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$* - \eta$-Ricci-Yamabe Solitons on Lorentzian Para- Sasakian Manifolds with Semi-symmetric Non-metric Connection

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Abstract

Objectives: The present paper is to study certain types of metric such as $* - \eta$-Ricci-Yamabe soliton on Lorentzian Para-Sasakian manifolds with respect to semi-symmetric non-metric connection. **Methods**: It includes Contraction, Lie derivative, Semi-symmetric non-metric connection, Gradient, Laplacian equation,$* - \eta$-Ricci-Yamabe soliton. **Findings**: We get some curvature properties of Lorentzian Para-Sasakian manifolds admitting semi-symmetric non-metric connection. Here, we develop the relation of soliton constant on Lorentzian Para- Sasakian manifold admitting $* - \eta$-Ricci-Yamabe soliton with respect to semi-symmetric non-metric connection. Later, we have acquired Laplacian equation from $* - \eta$-Ricci-Yamabe soliton when the potential vector field ξ of the soliton is of gradient types. Finally, we have shown the nature of the solitons when the vector field is conformally killing admitting semi-symmetric non-metric connection. **Novelty**: This work has not been done by any other authors.

Keywords: Ricci Yamabe solitons, $* - \eta$-Ricci-Yamabe soliton, conformal killing vector field, $* - \eta$-Einstien soliton, Lorentzian Para-Sasakian manifold
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1 Introduction

The study of Ricci solitons on Riemannian manifolds is very essential in differential geometry. In the past few years, Ricci solitons and their generalization have been studied by many geometers by providing new techniques for a better understanding of the geometry of Riemannian manifolds. The importance of studying Ricci solitons and their generalizations in differential geometry is increasing due to their connection to general relativity. Recently, Roy *et al.*[(1)] studied the nature of $* - \eta$-Ricci -Yamabe soliton on α-Cosymplectic manifolds with respect to quarter symmetric metric connection. In 2019, Guler and Crasmareanu[(2)] established a geometric flow, which is union of called as Ricci-Yamabe map and it is said to be Ricci-Yamabe flow of type (p,q). Recently, Roy *et al.*[(1)] studied the $* - \eta$-Ricci-Yamabe solitons on α-Cosymplectic manifolds with a quarter-symmetric metric connection.

This work is very new and is not studied by other authors in different manifolds.

Let M^n be a $n-$ dimensional Riemannian manifold and $T_2^s(M^n)$ be the linear space of its symmetric tensor fields of type $(0,2)$ and $RiemM^n \subsetneq T_2^s(M^n)$ be the infinite space of its Riemannian metrics.

Definition1: The map $RY^{(p,q,g)}: I \to T_2^S(M^n)$ given as:

$RY^{(p,q,g)} = \frac{\partial}{\partial t}g(t) + 2pRic(t) + qr(t)g(t),$ is said to be (p,q)-Ricci-Yamabe map of the Riemannian flow, where p,q are some constants and I is an open interval. If $RY^{(p,q,g)} = 0$ then $g\,(.)$ will be known as (p,q)-Ricci-Yamabe flow[2].

The (p,q)-Ricci-Yamabe flow is

(i) Ricci flow[3], when $p=1,\ q=0.$

(ii) Yamabe flow[4], when $p=0,\ q=1.$

(iii) Einstein flow[5], when $p=1,\ q=-1.$

If Ricci-Yamabe flow moves only by one parameter group of diffeomorphism and scaling, then the soliton is said to be Ricci-Yamabe soliton[6]. If this soliton satisfies the equation,

$$£_V g + 2pS = [2\Lambda - qr]g \tag{1}$$

where $£_V$ is the Lie derivative, along the vector field V, S is the Ricci tensor, R is the scalar curvature and $\wedge, p, q$ are constants. Then, the metric g is said to admit (p,q)-Ricci-Yamabe soliton or simply Ricci-Yamabe soliton(RYS) $(g,V,\wedge,p,q)$.

If $\wedge$is negative, zero, positive, then the Ricci-Yamabe soliton is called as shrinking, steady and expanding respectively. Moreover, if they are smooth function then (1) is called almost Ricci-Yamabe soliton. A few years later, Siddiqi and Akyol[6] studied the η-Ricci-Yamabe soliton which is generalization of Ricci-Yamabe soliton and is given as

$$£_V g + 2pS + (\wedge - qr]g + 2\mu\eta \otimes \eta = 0 \tag{2}$$

where η is 1-form and μ is scalar.

Dey and Roy[7] defined $*-\eta$-Ricci soliton as a generalization of η-Ricci soliton and found as

$$£_V g + 2S^* + 2\Lambda g + 2\mu\eta \otimes \eta = 0 \tag{3}$$

where S^* is the *-Ricci tensor[8].

Definition 2: An n-dimensional Riemannian manifold M^n is said to admit $*-\eta$-Ricci-Yamabe soliton if

$$£_\xi g + 2pS^* + 2(\Lambda - qr^*]g + 2\mu\eta \otimes \eta = 0 \tag{4}$$

where $r^* = trace(S*)$.

If we consider the vector field ξ as the gradient of a smooth function f, then the above equation is known as gradient $*-\eta$-Ricci-Yamabe soliton and is written as

$$Hess\, f + pS^* + \left[\Lambda - \frac{qr*}{2}\right]g + \mu\eta \otimes \eta = 0 \tag{5}$$

where $Hessf$ is the Hessian of the smooth function f.

In this paper, we study the nature of $*-\eta$-Ricci-Yamabe soiton on Lorentzian Para-Sasakian manifolds with semi-symmetric non-metric connection. This paper is organized as follows. After introduction we have preliminaries about Lorentzian Para-Sasakian manifold[9]. In the next section, we find the curvature tensor of the manifold with respect to semi-symmetric non-metric connection. In the fourth section, we establish the relation of the soliton constants of the manifold admitting $*-\eta$-Ricci- Yamabe soliton with semi-symmetric non-metric connection. In section five, we find the Laplacian equation satisfied by f, where the vector field ξ, associated with the soliton is of the form grad(f) in the manifold admitting $*-\eta$-Ricci- Yamabe soliton with semi-symmetric non-metric connection. Finally, in the last section, we also discuss the nature of $*-\eta$-Eintein soliton with semi-symmetric non-metric connection on the manifold.

2 Preliminaries

An n-dimensional differentiable manifold M^n is said to be Lorentzian Para-Sasakian manifold [in brief LP-Sasakian manifold][10] if it admits a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and lorentzian metric g which satisfy

$$\phi^2 X_1 = X_1 + \eta(X_1)\xi, \tag{6}$$

$$\eta(\xi) = -1, \tag{7}$$

$$g(\phi X_1, \phi X_2) = g(X_1, X_2) + \eta(X_1)\eta(X_2) \tag{8}$$

$$(i)\ g(X_1, \xi) = \eta(X_1), \quad (ii) \quad \nabla_{x_1}\xi = \phi X_1, \tag{9}$$

$$(\nabla_{X_1}\phi)(X_2) = g(X_1, X_2)\xi + \eta(X_2)X_1 + 2\eta(X_1)\eta(X_2)\xi \tag{10}$$

where ∇ is the covariant differentiation with respect to g.

In LP-Sasakian manifolds, the following relations hold:

$$(i)\ \phi\xi = 0\ ,\ (ii)\ \eta\ (\phi X_1) = 0 \tag{11}$$

$$rank\ \phi = n - 1 \tag{12}$$

If we put

$$F\ (X_1, X_2) = g(X_1, \phi X_2) \tag{13}$$

for any vector field $X_1,\ X_2$ and the tensor $F(X_1, X_2)$ is a symmetric $(0,2)$-type tensor field.

And since η is closed in a LP-Sasakian manifold, we have

$$(i)\ (\nabla_{X_1}\eta)\,(X_2) = F(X_1, X_2)\ ,\ (ii)\ F\ (X_1, \xi) = 0. \tag{14}$$

Also, in LP-Sasakian manifold M^n with structure $(\phi,\ \xi, \eta,\ g)$ the following hold

$$\begin{aligned} &g(R(X_1, X_2)X_3, \xi) = \\ &\eta\ (R(X_1, X_2)X_3\ ) = \\ &g\ (X_2,\ X_3\ )\,\eta\ (X_1) - \\ &g(X_1, X_3\ )\eta(X_2), \end{aligned} \tag{15}$$

$$R\,(\xi, X_1)\,X_2 = g\,(X_1, X_2)\,\xi - \eta\ (X_2)\,X_1, \tag{16}$$

$$R(X_1, X_2)\xi = \eta\ (X_2)\,X_1 - \eta\ (X_1)\,X_2, \tag{17}$$

$$S\,(X_1,\ \xi) = (n-1)\eta(X_1) \tag{18}$$

$$S\,(\phi X_1,\ \phi X_2) = S\,(X_1, X_2) + (n-1)\eta\ (X_1)\ \eta\ (X_2)\,, \tag{19}$$

for any vector fields $X_1,\ X_2$ and X_3. Here, R is the Riemannian curvature tensor and S is the Ricci tensor on M^n.

In the paper [11], Lemma 2.3, the $*-$Ricci tensor of an n-dimensional LP-Sasakian manifold is obtained as

$$S^*\,(X_2, X_3) = S\,(X_2, X_3) + (n-2)\,g\,(X_2, X_3) - g\,(X_2, \phi X_3)\,a + (2n-3)\eta(X_2)\eta(X_3) \tag{20}$$

where $a = trace\ \phi$ and S^* is the $*-$Ricci tensor of type $(0,2)$ on M^n.

Putting $X_2 = X_3 = e_i$, where $e_i^{'}s$ are the orthonormal basis of $T_p(M^n)$ for $i = 1, 2, \ldots, n$, we obtain

$$r^* = r + n\,(n-2) + (2n-3) - a^2 \tag{21}$$

where $r^* = tr(S^*)$, is the $*-$scalar curvature and r is the scalar curvature.

3 Results

Let $\widetilde{\nabla}$ be a semi-symmetric non-metric connection in LP-Sasakian manifold such that

$$\widetilde{\nabla}_{X_1}X_2 = \nabla_{X_1}X_2 + \eta(X_2)X_1 \tag{22}$$

Then, we have [9]

$$\widetilde{R}(X_1,X_2)X_3 = R(X_1,X_2)X_3 + \alpha(X_1,X_3)X_2 - \alpha(X_2,X_3)X_1 \tag{23}$$

where $\alpha(X_2,X_3) = (\nabla_{X_2}\eta)X_3 - \eta(X_2)\eta(X_3)$, $\widetilde{R}$ and R are the curvature tensors with respect to semi-symmetric non-metric connection and Riemannian connection respectively.Taking inner product of (23) with X_4, we have

$$\begin{aligned} {}'\widetilde{R}(X_1,X_2,X_3,X_4) &= {}'R(X_1,X_2,X_3,X_4) + g(\phi X_1,X_3)g(X_2,X_4) \\ &-\eta(X_1)\eta(X_3)g(X_2,X_4) - g(\phi X_2,X_3)g(X_1,X_4) + \eta(X_2)\eta(X_3)g(X_1,X_4), \end{aligned} \tag{24}$$

By contracting (24) over X_1 and X_4, we get

$$\widetilde{S}(X_2,X_3) = S(X_2,X_3) - (n-1)g(\phi X_2,X_3) + (n-1)\eta(X_2)\eta(X_3) \tag{25}$$

where $\widetilde{S}$ and S are the Ricci tensors of the connections $\widetilde{\nabla}$ and ∇ respectively.

So in an LP-Sasakian manifold, the Ricci tensor of the semi-symmetric non-metric connection is symmetric.

Again, contracting (25) over X_2, X_3, we obtain

$$\widetilde{r} = r - (n-1)(a+1). \tag{26}$$

By putting $X_3 = \xi$ in (25) and using (7), (11) we get

$$\widetilde{S}(X_2,\xi) = -(n-1)\eta(X_2). \tag{27}$$

Theorem 1

If the metric of an n -dimensional LP- Sasakian manifold admits a $* - \eta$- Ricci -Yamabe solitons with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$, then the solitons constants μ and Λ are related by the equation such that [6]

$$\Lambda - \mu = 1 - p(3n-5) + \frac{rq}{2} - \frac{q}{2}\left[na + n - a - 4 + n^2 - a^2\right]. \tag{28}$$

Proof: Let us consider an n-dimensional LP-sasakian manifold M, admitting a $* - \eta$ -Ricci-Yamabe soliton.

Then from (4), we have

$$(\pounds_\xi g)(X_2,X_3) + 2pS^*(X_2,X_3) + (2\Lambda - qr^*]g(X_2,X_3) + 2\mu\eta(X_2)\eta(X_3) = \tag{29}$$

0 for any vector fields X_2, X_3 on M^n.

Using (20) and (21), the above equation takes the form ,

$$\begin{aligned} &\pounds_\xi g(X_2,X_3) + 2pS(X_2,X_3) + g(X_2,X_3)[2\Lambda + 2p(n-2) - rq - q(n-2)n\big(-q(2n-3) + \big) \\ &a^2q + \eta(X_2)\eta(X_3)[2\mu - 2p(2n-3) - 2pag(X_2,\phi X_3)] = 0 \end{aligned} \tag{30}$$

Consider the Lie derivative along V with respect to semi-symmetric non-metric connection $\widetilde{\nabla}$. Now if an n-dimensional LP-Sasakian Manifold M admits a $* - \eta$-Ricci-Yamabe soliton with respect to the semi-symmetric non-metric connection $\widetilde{\nabla}$,we have

$$\begin{aligned} &(\widetilde{\pounds}_\xi g)(X_2,X_3) + 2p\bar{S}(X_2,X_3) + \left(2\Lambda + 2p(n-2) - \bar{r}q - q(n-2)n - q(2n-3) + a^2q\right]g(X_2,X_3) + \\ &\eta(X_2)\eta(X_3)(2\mu - 2p(2n-3) - 2pag(X_2,\phi X_3)] = 0 \end{aligned} \tag{31}$$

In view of (11), it gives

$(\widetilde{\pounds}_V g)(X_1,X_2) = g\left(\widetilde{\nabla}_{X_1}V, X_2\right) + g\left(X_1, \widetilde{\nabla}_{X_2}V\right)$

$= (_V g)(X_1, X_2) - 2g(X_1, X_2)$.

Using (25), (26) and (31) in the above equation, we get,

$$\begin{aligned}&(\pounds_\xi g)(X_2, X_3) - 2g(X_2, X_3) + 2p(S(X_2, X_3) - (n-1)g(\phi X_2, X_3) + (n-1)\eta(X_2)\eta(ZX_3)] + \\ &\{2\Lambda + 2p(n-2) - (r-(n-1)(a+1)]q - q(n-2)n - q(2n-3) + \\ &a^2 q\} g(X_2, X_3 \eta(X_2)\eta(X_3)[2\mu - 2p(2n-3) - 2pag(X_2, \phi X_3)] = 0\end{aligned} \tag{32}$$

Now in an n-dimensional LP- Sasakian manifold, from (9), we obtain,

$$(\pounds_\xi g)(X_2, X_3) = g(\nabla_{X_2}\xi, X_3) + g(X_2, \nabla_{X_3}\xi) = 2g(\phi X_2, X_3) \tag{33}$$

Putting this value in (33) and taking $X_3 = \xi$, we get

$$2pS(X_2, \xi) - \left(2\Lambda - 2p(n-2) - qr + q(n-1)(a+1) - qn(n-2) - q(2n-3) + a^2 q\right]\eta(X_2) - 2p(n-1)\eta(X_2) = 0 \tag{34}$$

In consequence of (18) and (34), it yields (28).

This completes the proof.

Theorem 2

If the metric of an n – dimensional LP- sasakian manifold admits *- η - Ricci-Yamabe solitons with respect to semi – symmetric non-metric connection $\widetilde{\nabla}$.Then the solitons constant $\wedge$ and μ takes the forms of the following equations, where div ξ is the divergence of the vector field ξ.

a) $\wedge = \frac{1}{(n-1)} + \frac{\rho(n-5)}{(n-1)} - \frac{rq}{(n-1)} + \frac{q}{(n-1)}$ [na - n - a - 4 + n^2 - a^2] - $\frac{div\xi}{2(n-1)} + \frac{\eta}{(n-1)} - \frac{pr}{(n-1)}$ + pa +p - $\frac{p(n-2)n^2}{(n-1)} + \frac{qn}{2(n-1)}$[r - (n-1) (a+1)] + $\frac{q(n-2)n^2}{2(n-1)} + \frac{qn(2n-3)}{2(n-1)} - \frac{qa^2}{2(n-1)} - \frac{p(2n-3)}{(n-1)} - \frac{pa^2}{(n-1)}$.

$$\begin{aligned}b)\ \mu = &\frac{1}{(n-1)} + \frac{\rho(n-5)}{(n-1)} - \frac{rq}{(n-1)} + \frac{q}{(n-1)}[na - n - a - 4 + n^2 - a^2] - \frac{div\xi}{2(n-1)} + \frac{n}{(n-1)} - \frac{pr}{(n-1)} + \\ &pa + p - \frac{p(n-2)n}{(n-1)} + \frac{qn}{2(n+1)}[r - (n-1)(a+1)] + \\ &\frac{q(n-2)n^2}{2(n-1)} - \frac{qn(2n-3)}{2(n-1)}\frac{a^2 q}{2(n-1)} - \frac{p(2n-3)}{(n-1)} - \frac{pa^2}{(n-1)} - 1 + p(3n-5) - \frac{rq}{2} + \frac{q}{2}[na + n - a - 4 + n^2 - a^2]\end{aligned} \tag{35}$$

Proof: Taking $X_2 = X_3 = e_i$, where $e_i's$ are the orthonormal basis of $T_p(M^n)$ for $i = 1, 2, 3, , , n$ we get,

$$\begin{aligned}&div\xi - 2n + 2pr - 2pa(n-1) - 2p(n-1) + 2\Lambda n + 2np(n-2) - n[r-(n-1)(a+1)]q - q(n-2)n^2 - \\ &q(2n-3)n + a^2 qn - 2\mu + 2 - p(2n-3) - 2pa^2 = 0\end{aligned} \tag{36}$$

where $div\xi$ is the divergence of the vector field.

Replacing the value of μ from Theorem 1, $\wedge$ takes the form.

$$\begin{aligned}\wedge = &\frac{1}{(n-1)} + \frac{p(n-5)}{(n-1)} - \frac{rq}{(n-1)} + \frac{q}{(n-1)}\left[na - n - a - 4 + n^2 - a^2\right] - \frac{div\xi}{2(n-1)} + \frac{\eta}{(n-1)} \\ &- \frac{pr}{(n-1)} + pa + p - \frac{p(n-2)n}{(n-1)} + \frac{qn}{2(n-1)}\left[r - (n-1)(a+1)\right] + \frac{q(n-1)n^2}{2(n-1)} + \frac{qn(2n-3)}{2(n-1)} - \frac{a^2 q}{2(n-1)} - \frac{p(2n-3)}{(n-1)} - \frac{pa^2}{(n-1)}.\end{aligned}$$

and

$$\begin{aligned}\mu = &\frac{1}{(n-1)} + \frac{p(n-5)}{(n-1)} - \frac{rq}{(n-1)} + \frac{q}{(n-1)}[na - n - a - 4 + n^2 - a^2] - \frac{div\xi}{2(n-1)} + \frac{n}{(n-1)} \\ &\frac{pr}{(n-1)} + pr + p - \frac{p(n-2)n}{(n-1)} + \frac{qn}{2(n+1)}\left[r - (n-1)(a+1)\right] + \frac{q}{2(n-1)}(n-2)n^2 \\ &+ \frac{qn(2n-3)}{2(n-1)} - \frac{a^2 q}{2(n-1)} - \frac{p(2n-3)}{(n-1)} - \frac{pa^2}{(n-1)} - 1 + p(3n-5) - \frac{rq}{2} + \frac{q}{2}[na + n - a - 4 + n^2 - a^2]\end{aligned}$$

Theorem 3

An n- dimensional LP-Sasakian manifold admits a $*-\eta$- Ricci -Yamabe Solitons with respect to semi-symmetric non-metric connection $\widetilde{\nabla}$. If the vector field ξ, associated with the solitons is of the form grad(f), where f is a smooth function. Then the Laplacian equation satisfied by f is given:

$$\begin{aligned}\Delta(f) = &2n - 2\rho r + 2pa(n-1) + 2p(n-1) - 2\Lambda n - 2np(n-2) + n[r-(n-1)(a+1)]q + \\ &q(n-2)n^2 + q(2n-3)n - a^2 qn + 2\mu - 2p(2n-3) + 2pa^2.\end{aligned} \tag{37}$$

Proof: If $\xi = grad(f)$, where $grad(f)$ is the smooth function f, then from (36) we obtain,

$$div\, f = \triangle(f).$$

As an application, we obtain the following results for $*-\eta$-Ricci solitons, *-η-Yamabe solitons, and *-η-Einstein solitons($p = 1, q = 0, p = 0, q = 1$ and $p = 1, q = -1$)[3–5]

Theorem 4.

Let an n-dimensional LP-Sasakian manifold admits $a *-\eta$-Ricci solitons with respect to a semi- symmetric non-metric connection $\widetilde{\nabla}$. If the vector field ξ, associated with the soliton is of the form grad(f), where f is a smooth function. Then the Laplacian equation satisfied by f becomes:

$$\triangle f = 2n - 2r + 2a(n-1) + 2(n-1) - 2\Lambda n - 2n(n-2) + 2\mu - 2(2n-3) + 2a^2. \tag{38}$$

Theorem 5

Let an n-dimensional LP-Sasakian manifold admits a $*-\eta$-Yamabe solitons with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$.If the vector field ξ, associated with the soliton is of the form grad(f), where f is a smooth function. Then the Laplacian equation satisfied by f becomes

$$\triangle f = 2n - 2\Lambda n + 2\mu \tag{39}$$

Theorem 6

Let an n- dimensional LP-Sasakian manifold admits a $*-\eta$-Einstein solitons with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$.If the vector field ξ, associated with the solitons is of the form grad(f), where f is a smooth function. Then the Laplacian equation satisfied by f becomes:

$$\begin{aligned}\triangle f = 2n - 2pr + 2pa(n-1) + 2p(n-1) - 2\Lambda n - 2np(n-2) - n[r-(n-1)(a+1)]q+ \\ q(n-2)n^2 + q(2n-3)n - a^2qn + 2\mu - 2p(2n-3) + 2pa^2.\end{aligned} \tag{40}$$

Definition3: A function $f : M^n \rightarrow R$ is said to be harmonic if $\triangle f = 0$, where $\triangle$ is the Laplacian operator on M^n [12].

Therefore, considering the fact that the vector field ξ is a gradient of a harmonic function f, then from theorem 4 ,we turn up the following conclusion:

Theorem 7

Let an n-dimensional LP-Sasakian manifold M^n admits a $*-\eta$-Ricci-Yamabe soliton with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$ and the vector field ξ, associated with the soliton is of the form grad(f), if f is expanding, steady and shrinking according as

$$\frac{q}{2}[r - na - n + a + n^2 - 2 - a^2] > -1 + \frac{1}{n}pr - pa(n-1) - p(n-1) - \mu - p(2n-3) - pa^2]. \tag{41}$$

$$\frac{q}{2}(r - na - n + a + n^2 - 2 - a^2] = -1 + \frac{1}{n}[pr - pa(n-1) - p(n-1) - \mu - p(2n-3) - pa^2]. \tag{42}$$

$$\frac{q}{2}[r - na - n + a + n^2 - 2 - a^2] < -1 + \frac{1}{n}[pr - pa(n-1) - p(n-1) - \mu - p(2n-3) - pa^2]. \tag{43}$$

Theorem 8

Let an n-dimensional LP-Sasakian manifold M^n admits a $*-\eta$- Ricci soliton with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$,and the vector field ξ, associated with the soliton is of the form grad(f), if f is a harmonic function on M^n. Then the $*-\eta$-Ricci soliton is expanding

$$3 - n + \frac{n-1}{n}[a+1] + \frac{1}{n}[a^2 - 2n + 3] > 0 \tag{44}$$

Theorem 9

Let an n-dimensional LP-Sasakian manifold M^n admits a $*-\eta$- Yamabe soliton with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$,and the vector field ξ, associated with the soliton is of the form grad(f), if f is a harmonic function on M^n. Then the $*-\eta$-Ricci soliton is expanding, steady and shrinking,

$$Expanding\ \Lambda > 0,\ 1+\frac{\mu}{n} > 0 \tag{45}$$

Steady $\Lambda = 0$,

$$1+\frac{\mu}{n} = 0 \tag{46}$$

Shrinking $\Lambda < 0$,

$$1+\frac{\mu}{} < 0 \tag{47}$$

eor

$\mu < -n$

Theorem 10

Let an n-dimensional LP-Sasakian manifold M^n admits a $*-\eta$ - Einstein soliton with respect to a semi-symmetric non-metric connection $\widetilde{\nabla}$,and the vector field ξ, associated with the soliton is of the form grad(f), If *f* is a harmonic function on M^n. hen the $*-\eta$ -Ricci soliton is expanding, steady and shrinking,

$$\mu > -n+r+[(n-1)(a+1)]-n(n-2]-\frac{n}{2}[r-(n-1)(a+1)]-\frac{(n-2)n^2}{2}-n(2n-3)[\frac{1}{2}+\frac{1}{n}]+na^2[\frac{1}{2}+\frac{1}{n}]. \tag{48}$$

$$\mu = -n+r+[(n-1)(a+1)]-n(n-2]-\frac{n}{2}[r-(n-1)(a+1)]-\frac{(n-2)n^2}{2}-n(2n-3)[1/2+1/n]+na^2[\frac{1}{2}+\frac{1}{n}]. \tag{49}$$

$$\mu < -n+r+[(n-1)(a+1)]-n(n-2]-\frac{n}{2}[r-(n-1)(a+1)]-\frac{(n-2)n^2}{2}-n(2n-3)\left(\frac{1}{2}+\frac{1}{n}\right]+na^2\left(\frac{1}{2}+\frac{1}{n}\right]. \tag{50}$$

Definition 4.

A vector field V is said to be a conformal Killing vector field iff the following relation holds:

$$(\pounds_V g)(X_2, X_3) = 2\theta\, g(X_2, X_3), \tag{51}$$

where θ is some function of the co-ordinates (conformal scalar). Moreover, if θ is not constant the conformal Killing vector field V is said to be proper .Also when θ is constant, V is called homothetic vector field and when the constant θ becomes non zero, V is said to be proper homothetic vector field. If $\theta = 0$, in the above equation, then V is called Killing vector field.

Theorem 11

Let an n-dimensional LP-Sasakian manifold admit a $*-\eta$ -Ricci-Yamabe soliton $(g, V, \Lambda, \mu, p, q)$ with respect to a semi-symmetric non-metric connection$\widetilde{\nabla}$. If V is a conformal Killing vector field, then the following equation holds:

$$\left(2\theta+2\Lambda+2\mu-2\eta(V)+6p(n-1)+qn(5n+a)-q(4+a-r)-q\left(n^2-a^2\right)-4\right]\eta(Y)=0 \tag{52}$$

Proof: Let us consider an n-dimensional LP-Sasakian manifold M^n, admitting a $*-\eta$-Ricci-Yamabe soliton, where $\xi = V$. Then we have,

$$\begin{aligned}&(\pounds_V g)(X_2, X_3)+2pS(X_2, X_3)-2pa(X_2,\phi X_3)+[2p(n-2)+2\Lambda-qr-qn(n-2)+\\&q(2n-3)-qa^2]g(X_2, X_3)+[2\mu-2p(2n-3)]\eta(X_2)\eta(X_3)=0\end{aligned} \tag{53}$$

Now if an n-dimensional LP-Sasakian manifold M^n admits a $*-\eta$-RicciYamabe soliton $(g, V, \Lambda, \mu, p, q)$ with respect to the semi-symmetric non-metric connection $\widetilde{\nabla}$, then we have,

$$\begin{aligned}&({}^{'}\pounds_V g)(X_2, X_3)+2pS'(X_2, X_3)-2pa(X_2, \phi X_3)+[2p(n-2)+2\Lambda-qr'-qn(n-2)\\&+q(2-3)-qa^2]g(X_2, X_3)+[2\mu-2p(2n-3)]\eta(X_2)\eta(X_3)=0.\end{aligned} \tag{54}$$

The above equation becomes:

$$\begin{aligned}&({}^{'}_{v}g)(X_2,X_3)+2\eta(V)g(X_2,X_3)+2p[S(X_2,X_3)-(n-1)g(\phi X_2,X_3)+\\&-1)\eta(X_2)\eta(X_2-2pag(X_2,\phi X_3)+[2p(n-2)+2\Lambda-q(r-(n-1)(a+1))-\\&qn(n-2)+q(2n-3)-qa^2]g(X_2,X_3)+[2\mu-2p(2n-3)]\eta(X_2)\eta(X_3)=0.)\end{aligned} \tag{55}$$

On taking $Z = \xi$ on (55), we get the required equation (52).

4 Conclusion

Some results of $*-\eta$-Ricci-Yamabe soliton have been studied in LP-Sasakian manifold which admits semi-symmetric non-metric connection. Among others, LP-Sasakian manifold admitting semi-symmetric non metric connection associated with the vector field ξ of the form grad (f), where f is a harmonic function, the $*-\eta$-Ricci-Yamabe soliton is expanding ,steady and shrinking according as

1. $\frac{q}{2}[r- na - n + a+n^2-2-a^2] > -1 + \frac{1}{n}[pr- pa(n-1)-p(n-1)- \mu - p(2n-3)-pa^2]$
2. $\frac{q}{2}[r- na - n +a +n^2-2-a^2] = -1 +\frac{1}{n}[pr - pa(n-1)-p(n-1)-\mu - p(2n-3)-pa^2]$
3. $\frac{q}{2}[r- na - n +a +n^2-2-a^2] < -1 +\frac{1}{n}[pr - pa(n-1)-p(n-1)-\mu - p(2n-3)-pa^2]$

In addition to this, in a LP-Sasakian manifold admitting *-η-Ricci-Yamabe soliton with respect to semi-symmetric non metric connection and the vector field ξ, associated with the soliton is of the form grad(f), where f is a harmonic function, then *-η-Ricci-Yamabe soliton is expanding.

5 Declaration

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