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A New Ratio and Product Type Estimator for Mean Under Stratified Random Sampling

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Abstract

Objectives: The present study deals with the formation of a modified ratio and product type estimator for estimating finite population mean using auxiliary variables under stratified random sampling. **Methods:** The expression for the bias and mean square error (MSE) of the suggested estimator was computed upto the first degree of approximation by using the ratio and product method of estimation. We used the data of Area and Production of Horticulture Crops, India obtained from the official website of National horticulture board. The another dataset was taken from a project that was undertaken by Department of Animal Husbandry of a state government, Punjab for comparison. An analysis was conducted to compare the suggested estimator with Chami's and other existing estimators. **Findings:** The optimum value of the proposed estimator has been obtained. The suggested estimator is reported to be more effective than the conventional mean, ratio, and product type estimators. We have used two data sets obtained through a stratified random sampling strategy to evaluate the effectiveness of the estimators discussed. The percent relative efficiency of envisaged estimator was obtained from population I is 130.05 and 141.56, from population II is 137.31 and 146.8 that is greater than existing estimators. The numerical illustration also showed that the new estimator is more efficient than simple mean, ratio, and product type estimators. **Novelty:** We proposed a new estimator that has practical implications. The comparative analysis has been done to set up the condition for which the suggested estimators are more efficient than other estimator with novelty.

Keywords: Estimator; Stratified; Random sampling; Mean Square Error; Bias

1 Introduction

Stratification is an approach used in contemporary surveys to increase the accuracy of estimates. Samples are often picked at random from each stratum in a stratified manner, in order to preserve homogeneity within each stratum, the entire population is divided into a specified number of strata. Sharma and Kumar[1] suggested a ratio-type estimator that uses many auxiliary variables to calculate the population mean of the variable under study. Tiwari et al.[2] provided an extensive collection of estimators for estimating the study variable's population mean in stratified random sampling. These estimators use auxiliary variables, particularly when the auxiliary variable's population mean is available.

Using a known coefficient of variation for the auxiliary variable, Garg and Pachori[3] suggested a new calibration estimator with a set of new calibration constraints for predicting the population mean in the stratified random sample. Later numerous authors, such as Zaman and Kadilar[4]; Iftikhar et al.[5]; Muneer et al.[6]; Husain et al.[7]; Cekim and Kadilar,[8]; Zaman[9]; Singh and Nigam[10] and Bhushan et al.[11] Kumar and Tiwari et al.[12]; Babatunde et al.[13]; Singh et al.[14]; Ahmad et al.[15] and Bhushan et al.[16] employed several well-known auxiliary variable parameters, including the variance and kurtosis coefficients, as well as a measure of the correlation in stratified random sampling between the auxiliary variable and the study variable. Koc and Koc[17] examined a novel class of estimators for finite population mean in stratified random sampling that is based on quantile regression ratios. The study's findings led to the conclusion that quantile regression allows us to produce more consistent results in data on air pollution, even in cases where asymmetrical distributions and outliers are frequent. This improves the entire forecasting process in practice by producing predictions that are stronger and more accurate. An effective class of estimators for finite population mean was also created by Singh et al.[18] utilizing auxiliary features in stratified random sampling. Their findings demonstrated that, under ideal circumstances, the proposed classes of estimators outperform other current approaches with the lowest MSE. Bhushan et al.[19] suggested some efficient combined and separate estimator classes for estimating population mean $\overline{Y}$ under stratified simple random sampling. They also carried out a simulation investigation over some synthetic symmetric and asymmetric populations, extending the theoretical conclusions.

For the most part, this study aims to perk up estimates of the mean of finite populations by collecting and analyzing data on an auxiliary variable for use in stratified random sampling.

The main point of the manuscript includes; Section 1 contains "Some existing estimators" that we studied from the literature, as well as notations and procedures. The "Proposed estimator" is given in section 2 which presents a new estimator based on a stratified random sampling strategy. Efficiency comparison, Numerical comparison, and optimum variance are discussed in section 3. In the last section "Conclusion" a numerical evaluation is offered to highlight the paper's contribution.

1.1 Notations

Consider a finite population $U = \left(U_1, U_2, \ldots, \ldots, U_N\right\}$ that is divided into ' r ' strata and has N unique and recognizable units. Let y and x be study auxiliary variables with values y_{ik} and x_{ik}, For the i, k_{th} unit $(k = 1, 2, \ldots, N \& I = 1, 2, \ldots, r)$ in the i_{th} stratum of N_i units, $\sum_{i=1}^{r} N_i = N$. For simplicity, let's say ni is assumed to be the size of the sample drawn from the ith stratum. We will achieve this by employing a basic random sample without replacement sampling strategy to ensure that $\sum_{i=1}^{r} n_i = n$.

Let's label different means for study variate and auxiliary variate as:

$N =$ Population size

$n =$ Sample size

$r =$ Number of strata

$N_i =$ Size of i^{th} stratum

Then $N = \sum_{i=1}^{r} N_i$

$n_i =$ Size of sample taken from i^{th} stratum

Then $n = \sum_{i=1}^{r} N_i$

$\overline{Y}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} y_{i,k}$ Stratum mean for study variate Y

$$\overline{Y}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} N_i \bar{Y}_i$$

Population mean of the study variate Y

$= \sum_{i=1}^{r} w_i \overline{Y_i}$

$W_i = \frac{N_i}{N}$ Stratum weight for i^{th} stratum

$\bar{Y}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} N_i \bar{Y}_i$ Population mean of the study variate for ith stratum.

$\bar{y} = \frac{1}{n} \sum_{i=1}^{r} n_i \bar{y}_i$ Sample mean of study variate Y.

$\bar{y}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} n_i \bar{y}_i$ Sample mean ith stratum of study variate Y.

1.2 Existing Estimators

The standard simple mean, ratio, and product type stratification estimators are as follows:

1.2.1 Simple Mean Estimator

The population mean $\overline{Y}$ in stratified sampling sample mean $\overline{y}_{st}$ is

$$\bar{y}_{st} = \sum_{i=1}^{r} w_i \bar{y}_i$$

and MSE is given by

$$MSE\left(\bar{y}_{st}\right) = \sum_{i=1}^{r} w_i^2 \bar{Y}_i^2 \frac{(1-f_i)}{n_i} C_{iy}^2$$

1.2.2 Ratio Estimator

The ratio estimator is provided below for calculating the population mean $\overline{Y}$ of the study variable Y :

$$\bar{y}_{rst} = \sum_{i=1}^{r} w_i \bar{y}_i \left(\frac{\bar{x}_i}{\bar{x}_i}\right) \tag{1}$$

MSE upto first-degree approximation is specified as:

$$MSE\left(\bar{y}_{rst}\right) = \sum_{i=1}^{r} w_i^2 \bar{Y}_i^2 \frac{(1-f_i)}{n_i} \left(C_{iy}^2 + C_{ix}^2 - 2C_{ixy}\right) \tag{2}$$

1.2.3 Product Estimator

The product estimator is presented below for calculating the population mean $\overline{Y}$ of study variable y:

$$\bar{y}_{pst} = \sum_{i=1}^{r} w_i \bar{y}_i \left(\bar{x}_i \bar{x}_i\right) \tag{3}$$

MSE upto first-degree approximation is given by:

$$MSE\left(\bar{y}_{pst}\right) = \sum_{i=1}^{r} w_i^2 \bar{Y}_i^2 \frac{(1-f_i)}{n_i} \left(C_{iy}^2 + C_{ix}^2 + 2C_{ixy}\right) \tag{4}$$

Chami et al. [20] a two-parameter ratio-product-ratio estimator using auxiliary information

$$\bar{y}_{\alpha,\beta} = \sum_{i=1}^{r} w_i \left[\alpha \left\{\frac{(1-\beta)\bar{x}_i + \beta \bar{x}_i}{\beta \bar{x}_i + (1-\beta)\bar{X}_i}\right\} \bar{y}_i + (1-\alpha) \left\{\frac{\beta \bar{x}_i + (1-\beta)\bar{x}_i}{(1-\beta)\bar{x}_i + \beta \bar{x}_i}\right\} \bar{y}_i\right]$$

MSE up to first-degree approximation is given by:

$$MSE\left(\bar{y}_{pst}\right) = \sum_{i=1}^{r} w_i^2 \bar{Y}_i^2 \frac{(1-f_i)}{n_i} \left[C_{iy}^2 + C_{ix}^2 (1-2\alpha)(1-2\beta)\{(1-2\alpha)(1-2\beta) - 2C\}\right] \tag{5}$$

2 Modified Ratio and Product Estimator in Stratified Sampling

2.1 Suggested Estimator in Stratified Sampling

Exponential estimators have been provided to calculate the populations mean by stratified random sampling, and they are as follows:

$$\bar{y}_{st,p\exp} = \sum_{i=1}^{r} w_i \left[m \exp\left\{\frac{(1-n)\bar{x}_i + (n-1)\bar{X}_i}{\bar{X}_i + \bar{x}_i}\right\} \bar{y}_i + (1-m) \exp\left\{\frac{(n-1)\bar{x}_i + (1-n)\bar{X}_i}{\bar{X}_i + \bar{x}_i}\right\} \bar{y}_i\right] \tag{6}$$

$$\bar{y}_{st,\text{Rexp}} = \sum_{i=1}^{r} w_i \left[m \exp\left\{ \frac{(1-n)\bar{X}_i + (n-1)\bar{x}_i}{\bar{X}_i + \bar{x}_i} \right\} \bar{y}_i + (1-m)\exp\left\{ \frac{(1-n)\bar{X}_i + (n-1)\bar{x}_i}{\bar{X}_i + \bar{x}_i} \right\} \bar{y}_i \right] \quad (7)$$

Expressing $\bar{y}_{st,pexp}$ for $\bar{x}_i$ and $\bar{y}_i$ in terms of e_o and e_1, we obtained:

$$\bar{y}_{st,pexp} = \sum_{i=1}^{r} w_i \left[mexp\left\{ \tfrac{(1-n)\overline{X}_i(1+e_1)+(n-1)\overline{X}_i}{\overline{X}_i+\overline{X}_i(1+e_1)} \right\} \overline{Y}_i(1+e_0) + (1-m)exp\left\{ \tfrac{(n-1)\overline{X}_i(1+e_1)+(1-n)\overline{X}_i}{\overline{X}_i+\overline{X}_i(1+e_1)} \right\} \overline{Y}_i(1-e_0) \right]$$
$$= \sum_{i=1}^{r} w_i \left[\overline{Y}_i \left(1 + e_0 - \tfrac{e_1}{2} + \tfrac{3}{8}e_1^2 + \tfrac{ne_1}{2} - \tfrac{ne_1^2}{2} + \tfrac{n^2e_1^2}{8} + \tfrac{ne_0e_1}{2} - \tfrac{e_0e_1}{2} + me_1 - \tfrac{1}{2}me_1^2 - mne_1 + me_0e_1 - mne_0e_1 + \tfrac{mne_1^2}{2} \right) \right]$$

$$Bias\left(\bar{y}_{st,pexp}\right) = E\left(\bar{y}_{st,pexp}\right] - \overline{Y}$$

$$= E\left[\sum_{i=1}^{r} w_i \left[\bar{Y}_i m \left(1 + e_0 - \frac{1}{2}(1-n)e_1 + \frac{1}{2}(1-n)e_0e_1 + \frac{1}{8}\left(n^2-1\right)e_1^2 \right) \right.\right.$$
$$\left.\left. + \bar{Y}_i(1-m)\left(1 + e_0 + \frac{1}{2}(n-1)e_1 + \frac{1}{2}(n-1)e_0e_1 - \frac{n}{2}e_1^2 + \frac{3}{8}e_1^2 + \frac{1}{8}n^2e_1^2 \right) \right] \right] - \bar{Y}$$

$$Bias\left(\overline{Y}_{st,pexp}\right) = \sum_{i=1}^{r} w_i \frac{(1-f_i)}{n_i} \left[\overline{Y}_i \left\{ \begin{array}{c} \frac{n}{2}c_{ijx} + \frac{3}{8}c_{ix}^2 - \frac{n}{2}c_{ix}^2 + \frac{n^2}{8}c_{ix}^2 - \frac{1}{2}mc_{ix}^2 \\ + \frac{mn}{2}c_{ix}^2 - \frac{1}{2}c_{ijx} + mc_{ijx} - mnc_{iyx} \end{array} \right\} \right] \quad (8)$$

M.S.E. $\left(\bar{y}_{st,pexp}\right) = E\left(\bar{y}_{st,pexp} - \overline{Y}\right]^2$

To the first degree of approximation, the MSE can be expressed as:

$$= \sum_{i=1}^{r} w_i^2 E[\overline{Y}_i \left\{ 1 + e_0 - \frac{e_1}{2} + \frac{3}{8}e_1^2 + \frac{ne_1}{2} - \frac{ne_1^2}{2} + \frac{n^2e_1^2}{8} + \frac{ne_0e_1}{2} - \frac{e_0e_1}{2} \right.$$
$$\left(\left(+ me_1 - \frac{1}{2}me_1^2 - mne_1 + me_0e_1 - mne_0e_1 + \frac{mne_1^2}{2} \right\} - \overline{Y}_i \right]^2$$
$$M.S.E.\left(\overline{Y}_{st,pexp}\right) = \sum_{i=1}^{r} w_i^2 \overline{Y}_i{}^2 \frac{(1-f_i)}{n_i} \left(C_{iy}^2 - 2mnC_{iyx} - mC_{ix}^2 + nC_{iyx} - C_{iyx} + 2mnC_{ix}^2 + 4mC_{iyx} \right)$$
$$\left(+ m^2C_{ix}^2 - 2m^2nC_{ix}^2 + m^2n^2C_{ix}^2 - mn^2C_{ix}^2 + \frac{1}{4}n^2C_{ix}^2 - \frac{1}{2}nC_{ix}^2 + \frac{1}{4}C_{ix}^2 \right) \quad (9)$$

Expressing $\bar{y}_{st,Rexp}$ for $\overline{x_i}$ and $\bar{y}_i$ in terms of e_0 and e_1, we get

$$\bar{y}_{st,rexp} = \sum_{i=1}^{r} w_i \left(mexp\left(\tfrac{(1-n)\overline{X}_i+(n-1)\overline{X}_i(1+e_1)}{\overline{X}_i(2+e_1)} \right\} \overline{Y}_i(1+e_0) + (1-m) \right.$$
$$\left(exp\left(\tfrac{(n-1)\overline{X}_i+(1-n)\overline{X}_i(1+e_1)}{\overline{X}_i+\overline{X}_i(1+e_1)} \right\} \overline{Y}_i(1+e_0) \right]$$
$$= \sum_{i=1}^{r} w_i[\left(\left(\left(\left(\begin{array}{c} 1 + e_0 - mn\frac{e_1^2}{2} - me_1 - \frac{1}{8}e_1^2 + mne_0e_1 - me_0e_1 + \frac{e_1}{2} \right)\right] \\ -n\frac{e_1}{2} + n^2\frac{e_1^2}{8} + \frac{e_0e_1}{8} - \frac{ne_0e_1}{2} + mne_1 + \frac{1}{2}me_1^2 \end{array} \right)\right]$$

$$Bias\left(\bar{y}_{st,Rexp}\right) = E\left[\left(\bar{y}_{st,Rexp}\right)\right] - \overline{Y}$$

$$= E\left(\sum_{i=1}^{r} w_i \left(\bar{Y}_i \left(\begin{array}{c} 1 + e_0 - mn\frac{e_1^2}{2} - me_1 - \frac{1}{8}e_1^2 + mne_0e_1 - me_0e_1 + \frac{e_1}{2} \\ -n\frac{e_1}{2} + n^2\frac{e_1^2}{8} + \frac{e_0e_1}{8} - \frac{ne_0e_1}{2} + mne_1 + \frac{1}{2}me_1^2 \end{array} \right)\right]\right] - \overline{Y}$$

$$\left[Bias\left(\bar{y}_{st,Rexp}\right) = \sum_{i=1}^{r} w_i \left(\frac{1-f_i}{n_i} \right) \left(\begin{array}{c} mnC_{iyx} - \frac{1}{2}mnC_{ix}^2 - \frac{1}{8}C_{ix}^2 - mC_{iyx} + \frac{1}{8}n^2C_{ix}^2 \\ \bar{Y}_i \\ + \frac{1}{2}C_{iyx} - \frac{1}{2}nC_{iyx} + \frac{1}{2}mC_{ix}^2 \end{array} \right) \right] \quad (10)$$

M.S.E. $\left(\overline{y}_{st,Rexp}\right)=E\left[\overline{y}_{\,st,Rexp}-\overline{Y}_i\right]^2$

Up to first-degree approximation, the mean square error is delineated by:

$$=\sum_{i=1}^{r}\ w_i^2E\left(\overline{Y}_i\left(1+e_0-\frac{1}{2}mne_1^2-me_1-\frac{1}{8}e_1^2+mne_0e_1-me_0e_1+\frac{e_1}{2}-n\frac{e_1}{2}\right.\right.$$
$$\left(+\frac{1}{8}n^2e_1^2+\frac{1}{2}e_0e_1-\frac{1}{2}ne_0e_1+mne_1+\frac{1}{2}me_1^2\right\}^2+\overline{Y}_i^2-2\overline{Y}_i\left(\overline{Y}_i\left(1+e_0-\frac{1}{2}mne_1^2-me_1\right)\right.$$
$$\left(\left(\left(+\frac{1}{8}e_1^2+mne_0e_1-me_0e_1+\frac{e_1}{2}-n\frac{e_1}{2}+\frac{1}{8}n^2e_1^2+\frac{1}{2}e_0e_1-\frac{1}{2}ne_0e_1+mne_1+\frac{1}{2}me_1^2\right)\right\}\right] \tag{11}$$
$$M.S.E.\left(\overline{y}_{st,Rexp}\right)=\sum_{i=1}^{r}\ w_i^2\overline{Y}_i^2\left(\frac{1-f_i}{n_i}\right)\left(2mnc_{ix}^2+2mnC_{iyx}-2mC_{iyx}+n^2C_{ix}^2+C_{iyx}-nC_{iyx}+C_{iy}^2\right)$$
$$\left(+m^2C_{ix}^2-\frac{1}{2}nC_{ix}^2-mn^2C_{ix}^2-2m^2nC_{ix}^2+m^2n^2C_{ix}^2+\frac{1}{4}C_{ix}^2-mC_{ix}^2\right)$$

3 Relative Efficiency

i) The given estimator $\overline{y}_{st,pexp}$is more proficient than simple mean $\overline{y_{st}}$

M.S.E. $\left(\overline{y}_{st,pexp}\right)<M.S.E.\left(\overline{y}_{st}\right)$ if $\rho_i<2(n-1)\left(m-\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

ii) The suggested estimator $\overline{y}_{st,pexp}$is more competent than ratio $\overline{y}_{rst}$

M.S.E. $\left(\overline{y}_{st,pexp}\right)<$ M.S.E. $\left(\overline{y}_{rst}\right)$ if $\rho_i>2(1-n)\left(m+\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

iii) The given estimator $\overline{y}_{st,pexp}$is more proficient than the product estimator $\overline{y}_{pst}$

M.S.E. $\left(\overline{y}_{st,pexp}\right)<M.S.E.\left(\overline{y}_{pst}\right)$ if $\rho_i<2(n+1)\left(m+\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

(iv) The suggested estimator $\left(\overline{y}_{st,Rexp}\right)$ is more proficient than the simple mean $\left(\overline{y}_{st}\right)$

M.S.E. $\left(\overline{y}_{st,Rexp}\right)<M.S.E.\left(\overline{y}_{st}\right)$ if $\rho_i>2(n-1)\left(m-\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

v) The suggested estimator $\overline{y}_{st,Rexp}$is more proficient than the ratio $\overline{y}_{rst}$

M.S.E. $\left(\overline{y}_{st,rexp}\right)<M.S.E.\left(\overline{y}_{rst}\right)$ if $\rho_i<2(1-n)\left(m+\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

vi) The proposed estimator $\overline{y}_{st,Rexp}$ is more efficient than the product $\overline{y}_{pst}$

M.S.E. $\left(\overline{y}_{st,Rexp}\right)<M.S.E.\left(\overline{y}_{pst}\right)$ if $\rho_i>2(n+1)\left(m+\frac{1}{2}\right)\frac{C_{ix}}{C_{iy}}$

Optimum Variance

The optimum estimator of the suggested estimator $\overline{y}_{st,pexp}$ is up to first degree approximation for which MSE is minimised respectively. Differentiate expression (9) w.r.t. *m* and *n* gives optimum values

$$m=\frac{\rho_i}{(1-n)}\frac{C_{iy}}{C_{ix}}+\frac{1}{2},n=\frac{-\rho_i}{\left(m-\frac{1}{2}\right)}\frac{C_{iy}}{C_{ix}}+1$$

Thus, the resulting minimum MSE of estimator $\overline{y}_{st,pexp}$ is

$$M.S.E.\left(\overline{y}_{st,pexp}\right)_{min}=\sum_{i=1}^{r}w_i^2\overline{Y}_i^2\left(\frac{1-f_i}{n_i}\right)C_{iy}^2\left(1-\rho_i^2\right)$$

The optimum estimator of the suggested estimator $\overline{y}_{st,Rexp}$ is up to first degree approximation for which MSE is minimised respectively. Differentiate expression (11) w.r.t. *m* and *n* which gives optimum values

$$m=\frac{\rho_i}{(n-1)}\frac{C_{iy}}{C_{ix}}+\frac{1}{2},n=\frac{\rho_i}{\left(m-\frac{1}{2}\right)}\frac{C_{iy}}{C_{ix}}+1$$

Thus, the resulting minimum MSE of estimator $\overline{y}_{st,Rexp}$ is

$$M.S.E.\left(\overline{y}_{st,Rexp}\right)_{min}=\sum_{i=1}^{r}w_i^2\overline{Y}_i^2\left(\frac{1-f_i}{n_i}\right)C_{iy}^2\left(1-\rho_i^2\right)$$

Numerical Comparison

In order to examine how well the aforementioned estimators perform in relation to the mean ratio and product, we take a look at the following data set:

Population I: (Singh and Mangat, 1996) [21]

X = The number of milch cows in 1990

Y = The number of milch cows in 1993

Parameters used in data set are given below:

Table 1.

$N_1 = 7$	$N_2 = 12$	$N_3 = 5$
$n_1 = 3$	$n_2 = 5$	$n_3 = 2$
$W_1 = 0.2916$	$W_2 = 0.5000$	$W_3 = 0.2083$
$X_1 = 15.2857$	$X_2 = 17.2500$	$X_3 = 20.6000$
$Y_1 = 17.4285$	$Y_2 = 20.4166$	$Y_3 = 17.8000$
$C_{x1} = 0.2991$	$C_{x2} = 0.3185$	$C_{x3} = 0.1770$
$C_{y1} = 0.2408$	$C_{y2} = 0.1997$	$C_{y3} = 0.1837$
$\rho = 0.29$	$n = 10$	$N = 24$

Population II (Tailor et al., 2014) [22]

y: Productivity (MT/Hector)

x : Production in ' 000 Tons

Parameters used in data set are given below:

Table 2.

Estimators	Stratum I	Stratum II
n_h	2.00	2.00
n'_h	4.00	4.00
N_h	10.00	10.00
$\bar{Y}_h$	1.70	3.67
$\bar{X}_h$	10.41	289.14
S_{yh}	0.50	1.41
S_{xh}	3.53	111.61
S_{yxh}	1.61	144.88
S_y^2	2.21	

4 Conclusion

Many authors have focused on the utilization of auxiliary information to improve the estimator accuracy. The present study was carried out by creating a novel estimator under SRSWOR. The bias and MSE equations for the recommended estimators have been examined. On a theoretical basis, the proposed estimators MSE were compared to the MSEs of the standard estimator $\bar{y}_{st}$, the ratio estimator $\bar{y}_{rst}$, the product estimator $\bar{y}_{pst}$, and Chami's suggested estimator $\bar{y}_{\alpha st}$, and the conditions for getting minimum MSE were also derived. The optimality of the estimators under consideration is compared theoretically and numerically with other estimators. The relative efficiency of several estimators in relation to $\bar{y}_{st}$ have been estimated and are shown in Table 3.

Under some circumstances, the suggested estimators have been observed to exhibit better performance compared to alternative estimators. The effectiveness of the proposed estimators is compared to that of other estimators using two population data sets. The suggested estimators have been shown to be more efficient numerically in all population sets compared to other estimators, the typical sample mean, ratio, product, and the estimator provided by Chami. As a result, it is advised to employ the presented estimators $\bar{y}_{st,pexp}$ and $\bar{y}_{st,Rexp}$ in a variety of real-world scenarios. To achieve even more efficient estimation results, we will further apply post stratification to our suggested estimator.

Table 3. PRE of various estimators w.r.t simple mean $\bar{y}_{st}$

Estimators	Population I	Population II
$\bar{y}_{st}$	100	100
$\bar{y}_{rst}$	122.25	125.7
$\bar{y}_{pst}$	102.45	101.64
$\bar{y}_{ost}$	128.55	135.4
$\bar{y}_{st,pexp}$	130.05	137.31
$\bar{y}_{st,Rexp}$	141.56	146.8

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