b*g\alpha}$- space
∵ {g, h} is αb*gα- CS but not α- CS in (S, τ).
Theorem 3.41. Any $T^*_{\alpha b*g\alpha}$- space is independent of *Tb*gα- space.
Proof: It can be seen by the following example.
Example 3.42. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}},
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}, αC(S, τ) = {S, ϕ, {h}, {f, g}},
G*C(S, τ) = {S, ϕ, {h},{f, g}}, B*G$\alpha C(S,\ \tau)$ = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
Here (S, τ) is a $T^*_{\alpha b*g\alpha}$- space but not *Tb*gα- space,
∵ {g} is b*gα- CS but not g*- CS in (S, τ).
Example 3.43. Let S = {f, g, h}, τ = {S, ϕ, {f}, {g, h}},
αB*GαC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}},
αC(S, τ) = {S, ϕ, {f}, {g, h}}, G*C(S, τ) = {S, ϕ, {f}, {g, h}},
B*G$\alpha C(S,\ \tau)$ = {S, ϕ, {f}, {g, h}}.
Here (S, τ) is a *Tb*gα- space but not $T^*_{\alpha b*g\alpha}$- space,
∵ {f, h} is αb*gα- CS but not α- CS in (S, τ).
Theorem 3.44. Any $*g\alpha T***_{1/2}$- space is independent of *Tb*gα- space.
Proof: It can be seen by the following example.
Example 3.45. Let S = {f, g, h}, τ = {S, ϕ, {h}, {f, g}},
αB*GαC(S, τ) = {S, ϕ, {h}, {f, g}}, $*G\alpha$C(S, τ) = {S, ϕ, {h}, {f, g}},
G*C(S, τ) = {S, ϕ, {h},{f, g}}, B*G$\alpha C(S,\ \tau)$ = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}}.
Here (S, τ) is a $*g\alpha T***_{1/2}$- space but not *Tb*gα- space,
∵ {g} is b*gα- CS but not g*- CS in (S, τ).
Example 3.46. Let S = {f, g, h}, τ = {S, ϕ, {f}, {g, h}},
αB*GαC(S, τ) = {S, ϕ, {f},{h}, {g}, {g, h}, {f, g}, {f, h}},

$*G\alpha$C(S, τ) = {S, ϕ, {f}, {g, h}}, G*C(S, τ) = {S, ϕ, {f}, {g, h},
B*G$\alpha C(S,\ \tau)$ = {S, ϕ, {f}, {g, h}}.
Here (S, τ) is a *Tb*gα- space but not $*g\alpha T***_{1/2}$- space,
$\because$ {f, g} is αb*gα- CS but not$*g\alpha$- CS in (S, τ).

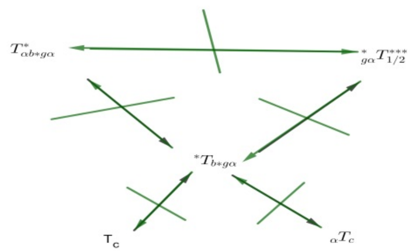

Fig 3. Diagram illustrates the independence among the defined spaces

The above diagram illustrates the independence among the defined spaces
Theorem 3.47. $T^*_{\alpha b*g\alpha}$- space is independent of Tc- space and αTc- space.
Proof: It can be seen by the following example.
Example 3.48. Let S = {f, g, h}, τ = {S, ϕ, {f}, {f, g}},
αB^*GαC(S, τ) = {S, ϕ, {g}, {h}, {g, h}}, αC(S, τ) = {S, ϕ, {g}, {h}, {g, h}},
G*C(S, τ) = {S, ϕ, {h}, {g, h}, {f, h}},
GSC(S, τ) = {S, ϕ, {h}, {g}, {g, h}, {f, h}} = $\alpha C(S,\ \tau)$.
Here (S, τ) is a $T^*_{\alpha b*g\alpha}$- space but not Tc- space and αTc- space.
$\because$ {g} is gs- CS and αg- CS but not g*- CS in (S, τ).
Example 3.49. Let S = {f, g, h}, τ = {S, ϕ, {f, h}}, αB^*GαC(S, τ) = {S, ϕ, {g}, {f, g}, {g, h}}, αC(S, τ) = {S, ϕ, {g}},G*C(S, τ) = {S, ϕ, {g}, {f, g}, {g, h}},GSC(S, τ) = {S, ϕ, {g}, {f, g}, {g, h}} = $\alpha C(S,\ \tau)$.Here (S, τ) is a Tc- space and αTc- space but not $T^*_{\alpha b*g\alpha}$- space,

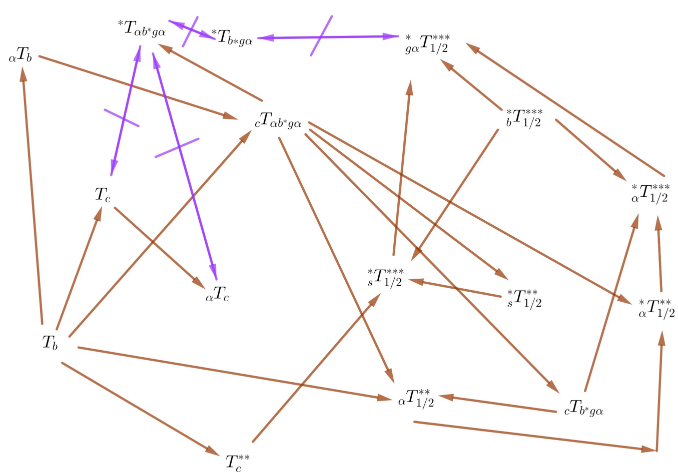

Fig 4. The red line indicates the relationship between the separation axioms of αb*gα- Closed Set and other existing spaces

$\because$(f, g) is $\alpha b^*g\alpha -$ CS but not $\alpha -$ CS in (S, τ)
In the above diagram, the red line indicates the relationship between the separation axioms of αb*gα- Closed Set and other existing spaces but not conversely and the blue line indicates the independence among the spaces.

4 Conclusion

In this paper, we have introduced the separations axioms using αb*gα- Closed Set in topological spaces and examined the properties and characteristics of $cT_{\alpha b*g\alpha}$- space, $T^{*}_{\alpha b*g\alpha}$- space,$*g\alpha T***_{1/2}$- Space by using $\alpha b*g\alpha$- closed set and comparison of the defined spaces with other existing spaces. In future, the article can be extended to find the compactness and connectedness properties of αb*gα- Closed Set in topological spaces.

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