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Triangular Intuitionistic Fuzzy Linear Programming Problem with a New Ranking Method based on Correlation Coefficient

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Abstract

Objectives: The study discusses solving Triangular Intuitionistic Fuzzy Linear Programming Problem (TIFLPP) with triangular intuitionistic fuzzy coefficients and variables using triangular intuitionistic fuzzy simplex method. **Method:** The triangular intuitionistic fuzzy information given for the linear programming problem is preserved and not converted to crisp values while solving the problem which will represent the complete information of the decision problem. **Findings:** Several definitions and theorems are given to justify the stability of the triangular intuitionistic fuzzy simplex method. In addition to the existing ratio ranking method for finding the minimum ratio a new method of correlation coefficient is proposed for finding the minimum ratio in the simplex method. The procedure of the simplex method is explained through a numerical illustration. **Novelty:** The proposed new correlation coefficient of triangular intuitionistic fuzzy with graded mean integration representation and triangular intuitionistic fuzzy simplex method with defuzzification method are compared and suggestions are given based on the results obtained.

Keywords: Triangular Intuitionistic Fuzzy Sets; Linear Programming Problem; Ratio Ranking; Intuitionistic Fuzzy Simplex Method; Correlation Coefficient of Triangular Intuitionistic Fuzzy Sets 1

1 Introduction

It has been discovered in various optimization situations that a little violation of provided constraints or conditions might lead to a more efficient solution. Several times, it is not practical to assign correct parameters because many of them are gained through approximation or some kind of direct perception. In an industrial optimization method, for example, it is not required that all of the manufactured items be of excellent quality and totally tradable at a certain cost. There is a chance that some of the items may be faulty and will not be able to be traded at the predetermined cost. Further, manufacturing costs as well as the spot price of completed goods may fluctuate owing to market surpluses or shortages caused by unpredictable circumstances. As a result, it is clear that pricing and compositions are not entirely stationary, but rather ambiguous or unstable and that such optimization issues must be solved using modern approaches. In a natural sense, modelling most real-world situations requiring optimization processes ends out to be a Linear Programming Problem (LPP). The Fuzzy Set (FS) is one of the ways for defining atypical circumstances in the field of uncertainty. Intuitionistic Fuzzy Sets (IFSs) are a concept established to integrate the unpredictable level in the membership function of FS. The Triangular Intuitionistic Fuzzy Set (TIFS), which has its membership function and non-membership function is applied by many researchers in decision-making theory. Many new researchers Pérez-Cañedo, & Concepción-Morales[1],Jayalakshmi et al.[2],Kabiraj et al.[3],Mohan et al.[4] and Kousar et al.[5] have proposed some methods for solving intuitionistic fuzzy linear programming problems. Abinaya & Amirtharaj[6] have applied fuzzy time cost trade off problems in Multiple Attribute Group Decision Making (MAGDM) problem. Augustine[7] have introduced correlation coefficient for IFSs in MAGDM problem. Li et al.[8] used intuitionistic fuzzy cross-entropy and comprehensive grey correlation in MAGDM problem. Robinson et al.[9] have introduced an automated decision support system miner algorithm to solve MAGDM problem using linear programming approach to fuzzy matrix games. The main aim of this paper is to propose a new ranking method for triangular intuitionistic fuzzy number based on correlation coefficient and to use it in the Intuitionistic Fuzzy Linear Programming Problem (IFLPP) to find the minimum ratio in the iteration of the simplex table. The TIFNs in the IFLPP are not converted into crisp values and solved using Triangular Intuitionistic Fuzzy Number (TIFN) arithmetic operations and the triangular intuitionistic fuzzy optimal solution is obtained. Whereas in all the previous researches the TIFNs in the IFLPP are converted into crisp values before solving. Several theorems are proved to show the stability of the proposed model. Three different methods of solving Triangular Intuitionistic Fuzzy Linear Programming Problem (TIFLPP) are compared and the conclusion is left to the user based on the utility of the problem. The methodology of a new correlation coefficient based on graded mean integration representation is described in section 2. In section 3 the construction of TIFLPP is explained and some theorems are proved to justify the model. A numerical illustration is given for solving TIFLPP with the new correlation coefficient in section 4. In section 5 the problem is solved using defuzzification by $\alpha-$ cut and $\beta-$cut methods. Comparisons of the numerical results based on the three ranking methods namely correlation coefficient, defuzzification and ratio ranking method are made to show the effectiveness of the proposed methods with the existing method.

2 Methodology

- **New Correlation Coefficient for TIFNs**

In the following section, we propose a new correlation coefficient for TIFNs based on graded mean integration representation.

- **Graded Mean Integration Representation (GMIR)**

Given a triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, the GMIR of $\widetilde{A}$ is defined as:

$$P\left(\widetilde{A}\right) = \frac{a_1 + a_2 + a_3}{6} \tag{1}$$

- **Correlation Coefficient for TIFNs Based on GMIR**

The correlation of TIFN is defined as follows with the help of triangular fuzzy GMIR together with the membership, non-membership grades of the TIFNs:

The triangular intuitionistic energy of $\widetilde{A} = (a_1, a_2, a_3; u_{\widetilde{A}}, v_{\widetilde{A}})$ and $\widetilde{B} = (b_1, b_2, b_3; u_{\widetilde{B}}, v_{\widetilde{B}})$ are described as follows:

$$E_{TIFN}(\widetilde{A}) = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{a_1 + 4a_2 + a_3}{6}\right]^2 (u_{\widetilde{A}^2(x_i) + v_{\widetilde{A}^2(x_i)})} \tag{2}$$

$$E_{TIFN}(\widetilde{B}) = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{b_1 + 4b_2 + b_3}{6}\right]^2 (u_{\widetilde{B}^2(x_i) + v_{\widetilde{B}^2(x_i)})} \tag{3}$$

Now the correlation between $\widetilde{A}$ and $\widetilde{B}$ is defined as:

(4)

Then the correlation coefficient between $\widetilde{A}$ and $\widetilde{B}$ is defined as:

$$K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = \frac{C_{TIFN}(\widetilde{A},\widetilde{B})}{\sqrt{E_{TIFN}\left(\widetilde{A}\right).E_{TIFN}\left(\widetilde{B}\right)}} \tag{5}$$

Proposition

For $\widetilde{A},\widetilde{B} \in$ TIFN(X), the following are true: (i) $0 \leq C_{TIFN}(\widetilde{A},\widetilde{B}) \leq 1$, (ii) $C_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = C_{TIFN}\left(\widetilde{B},\widetilde{A}\right)$, (iii) $K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = K_{TIFN}(\widetilde{A},\widetilde{B})$, (iv) $K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = 1$ if $\widetilde{A} = \widetilde{B}$.

Theorem 2.1 $0 \leq K_{TIFN}(\widetilde{A},\widetilde{B}) \leq 1$, when $\widetilde{A},\widetilde{B} \in$ TIFN(X).

Proof . $K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) \leq 1$ is true since $C_{TIFN}\left(\widetilde{A},\widetilde{B}\right) \geq 0$. The following inequality is true for any arbitrary real number ξ, :

$$0 \leq \sum_{t=1}^{n} \left\{ \begin{array}{l} \left(\left[\frac{a_1+4a_2+a_3}{6}\right] - \xi\left[\frac{b_1+4b_2+b_3}{6}\right]\right)^2 \times \\ \left[\left(\left(u_{\widetilde{A}\,(x_i)} - \xi u_{\widetilde{B}\,(x_i)}\right)^2 + \left(v_{\widetilde{A}\,(x_i)} - \xi v_{\widetilde{B}\,(x_i)}\right)^2\right)\right] \end{array} \right.$$

Hence,

$$\left(\sum_{i=1}^{n}\left(\left[\frac{a_1+4a_2+a_3}{6}\right]\left[\frac{b_1+4b_2+b_3}{6}\right]\right) \times \left[\left(u_{\widetilde{A}(x_i)u_{\widetilde{B}(x_i)}}\right) + \left(v_{\widetilde{A}(x_i)v_{\widetilde{B}(x_i)}}\right)\right]\right\}^2$$

$$\leq \left(\sum_{i=1}^{n}\left(\left[\frac{a_1+4a_2+a_3}{6}\right]^2 \times \left(u_{\widetilde{A}^2(x_i)+v_{\widetilde{A}^2(x_i)}}\right)\right\} \times \sum_{i=1}^{n}\left(\left[\frac{b_1+4b_2+b_3}{6}\right]^2 \times \left(u_{\widetilde{B}}^2(x_i) + v_{\widetilde{B}^2(x_i)}\right)\right\}\right)$$

The above inequality can be written as:

$$\frac{\left(\sum_{i=1}^{n}\left(\left[\frac{a_1+4a_2+a_3}{6}\right]\left[\frac{b_1+4b_2+b_3}{6}\right]\right) \times \left(\left(u_{\widetilde{A}(x_i)u_{\widetilde{B}(x_i)}}\right) + \left(v_{\widetilde{A}(x_i)v_{\widetilde{B}(x_i)}}\right)\right]\right\}^2}{\sum_{i=1}^{n}\left(\left[\frac{a_1+4a_2+a_3}{6}\right]^2 \times \left(u_{\widetilde{A}^2(x_i)+v_{\widetilde{A}^2(x_i)}}\right)\right\} \times \sum_{i=1}^{n}\left(\left[\frac{b_1+4b_2+b_3}{6}\right]^2 \times \left(u_{\widetilde{B}}^2(x_i) + v_{\widetilde{B}^2(x_i)}\right)\right\}} \leq 1$$

Therefore, $\frac{\left(C_{TIFN}(\widetilde{A},\widetilde{B})\right\}^2}{E_{TIFN}(\widetilde{A}).E_{TIFN}(\widetilde{B})} \leq 1$. Hence, $K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = \frac{C_{TIFN}(\widetilde{A},\widetilde{B})}{\sqrt{E_{TIFN}(\widetilde{A}).E_{TIFN}(\widetilde{B})}} \leq 1$.

Theorem 2.2 If $K_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = 1$ if and only if $\widetilde{A} = \widetilde{B}$.

Proof . The equality holds if and only if the following conditions are satisfied when considering the inequality in the proof of Theorem 2.1.: (i) $\left[\frac{a_1+4a_2+a_3}{6}\right] = \xi\left[\frac{b_1+4b_2+b_3}{6}\right]$, (ii) $u_{\widetilde{A}}(x_i) = \xi u_{\widetilde{B}}(x_i)$, (iii) $v_{\widetilde{A}}(x_i) = \xi v_{\widetilde{B}}(x_i)$, for some positive real ξ. As $u_{\widetilde{A}}(x_i) + v_{\widetilde{A}}(x_i) = u_{\widetilde{B}}(x_i) + v_{\widetilde{B}}(x_i) = 1$, then it means ξ =1, and therefore $\widetilde{A} = \widetilde{B}$.

Theorem 2.3 If $\widetilde{A}$ and $\widetilde{B}$ are non-fuzzy sets that satisfies the condition $u_{\widetilde{A}}(x_i) + u_{\widetilde{B}}(x_i)$=1 or $v_{\widetilde{A}}(x_i) + v_{\widetilde{B}}(x_i) = 1\ \forall\, x_i \in X$ then $C_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = 0$.

Proof . For all $x_i \in X$, $u_{\widetilde{A}}(x_i)\,u_{\widetilde{B}}(x_i) + v_{\widetilde{A}}(x_i)\,v_{\widetilde{B}}(x_i) \geq 0$. If $C_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = 0$ for all $x_i \in X$, then it should be that: $u_{\widetilde{A}}(x_i)\,u_{\widetilde{B}}(x_i) = 0$ and $v_{\widetilde{A}}(x_i)\,v_{\widetilde{B}}(x_i) = 0$. If $u_{\widetilde{A}}(x_i) = 1$ then $u_{\widetilde{B}}(x_i) = 0$ and $v_{\widetilde{A}}(x_i) = 0$. If $u_{\widetilde{B}}(x_i) = 1$ then $u_{\widetilde{A}}(x_i) = 0$ and $v_{\widetilde{B}}(x_i) = 0$. Hence, $u_{\widetilde{A}}(x_i) + u_{\widetilde{B}}(x_i) = 1$. Conversely, $u_{\widetilde{A}}(x_i) + u_{\widetilde{B}}(x_i) = 1$, When $\widetilde{A}$ and $\widetilde{B}$ are non-fuzzy sets. If $u_{\widetilde{A}}(x_i) = 1$ then $u_{\widetilde{B}}(x_i) = 0$ and $v_{\widetilde{A}}(x_i) = 0$ If $u_{\widetilde{B}}(x_i) = 1$ then $u_{\widetilde{A}}(x_i) = 0$ and $v_{\widetilde{B}}(x_i) = 0$ Therefore $C_{TIFN}\left(\widetilde{A},\widetilde{B}\right) = 0$.

Theorem 2.4 When $\widetilde{A}$ is a non-fuzzy set then $E_{TIFN}\left(\widetilde{A}\right) = 1$.

Proof. Assume $\widetilde{A}$ is not a non-fuzzy set. Then $0 \leq u_{\widetilde{A}}(x_i) < 1$ and $0 \leq v_{\widetilde{A}}(x_i) < 1$ for some x_i. Hence, $u_{\widetilde{A}}^2(x_i) + v_{\widetilde{A}}^2(x_i) < 1$. Then $E_{TIFN}\left(\widetilde{A}\right) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_1+4a_2+a_3}{6}\right]^2 \times \left(u_{\widetilde{A}^2(x_i)+v_{\widetilde{A}^2(x_i)}}\right) < 1$. This contradiction occurs because $\widetilde{A}$ is a non-fuzzy set.

3 Triangular Intuitionistic Fuzzy Linear Programming Problem (TIFLPP)

Using triangular intuitionistic fuzzy variables, the intuitionistic fuzzy linear programming problem is formulated as follows:

Max $\widetilde{Z}^{IF} \approx \sum_{j=1}^{n} \widetilde{C}_j^{IF} \widetilde{X}_j^{IF}$; Subject to $\sum_{j=1}^{n} \widetilde{A}_j^{IF} \widetilde{X}_j^{IF} \leq \widetilde{b}_i^{IF}$ for all i=1,2,…,m. Where $\widetilde{A}_{ij}^{IF} = \left((a_{ij})_1, (a_{ij})_2, (a_{ij})_3; u_{\widetilde{A}_{ij}}, v_{\widetilde{A}_{ij}}\right)$, $\widetilde{C}^{IF}$ and $\widetilde{X}^{IF} \in (IF(R))^n$ and $\widetilde{b}^{IF} \in (IF(R))^m$, where IF(R) is the set of all TIFNs and $\widetilde{X}_j^{IF} \geq \widetilde{0}_j^{IF}$. The above problem can be written as

Max $(z_1, z_2, z_3; u_{\widetilde{z}}, v_{\widetilde{z}}) \approx \sum_{j=1}^{n} \left((c_j)_1, (c_j)_2, (c_j)_3; u_{\widetilde{C}}, v_{\widetilde{C}}\right) \left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right)$;

Subject to $\left((a_{ij})_1, (a_{ij})_2, (a_{ij})_3; u_{\widetilde{A}_{ij}}, v_{\widetilde{A}_{ij}}\right) \left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right) \leq \left((b_i)_1, (b_i)_2, (b_i)_3; u_{\widetilde{b}}, v_{\widetilde{b}}\right)$

For all i=1,2,…,m. $\left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right) \geq (0,0,0;1,0)$.

Triangular Intuitionistic Fuzzy Feasible Solution

If for each $\widetilde{x}_j^{IF}$ in $\widetilde{X}^{IF}$ satisfies the constraints and non-negativity restriction then $\widetilde{X}^{IF}$ is said to be the triangular intuitionistic fuzzy feasible solution.

Triangular Intuitionistic Fuzzy Optimum Solution

Any triangular intuitionistic fuzzy feasible solution that optimizes the objective function of the TIFLPP is called triangular intuitionistic fuzzy optimum solution.

Triangular Intuitionistic Fuzzy Slack Variables

Let the constraints of TIFLPP be $\sum_{j=1}^{m} \widetilde{a}_{ij}^{IF} \widetilde{x}_j^{IF} \leq \widetilde{b}_i^{IF}$, i=1,2,…n. Then the non-negative triangular intuitionistic fuzzy variables $\widetilde{s}_k^{IF}$, which satisfies $\sum_{j=1}^{m} \widetilde{a}_{ij}^{IF} \widetilde{x}_j^{IF} + \widetilde{s}_k^{IF} \approx \widetilde{b}_i^{IF}$ k=1,2,…r, are called triangular intuitionistic fuzzy slack variables.

Standard form of TIFLPP

Standard form of TIFLPP is defined as:

Max $(z_1, z_2, z_3; u_{\widetilde{z}}, v_{\widetilde{z}}) \approx \sum_{j=1}^{n} \left((c_j)_1, (c_j)_2, (c_j)_3; u_{\widetilde{C}}, v_{\widetilde{C}}\right) \left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right)$;

Subject to $\sum_{j=1}^{n} \left((a_{ij})_1, (a_{ij})_2, (a_{ij})_3; u_{\widetilde{A}_{ij}}, v_{\widetilde{A}_{ij}}\right) \left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right) \approx \left((b_i)_1, (b_i)_2, (b_i)_3; u_{\widetilde{b}}, v_{\widetilde{b}}\right)$,

And $\left((x_j)_1, (x_j)_2, (x_j)_3; u_{\widetilde{X}}, v_{\widetilde{X}}\right) \geq (0,0,0;1,0)$.

Here we consider rank of $\widetilde{A}^{IF} = m$. Let the columns of $\widetilde{A}^{IF}$ be $\widetilde{a}_j^{IF}$. Let $\widetilde{D}^{IF}$ be a basis set for the column $\widetilde{a}_j^{IF}$.

Theorem 3.1 (Reduction of a TIF feasible solution to a TIF basic feasible solution)

If a TIFLPP has a triangular intuitionistic fuzzy feasible solution, then it also has a triangular intuitionistic fuzzy basic feasible solution.

Proof. Let the TIFLPP be to determine $\widetilde{x}^{IF}$ to maximize $\widetilde{z}^{IF} = \widetilde{c}^{IF} \widetilde{x}^{IF}$, Subject to the constraints $\widetilde{A}^{IF} \widetilde{x}^{IF} = \widetilde{b}^{IF}, \widetilde{x}^{IF} \geq \widetilde{0}^{IF}$, Where $\widetilde{A}^{IF}$ is an $n \times m$ matrix and $\widetilde{b}^{IF}, \widetilde{c}^{IF}$ are $m \times 1$ and $1 \times n$ triangular intuitionistic fuzzy matrix respectively, let rank of $\widetilde{A}^{IF} = n$. Since there does exist a triangular intuitionistic fuzzy feasible solution, we must have rank of ($\widetilde{A}^{IF} \widetilde{b}^{IF}$) = rank of $\widetilde{A}^{IF}$ and $n < m$. Let $\widetilde{x}^{IF} = \left(\widetilde{x}_1^{IF}, \widetilde{x}_2^{IF}, \ldots, \widetilde{x}_n^{IF}\right)$ be a triangular intuitionistic fuzzy feasible solution so that $\widetilde{x}_j^{IF} \geq \widetilde{0}^{IF}$ for all j. Suppose that $\widetilde{x}^{IF}$ has p positive components and let the remaining n-p components be zero. The first p components and the columns of $\widetilde{A}^{IF}$ are $\widetilde{a}_1^{IF} \widetilde{x}_1^{IF} + \widetilde{a}_2^{IF} \widetilde{x}_2^{IF} + \cdots + \widetilde{a}_p^{IF} \widetilde{x}_p^{IF} = \widetilde{b}^{IF}$. Where, $\widetilde{a}_1^{IF}, \widetilde{a}_2^{IF}, \ldots, \widetilde{a}_p^{IF}$ are the first p columns of $\widetilde{A}^{IF}$. Two cases now do arise: (i) The vectors $\widetilde{a}_1^{IF}, \widetilde{a}_2^{IF}, \ldots, \widetilde{a}_p^{IF}$ form a linearly independent set. Then $p \leq n$. If p = n, the given solution is a non-degenerate basic feasible solution, with $\widetilde{x}_1^{IF}, \widetilde{x}_2^{IF}, \ldots, \widetilde{x}_p^{IF}$ as the basic variables. If p < n, then the set $\widetilde{a}_1^{IF}, \widetilde{a}_2^{IF}, \ldots, \widetilde{a}_p^{IF}$ can be extended to $\widetilde{a}_1^{IF}, \widetilde{a}_2^{IF}, \ldots, \widetilde{a}_p^{IF}, \widetilde{a}_{p+1}^{IF}, \ldots, \widetilde{a}_n^{IF}$ to form a basis for the columns of $\widetilde{A}^{IF}$. Then, $\widetilde{a}_1^{IF} \widetilde{x}_1^{IF} + \widetilde{a}_2^{IF} \widetilde{x}_2^{IF} + \cdots + \widetilde{a}_n^{IF} \widetilde{x}_n^{IF} = \widetilde{b}^{IF}$. Where $\widetilde{x}_j^{IF} = \widetilde{0}^{IF}$ for j=p+1,p+2,…,n. Thus, in this case, a degenerate basic feasible solution with n-p of the basic variables zero. (ii) The set $\widetilde{a}_1^{IF}, \widetilde{a}_2^{IF}, \ldots, \widetilde{a}_p^{IF}$ is linearly dependent. Obviously p > n. Let $\left(\widetilde{\alpha}_1^{IF}, \widetilde{\alpha}_2^{IF}, \ldots, \widetilde{\alpha}_p^{IF}\right\}$ be a set of constants (not all zero) such that $\widetilde{\alpha}_1^{IF} \widetilde{a}_1^{IF} + \widetilde{\alpha}_2^{IF} \widetilde{a}_2^{IF} + \cdots + \widetilde{\alpha}_p^{IF} \widetilde{a}_p^{IF} = \widetilde{0}^{IF}$. Suppose that for any index r,$\widetilde{\alpha}_r \neq 0$, then $\widetilde{\alpha}_r^{IF} = \sum_{j \neq r}^{p} \frac{\widetilde{\alpha}_j^{IF}}{\widetilde{\alpha}_r^{IF}} \widetilde{a}_j^{IF}$. $\sum_{j \neq r}^{p} \widetilde{a}_j^{IF} \widetilde{x}_j^{IF} + \left(-\sum_{j \neq r}^{p} \frac{\widetilde{\alpha}_j^{IF}}{\widetilde{\alpha}_r^{IF}} \widetilde{a}_j^{IF}\right) \widetilde{x}_r^{IF} = \widetilde{b}^{IF}$, $\sum_{j \neq r}^{p} \left(\widetilde{x}_j^{IF} - \widetilde{x}_r^{IF} \frac{\widetilde{\alpha}_j^{IF}}{\widetilde{\alpha}_r^{IF}}\right) \widetilde{a}_j^{IF} = \widetilde{b}^{IF}$. Thus, we have a solution with not more than p-1 non-zero components. To ensure that these are positive, choose $\widetilde{a}_r^{IF}$ in such a way that $\widetilde{x}_j^{IF} - \widetilde{x}_r^{IF} \frac{\widetilde{\alpha}_j^{IF}}{\widetilde{\alpha}_r^{IF}} \geq \widetilde{0}^{IF}$ for all $j \neq r$. This requires that either $\widetilde{\alpha}_j^{IF} = \widetilde{0}^{IF}$ or $\frac{\widetilde{x}_j^{IF}}{\widetilde{\alpha}_j^{IF}} \geq \frac{\widetilde{x}_r^{IF}}{\widetilde{\alpha}_r^{IF}}$ if $\widetilde{\alpha}_j^{IF} > \widetilde{0}^{IF}$ and $\frac{\widetilde{x}_j^{IF}}{\widetilde{\alpha}_j^{IF}} \leq \frac{\widetilde{x}_r^{IF}}{\widetilde{\alpha}_r^{IF}}$ if $\widetilde{\alpha}_j^{IF} < \widetilde{0}^{IF}$. Thus, select $\widetilde{a}_r^{IF}$ such that $\frac{\widetilde{x}_r^{IF}}{\widetilde{\alpha}_r^{IF}} = \min_j \left\{ \frac{\widetilde{x}_j^{IF}}{\widetilde{\alpha}_j^{IF}}, \widetilde{\alpha}_j > 0 \right\}$. Then for each of the p-1 variables, $\widetilde{x}_j^{IF} - \widetilde{x}_r^{IF} \frac{\widetilde{\alpha}_j^{IF}}{\widetilde{\alpha}_r^{IF}}$ is non-negative, and so we have a triangular intuitionistic fuzzy feasible solution with not more than p-1 non-zero components. Consider now this new triangular intuitionistic fuzzy feasible solution with not more than p-1 non-zero components. If the corresponding set of p-1 columns of $\widetilde{A}^{IF}$ is linearly independent, case (i) applies and the triangular intuitionistic fuzzy basic feasible solution is arrived. If this set is again linearly dependent, repeat the process to arrive at a triangular intuitionistic fuzzy feasible solution with not more than p-2 non-zero components. Ultimately, we get a triangular intuitionistic fuzzy feasible solution with associated set of column vectors of $\widetilde{A}^{IF}$ which is linearly independent. The discussion of case (i) then applies, and we do get a triangular intuitionistic fuzzy basic feasible solution.

Theorem 3.2 (Replacement of a TIF basic vector)

A triangular intuitionistic fuzzy basic feasible solution is also achieved if we substitute one of the basis vectors in the intuitionistic fuzzy basis set for a non-basis vector.

Proof. Let $\widetilde{x}_B^{IF}$ be the triangular intuitionistic fuzzy basic feasible solution, so that $\widetilde{D}^{IF}\widetilde{x}_B^{IF} = \widetilde{b}^{IF}, \widetilde{x}^{IF} \geq \widetilde{0}^{IF}$. Where $\widetilde{D}^{IF}$ forms a basis set for the column vector of $\widetilde{A}^{IF}$. For any column vector $\widetilde{a}_j^{IF} \in \widetilde{A}^{IF}$, we have: $\widetilde{a}_j^{IF} = \widetilde{y}_{1j}^{IF}\widetilde{d}_1^{IF} + \widetilde{y}_{2j}^{IF}\widetilde{d}_2^{IF} + \ldots + \widetilde{y}_{mj}^{IF}\widetilde{d}_m^{IF} = \widetilde{D}^{IF}\widetilde{y}_j^{IF}$, where $\widetilde{d}_j^{IF} \in \widetilde{D}^{IF}$ and $\widetilde{y}_{ij}^{IF}$ are suitable scalars. If the basis vector $\widetilde{d}_r^{IF}$, for which the non-zero coefficient $\widetilde{y}_{rj}^{IF}$ is substituted by $\widetilde{a}_j^{IF} \in \widetilde{A}^{IF}$, we get the new set of vectors also forms a basis. For $\widetilde{y}_{rj}^{IF} \neq 0$,

$\widetilde{d}_r^{IF} = \frac{\widetilde{a}_j^{IF}}{\widetilde{y}_{rj}^{IF}} - \sum_{i=1}^{m}\left(\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}\right)\widetilde{d}_j^{IF}, i \neq r.$

$\widetilde{b}^{IF} = \sum_{i=1}^{m}\widetilde{x}_{Bi}^{IF}\widetilde{d}_i^{IF} + \widetilde{x}_{Br}^{IF}\left[\frac{\widetilde{a}_j^{IF}}{\widetilde{y}_{rj}^{IF}}\sum_{i=1}^{m}\left(\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}\right)\widetilde{d}_i^{IF}\right] = \sum_{i=1}^{m}\left[\widetilde{x}_{Bi}^{IF} + \widetilde{x}_{Br}^{IF}\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}\right]\widetilde{d}_i^{IF} + \frac{\widetilde{x}_{br}^{IF}}{\widetilde{y}_{rj}^{IF}}\widetilde{a}_j^{IF}.$

Thus, the new triangular intuitionistic fuzzy basic solution is $\widetilde{x}_B^{*IF}$ having as its components, the variables $\widetilde{x}_B^{*IF} = \widetilde{x}_{Bi}^{IF} + \widetilde{x}_{Br}^{IF}\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}$ and $\widetilde{x}_{Br}^{*IF} = \frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}$. We shall now show that $\widetilde{x}_B^{*IF}$ is also feasible, that is, the new triangular intuitionistic fuzzy basic variables $\widetilde{x}_{Bi}^{*IF}$ are also non negative. Now two cases arises: Case (i) $\widetilde{x}_{Br}^{IF} = \widetilde{0}^{IF}$. The newly introduced set of triangular intuitionistic fuzzy basic variables is clearly non-negative. Since we have assumed the existence of a triangular intuitionistic fuzzy basic feasible solution $\widetilde{x}_B^{*IF}$. Case (ii) $\widetilde{x}_{Bi}^{*IF} \neq \widetilde{0}^{IF}$. We must have $\widetilde{y}_{rj}^{IF} > \widetilde{0}^{IF}$. This requires that for the remaining $\widetilde{y}_{ij}^{IF}\,(i \neq r)$ either $\widetilde{y}_{rj}^{IF} = \widetilde{0}^{IF}$ for i=r, or $\frac{\widetilde{x}_{Bi}^{IF}}{\widetilde{y}_{ij}^{IF}} \geq \frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}$ for $\widetilde{y}_{rj}^{IF} > \widetilde{0}^{IF}$ and i$\neq r$ and $\frac{\widetilde{x}_{Bi}^{IF}}{\widetilde{y}_{ij}^{IF}} \leq \frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}$ for $\widetilde{y}_{rj}^{IF} < \widetilde{0}^{IF}$ and i$\neq r$. So if we select the index r ($\widetilde{y}_{rj}^{IF} \neq \widetilde{0}^{IF}$) in such a way that $\frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}} = \underset{i}{Min}\left\{\frac{\widetilde{x}_{Bi}^{IF}}{\widetilde{\alpha}_{ij}^{IF}}, \widetilde{y}_{ij}^{IF} > 0, i \neq r\right\}$ then, the set of triangular intuitionistic fuzzy basic variables that was newly introduced is non-negative. Hence, it is possible to find the triangular intuitionistic fuzzy basic solution.

Theorem 3.3 (Improved TIF basic feasible solution)

Let $\widetilde{x}_B^{IF}$ be a triangular intuitionistic fuzzy basic feasible solution for the TIFLPP max $\widetilde{Z}^{IF} \approx \widetilde{C}^{IF}\widetilde{x}^{IF}$ subject to $\widetilde{A}^{IF}\widetilde{x}^{IF} \approx \widetilde{b}^{IF}, \widetilde{x}^{IF} \geq \widetilde{0}^{IF}$, Let the another triangular intuitionistic fuzzy basic feasible solution acquired by admitting a non-basis column vector $\widetilde{a}_j^{IF}$ in the basis be $\widetilde{x}_B^{IF^*}$, for which the $\widetilde{Z}_j^{IF} - \widetilde{C}_j^{IF}$ is negative. Then the improved triangular intuitionistic fuzzy basic feasible solution is $\widetilde{x}_B^{IF^*}$, that is $\widetilde{c}_B^{IF^*}\widetilde{x}_B^{IF^*} > \widetilde{c}_B^{IF}\widetilde{x}_B^{IF}$.

Proof. The TIFLPP is to determine $\widetilde{x}^{IF}$ to max $\widetilde{Z}^{IF} \approx \widetilde{C}^{IF}\widetilde{x}^{IF}$ where $\widetilde{C}^{IF}, \widetilde{x}^{IF} \in (IF(R))^m$, subject to $\widetilde{A}^{IF}\widetilde{x}^{IF} \approx \widetilde{b}^{IF}$ and $\widetilde{x}^{IF} \geq \widetilde{0}^{IF}$, where $\widetilde{b}^{IF} \in (IF(R))^n, \widetilde{A}^{IF}$ is a $m \times n$ matrix. Let $\widetilde{x}_B^{IF}$ is an TIF basic feasible solution, then $\widetilde{z}_0^{IF} = \widetilde{c}_B^{IF}\widetilde{x}_B^{IF}$. If the vector eliminated from the basis be $\widetilde{b}_r^{IF}$ and let the new intuitionistic fuzzy basic feasible solution be $\widetilde{x}_B^{IF^*}$, then $\widetilde{x}_{Bi}^{IF^*} = \widetilde{x}_{Bi}^{IF} + \widetilde{x}_{Br}^{IF}\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}$ and $\widetilde{x}_{Br}^{IF^*} = \frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}$. Hence, the new value of the objective function is

$$\widetilde{Z}^{IF} = \sum_{i=1}^{m}\widetilde{c}_{Bi}^{IF}\widetilde{x}_{Bi}^{IF^*} = \sum_{i=1}^{m}\widetilde{c}_{Bi}^{IF}\left(\widetilde{x}_{Bi}^{IF} + \widetilde{x}_{Br}^{IF}\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}\right) + \widetilde{c}_{Br}^{IF}\frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}$$

$$= \sum_{i=1}^{m}\widetilde{c}_{Bi}^{IF}\left(\widetilde{x}_{Bi}^{IF} + \widetilde{x}_{Br}^{IF}\frac{\widetilde{y}_{ij}^{IF}}{\widetilde{y}_{rj}^{IF}}\right) + \widetilde{c}_j^{IF}\frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}\ \left(\widetilde{c}_{Br}^{IF} = \widetilde{c}_j^{IF}\right)$$

$$= \widetilde{z}_0^{IF} - \left(\widetilde{z}_j^{IF} - \widetilde{c}_j^{IF}\right)\frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}}\ \left(\frac{\widetilde{x}_{Br}^{IF}}{\widetilde{y}_{rj}^{IF}} > \widetilde{0}^{IF}\right)$$

$> \tilde{z}_0^{IF}$.

Hence, the improved value of the objective function is given by the new TIF basic feasible solution.

Theorem 3.4 (Unbounded TIF solution)

Let there exist a TIF basic feasible solution to a given TIFLPP. If for at least one j, for which $\tilde{y}_{ij}^{IF} \le \tilde{0}^{IF}$ and $\tilde{Z}_j^{IF} - \tilde{C}_j^{IF}$ is negative, then there does not exist any optimum solution to this TIFLPP.

Proof. Let a TIF basic feasible solution to this problem be $\tilde{x}_B^{IF}$, so that $\tilde{D}^{IF}\tilde{x}_B^{IF} = \tilde{b}^{IF}$ and $\tilde{x}_B^{IF} \ge \tilde{0}^{IF}$ with the value of the objective function $\tilde{Z}_0^{IF} = \sum_{j=1}^{n} \tilde{c}_{Bj}^{IF}\tilde{x}_{Bj}^{IF}$. Now we can write:

$$\tilde{b}^{IF} = \tilde{D}^{IF}\tilde{x}_B^{IF} + \xi\tilde{a}_j^{IF} - \xi\tilde{a}_j^{IF}$$

$$= \sum_{j=1}^{n} \tilde{x}_{Bj}^{IF}\tilde{d}_j^{IF} + \xi\tilde{a}_j^{IF} - \xi\sum_{j=1}^{n} \tilde{y}_{ij}^{IF}\tilde{d}_j^{IF} \quad \left(\tilde{a}_j^{IF} \in \tilde{A}^{IF}, \xi \textit{ is a scalar.}\right)$$

$$= \sum_{j=1}^{n} (\tilde{x}_{Bj}^{IF} - \xi\tilde{y}_{ij}^{IF})\,\tilde{d}_j^{IF} + \xi\tilde{a}_j^{IF}$$

If $\xi > 0$ then $(\tilde{x}_{Bj}^{IF} - \xi\tilde{y}_{ij}^{IF}) \ge \tilde{0}^{IF}$ since $\tilde{y}_{ij}^{IF} \le \tilde{0}^{IF}$. This shows that there exists a triangular intuitionistic fuzzy feasible solution whose n+1 components may be strictly positive. In general, it may not be a TIF basic solution. The value of the objective function for these n+1 variables is given by:

$$\tilde{Z}^{IF^*} = \sum_{j=1}^{n} \tilde{c}_{Bj}^{IF}(\tilde{x}_{Bj}^{IF} - \xi\tilde{y}_{ij}^{IF}) + \xi\tilde{c}_j^{IF}$$

$$= \sum_{j=1}^{n} \tilde{c}_{Bj}^{IF}\tilde{x}_{Bj}^{IF} - \xi\left(\sum_{j=1}^{n} \tilde{c}_{Bj}^{IF}\tilde{y}_{ij}^{IF} - \tilde{c}_j^{IF}\right)$$

$$= \tilde{Z}_0^{IF} - \xi\,(\tilde{z}_j^{IF} - \tilde{c}_j^{IF}).$$

But $\tilde{z}_j^{IF} - \tilde{c}_j^{IF} < \tilde{0}^{IF}$ and $\xi > 0$. Therefore, $\tilde{Z}^{IF^*} \to \infty$ as $\xi \to \infty$. Hence, there is no limit to the optimum value of $\tilde{Z}^{IF}$ and hence the unbounded solution to the given TIFLPP exists.

Theorem 3.5(Conditions of Optimality)

A sufficient condition for an intuitionistic fuzzy basic feasible solution to a TIFLPP to be an intuitionistic fuzzy optimum solution is that $\tilde{z}_j^{IF} - \tilde{c}_j^{IF} \ge \tilde{0}^{IF}$ for all j for which the column vector $\tilde{a}_j^{IF} \in \tilde{A}^{IF}$ is not in the basis $\tilde{B}^{IF}$.

Proof. Let us assume that there exists an intuitionistic fuzzy basic feasible solution $\tilde{x}_B^{IF}$ to this TIFLPP. Let $\tilde{c}_B^{IF}$ be the cost vector corresponding to the basis variables. $\tilde{B}^{IF}\tilde{x}_B^{IF} = \tilde{b}^{IF}$,

$\tilde{x}_B^{IF} \ge \tilde{0}^{IF}$ and $\tilde{z}_0^{IF} = \tilde{c}_B^{IF}\tilde{x}_B^{IF}$. For all j for which $\tilde{a}_j^{IF} \notin \tilde{B}^{IF}$, we are given that $\tilde{z}_j^{IF} - \tilde{c}_j^{IF} \ge \tilde{0}^{IF}$. Let $\tilde{a}_j^{IF} = \tilde{b}_j^{IF}$ for all such j for which $\tilde{a}_j^{IF} \in \tilde{B}^{IF}$. Then $\tilde{y}_j^{IF} = \tilde{B}^{IF^{-1}}\tilde{b}_j^{IF}$,

since $\tilde{y}_j^{IF} = \tilde{B}^{IF^{-1}}\tilde{a}_j^{IF}$ and $\tilde{e}_j^{IF}$ is the unit vector. $\tilde{z}_j^{IF} - \tilde{c}_j^{IF} = \tilde{c}_B^{IF}\tilde{e}_j^{IF} - \tilde{c}_j^{IF} = \tilde{c}_{Bj}^{IF} - \tilde{c}_j^{IF} = \tilde{0}^{IF}$. Since $\tilde{a}_j^{IF} \in \tilde{B}^{IF}, \tilde{c}_{Bj}^{IF} = \tilde{c}_j^{IF}$. Now, let $\tilde{x}^{IF}$ be an intuitionistic fuzzy feasible solution. Then $\sum_{i=1}^{n}\left(\tilde{z}_j^{IF} - \tilde{c}_j^{IF}\right)\tilde{x}_i^{IF} \ge \tilde{0}^{IF}$, since $\tilde{x}_j^{IF} \ge \tilde{0}^{IF}$, $\sum_{j=1}^{n}\tilde{z}_j^{IF}\tilde{x}_j^{IF} \ge \sum_{j=1}^{n}\tilde{c}_j^{IF}\tilde{x}_j^{IF}$. Since $\tilde{z}_j^{IF} = \tilde{c}_B^{IF}\tilde{y}_j^{IF}$, $\sum_{j=1}^{n}\tilde{c}_B^{IF}\tilde{y}_j^{IF}\tilde{x}_j^{IF} \ge \sum_{j=1}^{n}\tilde{c}_j^{IF}\tilde{x}_j^{IF}$. Since $\tilde{c}_B^{IF}\tilde{y}_j^{IF} = \sum_{i=1}^{m}\tilde{c}_{Bj}^{IF}\tilde{y}_{ij}^{IF}$,

$\sum_{i=1}^{m}\tilde{c}_{Bi}^{IF}\sum_{j=1}^{n}\tilde{y}_{ij}^{IF}\tilde{x}_j^{IF} \ge \sum_{j=1}^{n}\tilde{c}_j^{IF}\tilde{x}_j^{IF}$. For all j for which $\tilde{a}_j^{IF} \notin \tilde{B}^{IF}$,

Now, since $\tilde{x}_B^{IF} = \tilde{B}^{IF^{-1}}\left(\tilde{A}^{IF}\tilde{x}^{IF}\right) = \left(\tilde{B}^{IF^{-1}}\tilde{A}^{IF}\right)\tilde{x}^{IF} = \tilde{Y}^{IF}\tilde{x}^{IF}$, $\tilde{x}_{Bi}^{IF} = \sum_{j=1}^{n}\tilde{y}_{ij}^{IF}\tilde{x}_j^{IF}$. $\sum_{i=1}^{m}\tilde{c}_{Bi}^{IF}\tilde{x}_{Bi}^{IF} \ge \sum_{j=1}^{n}\tilde{c}_j^{IF}\tilde{x}_j^{IF}$, $\tilde{c}_B^{IF}\tilde{x}_B^{IF} \ge \tilde{c}^{IF}\tilde{x}^{IF}$, $\tilde{z}_0^{IF} \ge \tilde{z}^{IF^*}$, where $\tilde{z}^{IF^*}$ is the objective function's value for the feasible solution. Hence, $\tilde{z}_0^{IF}$ is an optimum for the basic feasible solution for which $\tilde{z}_j^{IF} - \tilde{c}_j^{IF} \ge \tilde{0}^{IF}$ for all j such that $\tilde{a}_j^{IF} \notin \tilde{D}^{IF}$.

4 Numerical Illustration: Solving TIFLPP with the New Correlation Coefficient

Consider a linear programming problem with triangular intuitionistic fuzzy variables and coefficients, Max $\widetilde{Z}$ =(10,12,13;0.5,0.4)$\widetilde{x}_1 + (2,3,5;0.8,0.1)\widetilde{x}_2 + (0.5,1,2;0.4,0.2)\widetilde{x}_3$

St. $(8,10,11;0.3,0.2)\widetilde{x}_1 + (1,2,3;0.7,0.3)\widetilde{x}_2 + (0.7,1,2;0.5,0.3)\widetilde{x}_3 \leq (98,100,101;0.6,0.3)$;

(6,7,10;0.4,0.3)$\widetilde{x}_1 + (1,3,4;0.2,0.1)\widetilde{x}_2 + (1,2,5;0.6,0.3)\widetilde{x}_3 \leq (75,77,78;0.7,0.2)$;

$$(1,2,5;0.5,0.3)\widetilde{x}_1 + (2,4,5;0.4,0.3)\widetilde{x}_2 + (0,1,3;0.7,0.1)\widetilde{x}_3 \leq (78,80,81;0.6,0.3).$$

Convert the given TIFLPP into the standard form of TIFLPP by adding TIF slack variables.

Max $\widetilde{Z}$ =(10,12,13;0.5,0.4)$\widetilde{x}_1 + (2,3,5;0.8,0.1)\widetilde{x}_2 + (0.5,1,2;0.4,0.2)\widetilde{x}_3 + \widetilde{0}\widetilde{s}_1 + \widetilde{0}\widetilde{s}_2 + \widetilde{0}\widetilde{s}_3$

St. $(8,10,11;0.3,0.2)\widetilde{x}_1 + (1,2,3;0.7,0.3)\widetilde{x}_2 + (0.7,1,2;0.5,0.3)\widetilde{x}_3 + \widetilde{s}_1 = (98,100,101;0.6,0.3)$;

(6,7,10;0.4,0.3)$\widetilde{x}_1 + (1,3,4;0.2,0.1)\widetilde{x}_2 + (1,2,5;0.6,0.3)\widetilde{x}_3 + \widetilde{s}_2 = (75,77,78;0.7,0.2)$;

$$(1,2,5;0.5,0.3)\widetilde{x}_1 + (2,4,5;0.4,0.3)\widetilde{x}_2 + (0,1,3;0.7,0.1)\widetilde{x}_3 + \widetilde{s}_3 = (78,80,81;0.6,0.3).$$

The initial table for TIFLPP with usual simplex procedure with TIFNs arithmetical operations is given in Table 1.

Table 1. Initial Table for Solving TIFLPP with New Correlation Coefficient

B	$\widetilde{x}_1^{IF}$	$\widetilde{x}_2^{IF}$	$\widetilde{x}_3^{IF}$	$\widetilde{s}_1^{IF}$	$\widetilde{s}_2^{IF}$	$\widetilde{s}_3^{IF}$	$\widetilde{X}_B^{IF}$	$\widetilde{\theta}^{IF}$
$\widetilde{s}_1^{IF}$	(8,10,11; 0.3,0.2)	(1,2,3; 0.7,0.3)	(0.7,1,2; 0.5,0.3)	$\widetilde{1}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	(98,100,101; 0.6,0.3)	(8.91,10,12.63; 0.3,0.3)
$\widetilde{s}_2^{IF}$	(6,7,10; 0.4,0.3)	(1,3,4; 0.2,0.1)	(1,2,5; 0.6,0.3)	$\widetilde{0}^{IF}$	$\widetilde{1}^{IF}$	$\widetilde{0}^{IF}$	(75,77,78; 0.7,0.2)	(7.5,11,13; 0.4,0.3)
$\widetilde{s}_3^{IF}$	(1,2,5; 0.5,0.3)	(2,4,5; 0.4,0.3)	(0,1,3; 0.7,0.1)	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{1}^{IF}$	(78,80,81; 0.6,0.3)	(15.6,40,81; 0.5,0.3)
$\widetilde{Z}_j^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$		
$\widetilde{Z}_j^{IF} - \widetilde{C}_j^{IF}$	(-13,-12,-10; 0.5,0.4)	(-5,-3,-2; 0.8,0.1)	(-2,-1,-0.5; 0.4,0.2)	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$		

Now, rank the ratios $\widetilde{\theta}_j^{IF}$ using the proposed correlation coefficient as follows: $K\left(\widetilde{\theta}_1^{IF}, \widetilde{i}^{+}\right) = 0.7017$, $K\left(\widetilde{\theta}_2^{IF}, \widetilde{i}^{+}\right) = 0.8000$ and $K\left(\widetilde{\theta}_3^{IF}, \widetilde{i}^{+}\right) = 0.8575$. Then the ranking is $\widetilde{\theta}_3^{IF} > \widetilde{\theta}_2^{IF} > \widetilde{\theta}_1^{IF}$. Proceeding with the minimum ratio $\widetilde{\theta}_1^{IF}$. Choose the first element in column $\widetilde{x}_1^{IF}$ as pivotal element.

Table 2. First Iteration for Solving TIFLPP with the New Correlation Coefficient

B	$\widetilde{x}_1^{IF}$	$\widetilde{x}_2^{IF}$	$\widetilde{x}_3^{IF}$	$\widetilde{s}_2^{IF}$	$\widetilde{s}_2^{IF}$	$\widetilde{s}_3^{IF}$	$\widetilde{x}_B^{IF}$	$\widetilde{\theta}^{IF}$
$\widetilde{x}_1^{IF}$	(0.73,1,1.38; 0.3,0.2)	(0.09,0.2,0.38; 0.3,0.3)	(0.06,0.1,0.25; 0.3,0.3)	(0.09,0.1,0.13; 0.3,0.2)	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$	(8.91,10, 12.63; 0.3,0.3)	(23.45,50, 140.33; 0.3,0.3)
$\widetilde{s}_2^{IF}$	(-7.75,0,5.64; 0.3,0.3)	(-2.75,1.6,3.45; 0.2,0.3)	(-1.5,1.3,4.62; 0.3,0.3)	(-1.25,-0.7, -0.55; 0.3,0.3)	$\widetilde{1}^{IF}$	$\widetilde{0}^{IF}$	(-51.25,7, 24.55; 0.3,0.3)	(-14.86,4.38, 18.64; 0.2,0.3)
$\widetilde{s}_3^{IF}$	(-5.88,0,4.27; 0.3,0.3)	(0.13,3.6, 4.91;0.3,0.3)	(-1.25,0.8, 2.94;0.3,0.3)	(-0.63,-0.2, -0.09; 0.3,0.3)	$\widetilde{0}^{IF}$	$\widetilde{1}^{IF}$	(14.88,60, 72.09; 0.3,0.3)	(3.03,16.67, 554.54; 0.3,0.3)
$\widetilde{Z}_j^{IF}$	(7.27,12,17.88; 0.3,0.4)	(0.91,2.4, 4.88;0.3,0.4)	(0.64,1.2, 3.25;0.3,0.4)	(0.91, 1.2,1.63; 0.3,0.4)	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$		
$\widetilde{Z}_j^{IF} - \widetilde{C}_j^{IF}$	(-5.73,0,7.88; 0.3,0.4)	(-4.09,-0.6, 2.88;0.3,0.4)	(-1.36, 0.2,2.75; 0.3,0.4)	(0.91, 1.2,1.63; 0.3,0.4)	$\widetilde{0}^{IF}$	$\widetilde{0}^{IF}$		

The computations of first iteration with TIFNs arithmetical operations are shown in Table 2. Since $\widetilde{Z}_j^{IF} - \widetilde{C}_j^{IF}$ of $\widetilde{x}_2^{IF}$ has the negative TIFN have to proceed for another iteration Now, rank the ratios $\widetilde{\theta}_j^{IF}$ using the proposed correlation coefficient as follows: $K\left(\widetilde{\theta}_1^{IF}, \widetilde{i}^{+}\right) = 0.7071$, $K\left(\widetilde{\theta}_2^{IF}, \widetilde{i}^{+}\right) = 0.5547$ and $K\left(\widetilde{\theta}_3^{IF}, \widetilde{i}^{+}\right) = 0.7071$. Then the ranking is $\widetilde{\theta}_3^{IF} > \widetilde{\theta}_1^{IF} > \widetilde{\theta}_2^{IF}$. Proceeding with the minimum ratio $\widetilde{\theta}_2^{IF}$. Choose the second element in column $\widetilde{x}_2^{IF}$ as pivotal element.

The computations of first iteration with TIFNs arithmetical operations are shown in Table 3. All $\widetilde{Z}_j^{IF} - \widetilde{C}_j^{IF}$ are positive then the optimal TIF solution is $\widetilde{x}_1^{IF}$ =(1.83,9.13,18.26;0.2,0.3), $\widetilde{x}_2^{IF} = (-14.84, 4.38, 18.64; 0.2, 0.3)$ and the objective is

$\widetilde{Z}^{IF} = (-55.90, 122.70, 330.58; 0.2, 0.4)$.

Table 3. Second Iteration for Solving TIFLPP with the New Correlation Coefficient

B	$\tilde{x}_1^{IF}$	$\tilde{x}_2^{IF}$	$\tilde{x}_3^{IF}$	$\tilde{s}_2^{IF}$	$\tilde{s}_2^{IF}$	$\tilde{s}_3^{IF}$	$\tilde{X}_B^{IF}$
$\tilde{x}_1^{IF}$	(-0.34,1,2.23; 0.2,0.3)	(-0.29,0,0.85; 0.2,0.3)	(-0.44,-0.06, 0.89;0.2,0.3)	(-0.08,0.19, 0.26;0.2,0.3)	(-0.11,-0.13, 0.14;0.2,0.3)	$\tilde{0}^{IF}$	(1.83,9.13, 18.26;0.2,0.3)
$\tilde{x}_2^{IF}$	(-2.24,0,2.82; 0.2,0.3)	(-1.26,1,1; 0.2,0.3)	(-1.68,0.81, 1.34;0.2,0.3)	(-0.36,-0.44, 0.45;0.2,0.3)	(-0.36,0.63, 0.29;0.2,0.3)	$\tilde{0}^{IF}$	(-14.84,4.38, 18.64;0.2,0.3)
$\tilde{s}_3^{IF}$	(-19.71,0,15.29; 0.2,0.3)	(-4.79,0,11.08; 0.2,0.3)	(-7.81,-2.13, 11.18;0.2,0.3)	(-2.86,1.38, 1.69;0.2,0.3)	(-1.42,-2.25, 1.79;0.2,0.3)	$\tilde{1}^{IF}$	(-76.63,44.25, 144.93;0.2,0.3)
$\tilde{Z}_j^{IF}$	(-15.68,12, 43.05;0.2,0.4)	(-10.04,3, 16.08;0.2,0.4)	(-14.17,1.69, 18.23;0.2,0.4	(-2.87,0.94, 5.69;0.2,0.4)	(-3.25,0.38, 3.24;0.2,0.4)	$\tilde{0}^{IF}$	
$\tilde{Z}_j^{IF} - \tilde{C}_j^{IF}$	(-28.68,0,33.05; 0.2,0.4)	(-15.04,0, 14.08;0.2,0.4)	(-16.17,0,0.69, 17.73;0.2,0.4)	(-2.87,0.94, 5.69;0.2,0.4)	(-3.25,0.38, 3.24;0.2,0.4)	$\tilde{0}^{IF}$	

5 Solving TIFLPP with Defuzzification of TIFNs

Consider the same TIFLPP given in section 4 and defuzzify using $\alpha-$cut for TIFNs taking $\alpha = 0.5$. Max $\widetilde{Z} = (12,12]\widetilde{x}_1 + (2.63,3.75]\widetilde{x}_2 + (1.13,0.75]\widetilde{x}_3$;

st. $(11.33,9.33]\widetilde{x}_1 + (1.71,2.29]\widetilde{x}_2 + (1,1]\widetilde{x}_3 \leq (99.67,100.17]$;

$$(7.25,6.25]\widetilde{x}_1 + (6,1.5]\widetilde{x}_2 + (1.83,2.5]\widetilde{x}_3 \leq (76.43,77.29];$$

$(2,2]\widetilde{x}_1 + (4.5,3.75]\widetilde{x}_2 + (0.71,1.57]\widetilde{x}_3 \leq [79.67,80.17]$.

Now, on taking average for the $\alpha-$cut intervals we get the crisp LPP,

Max $\widetilde{Z} = 12\widetilde{x}_1 + 3.19\widetilde{x}_2 + 0.94\widetilde{x}_3$;

st. $10.33\widetilde{x}_1 + 2\widetilde{x}_2 + \widetilde{x}_3 \leq 99.92$; $6.75\widetilde{x}_1 + 3.75\widetilde{x}_2 + 2.17\widetilde{x}_3 \leq 76.86$;

$2\widetilde{x}_1 + 4.13\widetilde{x}_2 + 1.14\widetilde{x}_3 \leq 79.92$. By solving the LPP with usual simplex method, the optimal solution is $x_1 = 8.76,\ x_2 = 4.74$ and the maximum objective Z=120.18. Now consider the same TIFLPP given in section 4 and defuzzify using $\beta-$cut for TIFNs taking $\beta = 0.5$.

Max $\widetilde{Z} = (11.67,12.17]\widetilde{x}_1 + (2.56,3.89]\widetilde{x}_2 + (0.81,1.38]\widetilde{x}_3$;

st. $(9.25,10.38]\widetilde{x}_1 + (1.71,2.29]\widetilde{x}_2 + (0.91,1.29]\widetilde{x}_3 \leq (99.43,100.29]$;

$$(6.71,7.86]\widetilde{x}_1 + (2.11,3.44]\widetilde{x}_2 + (1.71,2.86]\widetilde{x}_3 \leq (76.25,77.38];$$

$(1.71,2.86]\widetilde{x}_1 + (3.43,4.29]\widetilde{x}_2 + (0.56,1.89]\widetilde{x}_3 \leq [79.43,80.29]$.

Now, on taking average for the $\beta-$cut intervals we get the crisp LPP,

Max $\widetilde{Z} = 11.92\widetilde{x}_1 + 3.22\widetilde{x}_2 + 1.09\widetilde{x}_3$;

st. $9.81\widetilde{x}_1 + 2\widetilde{x}_2 + 1.10\widetilde{x}_3 \leq 99.86$; $7.29\widetilde{x}_1 + 2.78\widetilde{x}_2 + 2.29\widetilde{x}_3 \leq 76.81$;

$2.29\widetilde{x}_1 + 3.86\widetilde{x}_2 + 1.22\widetilde{x}_3 \leq 79.86$. By solving the LPP with usual simplex method, the optimal solution is $x_1 = 9.77,\ x_2 = 2.01$ and the maximum objective Z=122.93.

Table 4. Comparison of optimal solution and objective

Optimization Method	Optimal Solution and Objective
TIFLPP with ratio ranking method [10]	$\tilde{x}_1^{IF}$ = (15.6,40,81;0.5,0.3); $\tilde{Z}^{IF}$ =(156,480,1053;0.5,0.4)
TIFLPP with proposed correlation coefficient	$\tilde{x}_1^{IF}$ =(1.83,9.13,18.26;0.2,0.3); $\tilde{x}_2^{IF}$ = (-14.84,4.38,18.64;0.2,0.3); $\tilde{Z}^{IF}$ =(-55.90,122.70,330.58;0.2,0.4)
TIFLPP with defuzzification of TIFNs with $\alpha-$cut	$x_1 = 8.76,\ x_2 = 4.74$ Z=120.18
TIFLPP with defuzzification of TIFNs with $\beta-$cut	$x_1 = 9.77,\ x_2 = 2.01$ Z=122.93

6 Discussion

When comparing the TIFLPP with ratio ranking method [10], TIFLPP with proposed correlation coefficient and TIFLPP with defuzzification of TIFNs in Table 4 we can clearly see that TIF optimal solution obtained from TIFLPP with ratio ranking method has only one TIF variable $\tilde{x}_1^{IF}$ in the basis, even though it gives the maximum objective with the single iteration. Whereas in TIFLPP with proposed correlation coefficient it has TIF variables $\tilde{x}_1^{IF}$ and $\tilde{x}_2^{IF}$ in the basis. In TIFLPP with defuzzification of TIFNs with $\alpha-$cut and $\beta-$cut, the given TIFN information are converted into crisp values. If the user/reader wants higher objective, independent of involving more variables, they can choose TIFLPP with ratio ranking method. If the user/reader wants to involve more variables even though it minimizes the objective they can choose TIFLPP with proposed correlation coefficient. If the user/reader prefers to defuzzify the TIFNs at the initial stage of the problem, then the method of section 5 can be followed.

7 Conclusion

The simplex method following the new correlation coefficient of triangular intuitionistic fuzzy sets with graded mean integration representation and triangular intuitionistic fuzzy simplex method with defuzzification method and ratio ranking method are compared and suggestions are given based on the results obtained. A new optimization method for TIFLPP, which is solved without defuzzifying the TIFNs to preserve the given information is introduced. Several theorems are proved for concepts like the improvement of the basic feasible solution according to the objective, finding whether the given TIFLPP has a bounded solution and the condition to get the optimal solution. The proposed optimization method is explained using numerical illustration. The results of the TIFLPP with ratio ranking method, TIFLPP with proposed correlation coefficient and TIFLPP with defuzzification of TIFNs with $\alpha-$cut and $\beta-$cut are compared and the superiority of the proposed model is discussed. The research can be extended by applying the TIFLPP in multi attribute group decision making methods and TIFLPP can be used in solving intuitionistic fuzzy matrix games.

8 Declaration

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