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RESEARCH ARTICLE

Proper d-Lucky Labeling of Corona Products of Certain Graphs

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Abstract

Objectives: The objective of this paper is to find exact proper d-lucky number for corona product graphs of certain graphs. **Methods:** The results are proved by method of elaboration and construction. **Findings:** Exact proper d-lucky numbers are obtained for corona products: $P_n \odot C_m$, $P_n \odot K_m$, $C_n \odot K_n$ and $K_n \odot K_n$. **Novelty:** Proper d-lucky labeling for the corona product of $P_n \odot C_m$, $P_n \odot K_m$, $C_n \odot K_n$ and $K_n \odot K_n$ are not studied by any other authors, thus the proper d-lucky numbers for the above graphs are new findings.

Keywords: Lucky labeling; Proper labeling; d-lucky labeling; Proper d-lucky labeling; Corona product

1 Introduction

A lot of research work is under progress in the area of graph labeling and in its variants. Lucky and Proper Lucky Labeling of Quadrilateral Snake Graphs were studied by T. V. Sateesh Kumar et al[1]. d- Lucky labeling was introduced by Mirka Miller et al[2]. A lower bound and several exact results on d-lucky number were found by Sandi Klavzar et al[3]. Some work on d-lucky labeling of Honeycomb network were done by A. Sahayamary et al[4]. Cordial labeling of certain corona product graphs was obtained by Elrokh, A. et al[5]. In this paper proper d-lucky number is defined and new notation for the same is introduced and proper d-lucky number for corona products of certain graphs are obtained.

2 Proper d-lucky labeling for Corona Product of $P_n \odot C_m, P_n \odot K_m, C_n \odot K_n$ and $K_n \odot K_n$

For a vertex u in a graph G, let $N(u) = \{v \in V(G)/uv \in E(G)$. Let $l\ : V(G) \to \{1, 2, \ldots, k\}$ be a labeling of vertices of a graph G by positive integers. Define $C(u) = \sum_{v \in N(u)} l(v) + d(u)$, where $d(u)$ denotes the degree of u. Define a labeling l as d-lucky if $C(u) \neq C(v)$, for every pair of adjacent vertices u and v in G. The d-lucky number of a graph G, denoted by $n_{dl}(G)$, is the least positive k such that G has a d-lucky labeling with $\{1, 2, \ldots, k\}$ as the set of labels[2]. A d-lucky labeling is said to be proper d-lucky labeling if for every pair of adjacent vertices u *and* v in G, $u \neq v$, and is denoted by $\eta_{pdl}(G)$.

The corona product of G and H is the graph $G \odot H$ obtained by taking one copy of G, called the center graph, $(V(G)|$ copies of H, called the outer graph, and making the ith vertex of G adjacent to every vertex of the i^{th} copy of H, where $1 \leq i \leq (V(G)|$. We take first corona product of path P_n with cycle graph C_n and P_n with complete graph K_n as $C_n \odot K_n$ and $K_n \odot K_n$ then compute the proper $d-$lucky labeling for them and obtain the proper d-lucky number for the same.

Theorem 1:

Corona product of $P_n \odot C_m$ admits proper d-lucky labeling and $\eta_{pdl}\left(P_n \odot C_m\right) = \begin{cases} 3, & \text{when } m \text{ is even} \\ 5, & \text{when } m \text{ is odd} \end{cases}$.

Proof:

The theorem is proved by construction. There are two cases.

Case 1: When m is even

Label the base vertices of path P_n with $1,2$ alternately.

If vertices of C_m is adjacent to a vertex which has label as 1 in the path, then, label all other vertices of C_n with $2,3$ alternately, if it is adjacent to vertex with label as 2 then label all other vertices of C_n with $1,3$ alternately.

The neighborhood sums $s(u)$'s and $c(u)^{'}s$ of C_m are calculated as follows:

The vertices with label as 2 have $s(u) = 7$ and $c(u) = 10$. The vertices with label as 3 which are adjacent to path vertex with label as 1 then, $s(u) = 5$ and $c(u) = 8$. The vertices with label as 1 have $s(u) = 8$ and $c(u) = 11$. The vertices with label as 3 which are adjacent to vertex with label as 2 in the path have $s(u) = 4$ and $c(u) = 7$.

In path P_n the vertices with label as 1 have $s(u) = \frac{5m+8}{2}$ and $c(u) = \frac{5m+20}{2}$ except the end vertices have, $s(u) = \frac{5m+4}{2}$ and $c(u) = \frac{7m+6}{2}$. In path the vertices with label as 2 have

$s(u) = 2m+2$ and $c(u) = 3m+4$, except the end vertex has $s(u) = 2m+1$ and $c(u) = 3m+2$.

Case 2: When m is odd

Label the vertices of base path P_n with $4,5$ alternately. Label the vertices of C_m if they are adjacent to the path vertex labeled as 4 then, with $1,3$ alternately and last vertex with 2. If they are adjacent to path vertex having label as 5 then label all other vertices as $1,3$ alternately and the last vertex as 2. (for illustration see Figure 1)

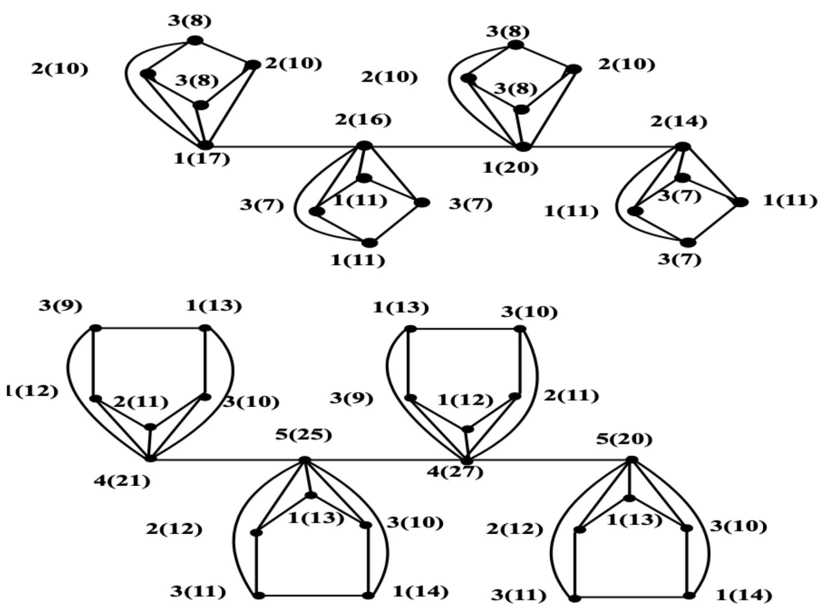

Fig 1. Proper d-lucky labeling of $P_4 \odot C_4$ *and* $P_4 \odot C_5$

The neighborhood sums $s(u)'s$ and $c(u)'s$ for C_m are calculated as follows:

The vertices which are adjacent to path vertex labeled as 4 or labeled as 1 have $s(u) = 9$ *or*10 and $c(u) = 12$ *or*13. The vertices labeled as 3 have $s(u) = 6$ *or*7 *and* $c(u) = 9$ *or* 10. The vertices labeled as 2 have $s(u) = 8$ *and* $c(u) = 11$.

If the vertices of C_m are adjacent to path vertex labeled as 5 then, the vertices in C_n labeled as 1 have $s(u) = 10$ *or* 11 and $c(u) = 13$ *or* 14. The vertices labeled as 3 have $s(u) = 7$ *or* 8 and $c(u) = 10$ *or* 11. the vertices labeled as 2 have $s(u) = 9$ and $c(u) = 11$.

In the path, vertices labeled as 4 have $s(u) = 2m+10$ and $c(u) = 3m+12$, except the end vertex which has $s(u) = 2m+5$ and $c(u) = 3m+6$.

The vertices labeled as 5 in the path have $s(u) = 2m+8$ and $c(u) = 3m+10$, except the end vertex which has $s(u) = 2m+4$ and $c(u) = 3m+5$.

It is observed that no two adjacent vertices have the same $c(u)'s$.

Hence Corona product of $P_n \odot C_m$ admits proper d-lucky labeling and $-$

Theorem 2 :

Corona product of $P_n \odot K_m$ admits proper d-lucky labeling and $n_{pdl}(P_n \odot K_m) = m+1$

Proof:

Label the vertices of the base path with $1,2,3$ in cyclic order. Vertices of K_m if they are adjacent to path vertex with label as 1 then label all other vertices of K_m with $2,3,4,\ldots,(m+1)$. If they are adjacent to vertex with label as 2 then label them with $3,4,5,\ldots,(m+1)$, 1, and if they are adjacent to vertex with label as 3 then label them with $4,5,6,\ldots,(m+1)$, 2, 1 etc.

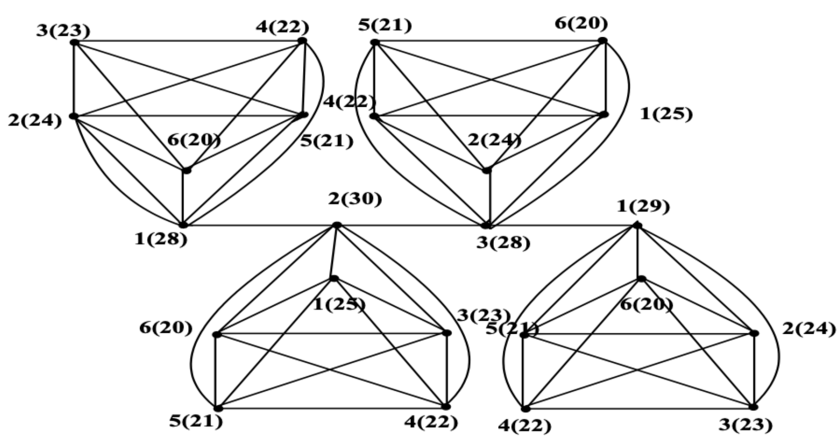

Fig 2. 2 proper d-lucky labeling of $P_4 \odot K_5$

The neighborhood sums $s(u)^{'}s$ and $c(u)'s$ for K_m are calculated as follows:

$$for\ i = 1,2,3,\ldots,,(m+1)$$

$$s(u_i) = \frac{(m+1)(m+2)-2i}{2}$$

$$c(u_i) = \frac{m^2+5m-2(i-1)}{2}$$

The neighborhood sums $s(u)'s$ and $c(u)'s$ for path vertices are calculated as follows:

The vertices with label as 1 have $s(u) = \frac{(m+1)(m+2)+8}{2}$

$c(u) = \frac{m^2+5m+14}{2}$, except the end vertex which is adjacent to 2 has $s(u) = \frac{(m+1)(m+2)+2}{2}$, $c(u) = \frac{m^2+5m+6}{2}$ and if the vertex is adjacent to 3 then $s(u) = \frac{(m+1)(m+2)+4}{2}$ and $c(u) = \frac{m^2+5m+8}{2}$.

The path vertices with label as 2 have $s(u) = \frac{(m+1)(m+2)+4}{2}$ and $c(u) = \frac{m^2+5m+10}{2}$ except the end vertex which has $s(u) = \frac{(m+1)(m+2)-2}{2}$ and $c(u) = \frac{m^2+5m+2}{2}$.

The path vertex with label as 3 has $(u) = \frac{(m+1)(m+2)}{2}$, $c(u) = \frac{m^2+5m+6}{2}$, except the end vertex , which has $s(u) = \frac{(m+1)(m+2)-2}{2}$ *and* $c(u) = \frac{m^2+5m+2}{2}$.

It is seen that no two adjacent vertices have the same $c(u)'s$.

Hence Corona product of $P_n \odot K_m$ admits proper d-lucky labeling and $n_{pdl}(P_n \odot K_m) = m+1$.

Theorem 3 :

The corona product of $C_n \odot K_n$ admits proper d-lucky labeling and $n_{pdl}(C_n \odot K_n) = n+1$.

Proof: label the base vertices of C_n with $1,2,3,\ldots,n$.

Label the vertices of K_n which are adjacent to vertex with labeled as 1 of C_n as $2,3,4,\ldots,(n+1)$. If the vertices are adjacent to vertex with label as 2 of base C_n then label the vertices of K_n as $3,4,5,\ldots,(n+1),1$ and so on till all the vertices of $K_n's$ are labelled.

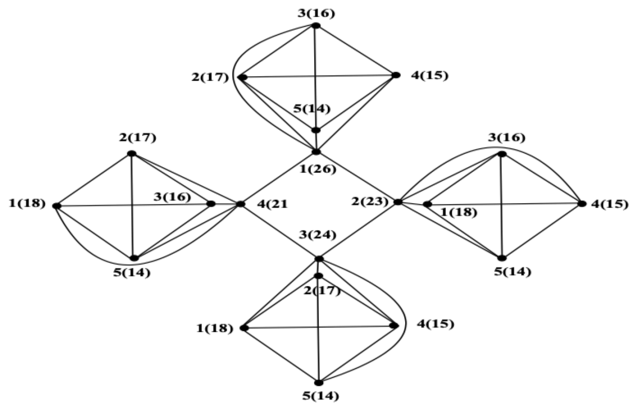

Fig 3. Proper d-lucky labeling of $C_4 \odot K_4$

The neighborhood sums $s(u)'s$ and $c(u)'s$ of $K_n's$ are calculated as follows:

$$for\ i = 1,2,3,\ldots(n+1)$$

$s(u_i) = \frac{(n+1)(n+2)-2i}{2}$ and $c(u_i) = \frac{n^2+5n-2(i-1)}{2}$.

The neighborhood sums $s(u)'s$ and $c(u)'s$ for the base vertices of $C_n's$ are calculated as follows:

The base vertices with label as 1 have $s(u) = \frac{n^2+5n+4}{2}$ and $c(u) = \frac{n^2+7n+8}{2}$ and the vertices with label as n have, $s(u) = \frac{(n+1)(n+2)}{2}$ and $c(u) = \frac{n^2+5n+6}{2}$.

For all other vertices of $C_n for\ i = 2,3,4,\ldots(n-1)$

$s(u_i) = \frac{(n+1)(n+2)+2i}{2}$ and $c(u_i) = \frac{n^2+5n+2(3+i)}{2}$.

It is observed that no two adjacent vertices have the same $c(u)'s$.

Hence the corona product of $C_n \odot K_n$ admits proper d-lucky labeling and $n_{pdl}(C_n \odot K_n) = n+1$.

Theorem 4:

The corona product of $K_n \odot K_n$ admits proper d-lucky labeling and $n_{pdl}(K_n \odot K_n) = n+1$.

Proof: Label the base vertices of K_n with $1,2,3,\ldots,n$.

The outer K_n vertices if they are adjacent to vertex with label as 1 then label them as $2,3,4,\ldots,(n+1)$. If they are adjacent to vertex with labeled as 2 then label them as $3,4,5,\ldots,(n+1),1$ and so on till all the outer vertices of $K_n's$ are labeled.

The neighborhood sums $s(u)$ and $c(u)'s$ for outer $K_n's$ are calculated as follows:

$$for\ i = 1,2,3,\ldots,\ (n+1)$$

$$s(u_i) = \frac{(n+1)(n+2)-2i}{2}$$

$$c(u_i) = \frac{n^2+5n-2(i-1)}{2}$$

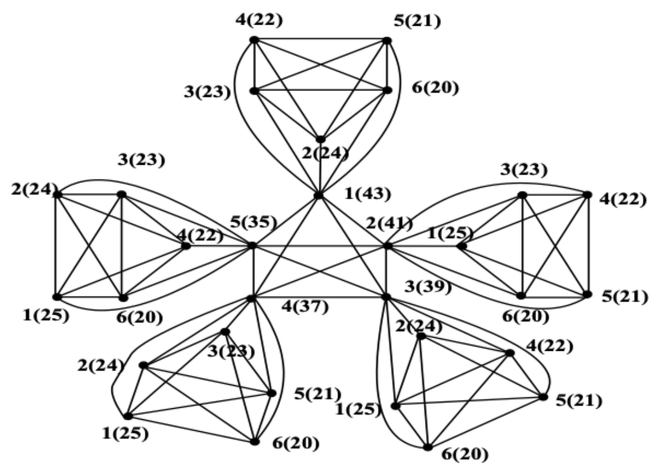

Fig 4. Proper d-lucky labeling of $K_5 \odot K_5$

For base K_n vertices the neighborhood sums $s(u)$ and $c(u)'s$ are given by

$$for\ i = 1, 2, 3, \ldots, n.$$

$$s(u_i) = (n+1)^2 - 2i$$

$$c(u_i) = n^2 + 4n - 2i$$

It is observed that no two adjacent vertices have the same $c(u)'s$.

Hence the corona product of $K_n \odot K_n$ admits proper d-lucky labeling and
$n_{pdl}(K_n \odot K_n) = n+1$.

3 Conclusion

In this paper proper d-lucky number for corona product graphs of $P_n \odot C_m$, $P_n \odot K_m$, $C_n \odot K_n$ and $K_n \odot K_n$ are computed and found as $\eta_{pdl}(P_n \odot C_m) = \begin{cases} 3, \text{ when } m \text{ is even} \\ 5, \text{ when } m \text{ is odd} \end{cases}$,

$n_{pdl}(P_n \odot K_m) = m+1$,
$n_{pdl}(C_n \odot K_n) = n+1$, and
$n_{pdl}(K_n \odot K_n) = n+1$.

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