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RESEARCH ARTICLE

# Analyze the Flow Behavior for MHD Power-Law Fluids

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## Abstract

**Objective:** This study investigates the Numerical solution of laminar boundary layer flow of Magnetohydrodynamics (MHD) model for power-law fluid over a continuous moving surface in the presence of a transverse magnetic. **Methods:** The governing partial differential equation for the flow was transformed into non-linear ordinary differential equation using the Group theoretic method. Firstly, we convert this non-linear ordinary differential equation (ODE) into linear by using quasilinearization process. This linear ODE was solved numerically by applying the Spline collocation method suggested by Bickley. **Findings:** The solution for displacement profile and velocity profile were obtained as functions of the magnetic parameters. The effect of the magnetic parameters was discussed graphically. We used MATLAB software for finding the outcomes. **Novelty:** The main goal of this article is to analyze boundary layer flow of Magneto hydrodynamics (MHD) model for power-law fluid over a continuous moving surface in the presence of a transverse magnetic. The conservation equations of mass, momentum and energy are converted into ordinary differential equations along with boundary conditions by appropriate similarity transformations and solved by applying Spline Collocation Method. The convergence of solutions is important for providing the developing linear functions of solutions, which is a benefit of the Spline Collocation Method. These research findings are applicable, for example, in predicting skin friction and heat transfer rate over a stretching sheet, which has implications in technological and manufacturing industries such as polymer extrusion. Comparisons with previously published works are made, and the results show a high level of agreement. This type of research is applicable to work in fire dynamics in insulation, solar collection systems, recovery of petroleum products, etc.

**Keywords:** Power-Law Fluids; Magnetic Field; Nonlinear Differential Equation; Quasilinearization; Bickley's Method; Linear Equations

## 1 Introduction

Magneto hydrodynamic (MHD) fluid flow has great utilization in industrial areas and manufacturing processes. MHD is a consolidation of three fundamental terms—magneto, hydro, and dynamics. Here, the term magneto refers to magnetic field, hydro refers to liquid/fluid, and dynamic refers to the evolution of particles.

The subject of boundary-layer flow on a continuously moving surface traveling through a quiet ambient fluid is important because of its relevance to several engineering processes. Flows due to a continuously moving surface are encountered in several engineering problems and have many applications, including rubber sheet manufacturing, production of glass fibers, petroleum industries, polymer processing and filament extruded continuously from die. The field of magneto hydrodynamics consists of the study of fluid behavior in an electrically conducting environment. The boundary layer flow of viscous incompressible fluid on moving surface with constant velocity was first studied by [1]. The boundary layer behavior on a continuous solid surface moving on both flat and the cylindrical surface was reviewed by [1–3]. The effects of suction or injection in a steady two-dimensional MHD boundary layer flow of on a flat plate were studied by [4]. A method for integrating the boundary layer equations through a region of reverse flow and applied it to the problem of uniform flow past a parallel flat plate of finite length whose surface has a constant velocity directed opposite to that of mainstream was studied by [5].

All of the above investigators restrict their analysis to the flow of Newtonian fluids. Most fluids such as molten plastics, artificial fibers, drilling of petroleum, blood and polymer solutions are considered non-Newtonian fluids. The concept of the boundary layer in the theory of non-Newtonian power-law fluids had been introduced by [6]. The boundary layer free convective unsteady flow of an incompressible micropolar fluid under a uniform magnetic field is considered with thermal radiation in symmetric and asymmetric boundary conditions was studied by [7]. A two-dimensional mixed convective MHD stagnation point flow of Carreau fluid past an infinite plate in a porous medium was studied by [8]. A continuously moving surface with a parallel free stream was discussed by [9]. The analytical solutions of hydro magnetic boundary-layer flow of a non-Newtonian power-law fluid past a continuously moving surface had been given by [10]. Irregularity in heat generation, chemical reaction and thermal radiation effect an unsteady micro polar fluid flow was examined by [11] using HAM.

Motion of power-law fluids in the presence of magnetic field has been studied earlier by several authors. The concept of triple diffusive flow with magnetic field effect toward a power law stretching sheet was studied by [12] using Galerkin finite-element simulation. MHD free convection of power-law fluids in a sinusoidally heated enclosure was investigated by [13] using the MRT-LBM. MHD flow of a power-law fluid over a rotating disk was studied by [14]. Analytical Solution for the MHD Flow of Non-Newtonian Fluids between Two Coaxial Cylinders was obtained by [15]. Lie Group Analysis of Double Diffusive MHD Tangent Hyperbolic Fluid Flow over a Stretching Sheet was studied by [16]. Velocity Slip Effect on MHD Power-Law Fluid over a Moving Surface with Heat Generation, Viscous Dissipation and Thermal Radiation was analyzed by [17]. A numerical approach to MHD flow of power-law fluid on a stretching sheet with non-uniform heat source was presented by [18]. Steady, two dimensional laminar incompressible boundary layer flows past a moving continuous flat surface was investigated by [19]. The MHD power-law fluid flow and heat transfer over a non-isothermal stretching sheet had been investigated by [20]. Partial differential equation into nonlinear ordinary differential equations using Group theoretic method was converted by [21].

The objective of the present study is to analyze the flow behavior of Magneto hydrodynamic boundary layer model for power law fluid in the presence of transverse magnetic field (i) when plate is stationary (ii) fluid and plate moves in same direction and same velocity (iii) fluid and plate moves in opposite direction. We also analyzed different parameters, including power law index, magnetic parameter and velocity profiles. All the important findings are illustrated graphically. A similar study was also carried out by [22] using an implicit finite difference scheme.

## 2 Governing Equation and Similarity Transformation

Consider a steady, two-dimensional laminar flow of a power-law fluid passing through a moving flat plate with constant velocity $U_w$, in the same or opposite direction to the free stream $U_\infty$. The $x-$axis extends parallel to the plate, while the y-axis extends upwards, normal to it. Also, a magnetic field of strength $B_0$is applied in the positive $y-$direction, which produces magnetic effect in the $x-$direction. The boundary layer equations governing the flow in a power-law fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

Where $u$ and $v$ are the velocity components along the $x$ and $y$ axes, $\tau_{xy}$is shear stress, $\sigma$ is electrical conductivity and $\rho$ is field density

The boundary conditions are:

$$\begin{array}{ll} y=0: & u=U_w, \quad v=0 \\ y=\infty: & u=U_\infty \end{array} \tag{3}$$

Where $\gamma\left|\frac{\partial u}{\partial y}\right|^{n-1}$denotes the kinematic viscosity, $K$ is the consistency coefficient $\gamma=\frac{K}{\rho}$ and $n$ is the power-law index, for $n<1$ pseudo plastic, for $n=1$the fluid is Newtonian, $n>1$ for dilatant fluid. The equation becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial}{\partial y}\left(\gamma\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}u \tag{4}$$

Partial differential equation converted into nonlinear ordinary differential equations using Group theoretic method by[21]. Similarity analysis by the deductive group-theoretic method is derived from theory of continuous group transformations. Recently, this theory is found to give more adequate treatment of boundary layer equations.

Consider the following transformation:

$$\psi(x,y)=ax^{\alpha}f(\eta), \quad \eta=b\frac{y}{x^b} \tag{5}$$

Where $a, b, \alpha$ and $\beta$ are real numbers, $\eta$ is similarity variable, $f(n)$ is the transformed dimensionless stream function.

Applying this similarity variable $\eta$ they derive

$$\begin{aligned} &\psi_x=ax^{\alpha-1}\left[\alpha f-\eta\beta f'\right] \\ &\psi_y=abf'x^{\alpha-\beta} \\ &\psi_{yy}=ab^2x^{\alpha-2\beta}f'' \\ &\psi_{yx}=abx^{\alpha-\beta-1}\left[\alpha f'-\beta f'-\beta\eta f'\right] \end{aligned} \tag{6}$$

Using the equation (5) along with (6) they get transformed nonlinear ordinary differential of the form

$$\left(|f''|^{n-1}f''\right)'-Mf'+\frac{1}{n+1}ff''=0 \tag{7}$$

With the transformed boundary conditions:

$$f(0)=0, \quad f'(0)=\in, f'(\infty)=1 \tag{8}$$

Putting $n=1$in the equation (7), it becomes

$$f'''-Mf'+\frac{1}{2}ff''=0 \tag{9}$$

With boundary conditions

$$f(0)=0, \quad f'(0)=\varepsilon, f'(\infty)=1. \tag{10}$$

Where $\varepsilon=\frac{U_w}{U_\infty}$ and $M=\frac{\sigma B_0^2}{\rho U_\infty}x$are called velocity parameter and magnetic parameter.

Here note that when $\varepsilon=0$ plate is stationary and, $\varepsilon=1$plate and fluid moves same direction and same velocity and for $\in<1$plate and fluid moves opposite direction.

## 2.1 Numerical Procedures

Since nonlinear differential equations cannot be solved by Bickley's[23] method, we convert nonlinear ODE using Quasilinearization into linear ODE. Applying the Quartic Spline collocation method for linear ODE, we get the system of a linear equation which we solve using software.

### 2.2 Quartic Spline Collocation Method

Consider equally spaced knots of partition : $\pi : a = x_0 < x_1 < x_2 < \ldots\ldots. < x_n = b$ on $[a,b]$. The quartic spline is defined by

$$s(x) = a_0 + b_0\left(x - x_0\right) + \frac{1}{2}c_0\left(x - x_0\right)^2 + \frac{1}{6}d_0\left(x - x_0\right)^3 + \frac{1}{24}\sum_{k=0}^{n-1} e_k\left(x - x_k\right)_+^4 \tag{11}$$

Where the powers function$(x - x_k)_+$

$$(x - x_k)_+ = \begin{cases} x - x_k, & x > x_k \\ 0, & x \le x_k \end{cases} \tag{12}$$

and the boundary value problem is given by

$$y'''(x) + p(x)y''(x) + q(x)y'(x) + r(x)y(x) = m(x) \tag{13}$$

Subject to boundary conditions

$$\begin{aligned} \alpha_0 y_0 + \beta_0 y_n' + \gamma_0 y_n'' &= \delta_0 \\ \alpha_1 y_0' + \beta_1 y_n + \gamma_1 y_n'' &= \delta_1 \\ \alpha_2 y_0'' + \beta_2 y_n + \gamma_2 y_n' &= \delta_2 \end{aligned} \tag{14}$$

To solve this boundary value problem substitute $s(x), s'(x), s''(x), s'''(x)$ from quartic spline, then the boundary value problem becomes

$$\begin{aligned} &\sum_{k=0}^{n-1} e_k\left\{(x_i - x_k)_+ + \frac{1}{2}p_i\left(x_i - x_k\right)_+^2 + \frac{1}{6}q_i\left(x_i - x_k\right)_+^3 + \frac{1}{24}r_i\left(x_i - x_k\right)_+^4\right\} \\ &+ d_0\left\{1 + p_i\left(x_i - x_0\right) + \frac{1}{2}q_i\left(x_i - x_0\right)^2 + \frac{1}{6}r_i\left(x_i - x_0\right)^3\right\} \\ &+ c_0\left\{p_i + q_i\left(x_i - x_0\right) + \frac{1}{2}r_i\left(x_i - x_0\right)^2\right\} \\ &+ b_0\left\{p_i + r_i\left(x_i - x_0\right)\right\} + a_0\left\{r_i\right\} = m\left\{x_i\right\}. \text{ Where i} = 0, 1, 2, \ldots\ldots \text{n}. \end{aligned} \tag{15}$$

Thus for quartic spline and third order boundary value problem we get nine linear algebraic equations in nine unknowns$a_0, b_0, c_0, d_0, e_0, e_1, \ldots. e_4$. The matrix form of this system is given by

$AX = B$

Where $X = [e_4, e_3, e_2, e_1, e_0, d_0, c_0, b_0, a_0]^T$ ,$B = [\delta_2, \delta_1, \delta_0, m_5, m_4, m_3, m_2, m_1, m_0]^T$

and the co-efficient matrix A is an upper Hessenberg matrix.This system of linear equation can easily solve by MATLAB.

### 2.3 Numerical Solution by Using Bickley Method

Our aim is to find numerical solution of the nonlinear ordinary differential equation

$$f''' - Mf' + \frac{1}{2}ff'' = 0 \tag{16}$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = \in, f'(1) = 1 \tag{17}$$

Solve this equation for three cases when $\varepsilon = 0$ plate is stationary, $\varepsilon = 1$ plate and fluid moves same direction and same velocity and for $\in < 1$ plate and fluid moves opposite direction.

We use quasilinearization technique to convert (16) into linear form with help of boundary conditions (17).

We get linear form as

$$f_{i+1}''' + \frac{1}{2}f_i f_{i+1}'' - Mf_{i+1}' + \frac{1}{2}f_i'' f_{i+1} = \frac{1}{2}f_i'' f_i \tag{18}$$

With boundary conditions (17)

The Quartic spline is given by

$$s(\eta) = a_0 + b_0(\eta - \eta_0) + \frac{1}{2}c_0(\eta - \eta_0)^2 + \frac{1}{6}d_0(\eta - \eta_0)^3 + \frac{1}{24}\sum_{k=0}^{n-1} e_k(\eta - \eta_k)_+^4 \tag{19}$$

Substitute (19) in (18), we get collocation as follows

$$\begin{aligned} &\sum_{k=0}^{n-1} e_k\left[(\eta_i - \eta_k) + \frac{f_i}{4}(\eta_i - \eta_k)^2 - \frac{M}{6}(\eta_i - \eta_k)^3 + \frac{f_i'}{48}(\eta_i - \eta_k)^4\right] \\ &+ d_0\left[1 + \frac{1}{2}f_i(\eta_i - \eta_0) - \frac{M}{2}(\eta_i - \eta_0)^2 + \frac{f_i''}{12}(\eta_i - \eta_0)^3\right] + c_0\left[\frac{1}{2}f_i - M(\eta_i - \eta_0) + \frac{f_i''}{4}(\eta_i - \eta_0)^2\right] \\ &+ b_0\left[-M + \frac{1}{2}f_i'(\eta_i - \eta_0)\right] + a_0\left[\frac{1}{2}f_i''\right] = \frac{1}{2}f_i''f_i \end{aligned} \tag{20}$$

**Case (i): $\varepsilon = 0$ plate is stationary**

To obtain the spline solution, begin with a assume function $f(\eta) = \frac{1}{2}\eta^2$ which satisfy given boundary conditions (17). For numerical solution of equation (16) along with boundary conditions (17), first we use $f(\eta) = \frac{1}{2}\eta^2$in (20).

We get graphical solution as follows:

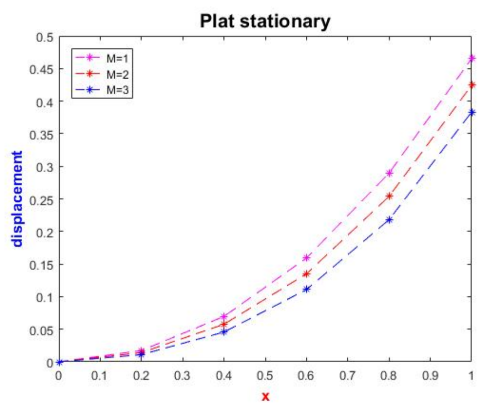


**Fig 1.** $f(\eta)$ versus $\eta$for different values of M.

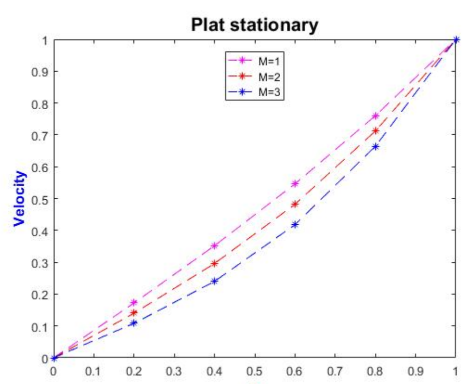


**Fig 2.** $f'(\eta)$ versus $\eta$for different values of M.

**Case (ii): $\varepsilon = 1$ plate and fluid moves same direction and same velocity**

To obtain the spline solution, begin with a assume function $f(\eta) = \eta$ which satisfy given boundary conditions (17). For numerical solution of equation (16) along with boundary conditions (17), first we use $f(\eta) = \eta$ in (20).

We get graphical solution as follows:

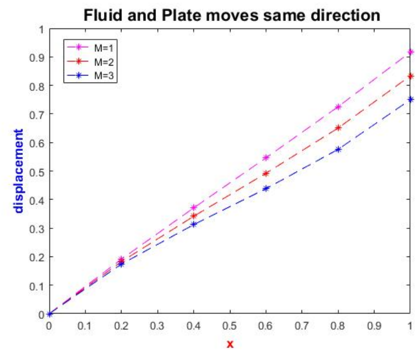


**Fig 3.** $f(\eta)$ versus $\eta$ for different values of M.

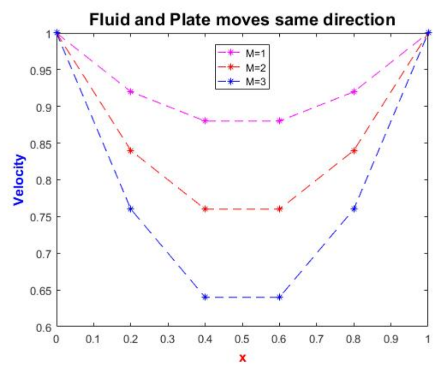


**Fig 4.** $f'(\eta)$ versus $\eta$ for different values of M.

**Case (iii):** $\in < 1$ **plate and fluid moves opposite direction**

Similarly, we use $f(\eta) = \eta^2 - \eta$ which satisfy given boundary conditions (17). For Numerical solution, we use $f(\eta) = \eta^2 - \eta$ in (20).

We get graphical solution as follows:

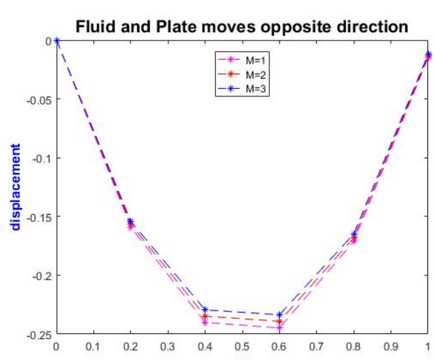


**Fig 5.** $f(\eta)$ versus $\eta$ for different values of M.

Here we presented the comparison of velocity and displacement when fluid and plate moves along the same direction and plate and fluid moves opposite direction.

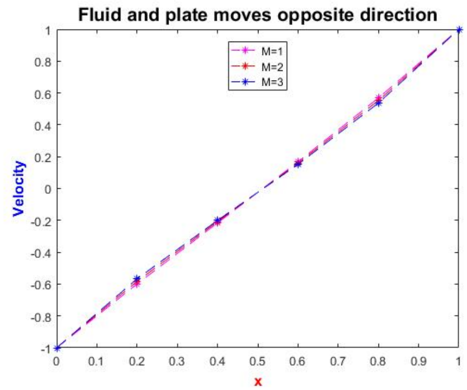


**Fig 6.** $f'(\eta)$ versus $\eta$ for different values of M.

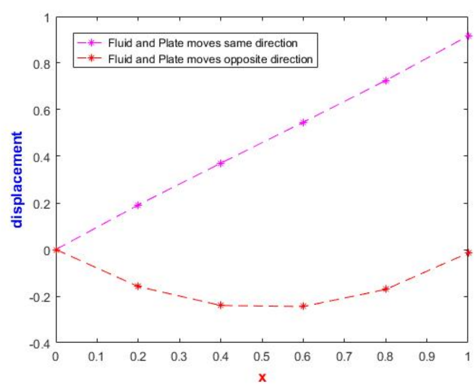


**Fig 7.** Comparison of $f(\eta)$ versus $\eta$ , when M=1.

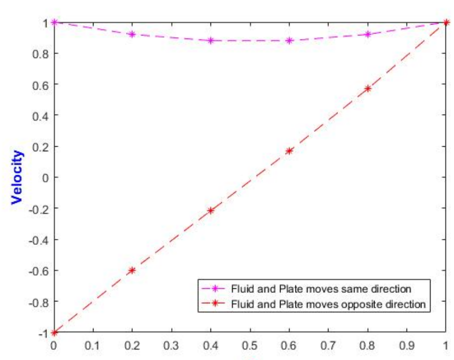


**Fig 8.** Comparison of $f'(\eta)$ versus $\eta$, when M=1.

## 3 Result and Discussion

Numerical calculations are performed to study the behavior of the adopted scheme. Differential Equation (16) subjected to the boundary conditions (17) are solved by adopting Bickley's Spline collocation method. The main reason behind solving the present problem is to determine the impact of magnetic parameter, velocity profile and displacement profile when fluid flow on a moving surface in different situations.

In the numerical method mentioned in the previous section, numerical computations are carried out at different values of magnetic parameters. The following three cases depict the graphical representations of the numerical results for different governing parameters influencing the proposed model's flow behavior.

**1.Case - 1 Plate is stationa ry**

In Figures 1 and 2, it is observed that the value of the magnetic parameter increases, then displacement and velocity decrease.

**2. Case -2 Plate and Fluid move in same direction**

Similarly, in Figures 3 and 4, it is found that the value of the magnetic parameter increases, then displacement and velocity also decrease.

**3.Case-3 Plate and Fluid move in opposite direction**

In the case of Figure 5, the increase in magnetic parameter displacement also increases, while in Figure 6, the increase in magnetic parameter resulted in steady behavior in velocity.

However, in Figures 7 and 8, Both displacement and velocity for Plate and Fluid moving in the opposite direction have lower values than Plate and Fluid moving in the same direction by keeping the value M=1 fixed.

## 4 Conclusion

The present study reflects the flow behavior of the Magneto hydrodynamic boundary layer model for power law fluid in the presence of transverse magnetic field (i) when the plate is stationary (ii) fluid and plate move in the same direction and at same velocity (iii) fluid and plate moves in the opposite direction. The outcomes of the current study incorporate the significance of MHD fluid flow on velocity profile, displacement profile and magnetic parameter. The key findings of the current analysis are summarized as

• The magnetic parameter is transversely proportional to velocity and displacement when the plate is stationary ( $\varepsilon = 0$ )

• The magnetic parameter is transversely proportional to velocity and displacement when the plate and fluid move same direction ($\varepsilon = 1$).

• When plate and fluid move in the opposite direction, there is no remarkable change in velocity even though the magnetic parameter changes ($\varepsilon < 1$).

This magnetism helps to control the rate of fluid velocity in manufacturing processes and industrial applications to obtain the desired quality of product.

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