INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 21-10-2023

Accepted: 09-11-2023

Published: 30-12-2023

Citation: Kalaiarasi K, Swathi S (2023) Arithmetic Operations of Trigonal Fuzzy Numbers Using Alpha Cuts: A Flexible Approach to Handling Uncertainty. Indian Journal of Science and Technology 16(47): 4585-4593. https://doi.org/10.17485/IJST/v16i47.2654

* **Corresponding author**.

kkalaiarasi.maths@cauverycollege.ac.in

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Arithmetic Operations of Trigonal Fuzzy Numbers Using Alpha Cuts: A Flexible Approach to Handling Uncertainty

K Kalaiarasi[1,2]*, S Swathi[3]

1 Assistant Professor, PG and Research Department of Mathematics, Cauvery college for Women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirappalli, Tamil Nadu, India
2 D. Sc (Mathematics) Researcher Fellow, Srinivas University, Surathkal, Mangaluru, 574146, Karnataka, India
3 Ph.D Research Scholar, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirappalli, 620018, Tamil Nadu, India

Abstract

Objective: To introduce a new concept in the realm of fuzzy number theory, the ”Trigonal Fuzzy Number” or ”30-gonal Fuzzy Number.” Unlike conventional fuzzy numbers with fixed and rigid membership functions, Trigonal Fuzzy Numbers present a versatile and adaptive framework for representing and managing uncertainty. **Method:** The study delves into the effective utilization of ”Alpha Cuts” as a precise tool for quantifying and navigating uncertainty levels within Trigonal Fuzzy Numbers. Furthermore, it showcases essential arithmetic operations, including addition, subtraction, and multiplication, performed on Trigonal Fuzzy Numbers using Alpha Cuts, thus extending the boundaries of fuzzy arithmetic. **Findings:** This innovative approach demonstrates real-life applications across various domains, emphasizing its practicality and potential to revolutionize decision-making processes under uncertain conditions. The presented framework not only enriches fuzzy number concepts but also serves as a valuable asset for addressing real-world problems characterized by imprecision and ambiguity. **Novelty:** Innovatively, the article demonstrates the real-life applications of this novel approach in various domains, illustrating its practicality and potential for revolutionizing decision-making processes under uncertain conditions. The presented framework is not only conceptually enriching but also offers a valuable asset for addressing real-world problems where imprecision and ambiguity are inherent, thereby bridging the gap between theoretical fuzzy number concepts and practical decision support systems.

Keywords: Trigonal Fuzzy Numbers; Alpha Cuts; Fuzzy Arithmetic; Uncertainty Management; Decision Making

1 Introduction

Uncertainty is an ever-present challenge in real-world problem-solving, demanding effective strategies for informed decision-making. Lotfi A. Zadeh's pioneering work in 1965 introduced fuzzy set theory, a pivotal tool for managing imprecise and uncertain information[1]. It allowed for the gradual assessment of membership in sets, providing a flexible framework to represent and reason with uncertainty. While traditional fuzzy numbers, typically defined by fixed membership functions, have found wide application in quantifying uncertainty, they may fall short in capturing the intricacies of complex scenarios.

In recent years, a series of significant advancements has pushed the boundaries of fuzzy number theory. Notable works, such as Juuso's exploration of nonlinear scaling and fuzzy systems for automation[2], and Pourabdollah, Mendel, and John's innovative defuzzification approach using alpha-cut representations[3], have addressed critical aspects of uncertainty management. Additionally, Abbasi and Allahviranloo's research on fuzzy arithmetic operations with pseudo-hexagonal fuzzy numbers expanded the horizons of system reliability analysis[4]. Ahmad and Cheng's introduction of alpha-cut triangular fuzzy numbers for control charting and process capability assessment further contributed to practical quality control[5].

Additionally, Leandry, Sosoma, and Koloseni's 2022 paper[6] focused on fundamental fuzzy arithmetic operations, specifically utilizing alpha-cuts with Gaussian membership functions. Understanding these mathematical foundations is essential for various applications across different domains. In Gowri and Sandhiya's research in 2022[7], they explored arithmetic operations with octagonal fuzzy numbers, using the "cut method." This work contributes to our understanding of specialized fuzzy number types and how to perform mathematical operations with them.

Kumar, Khepar, Yadav, and their collaborators' systematic review in 2022[8] provides a comprehensive overview of generalized fuzzy numbers and their applications across different fields. This review serves as a valuable resource for researchers interested in the historical development and future potential of fuzzy number theory. Chen, Wang, and Chiu's work in 2023[9] introduced an efficient approach for deriving fuzzy priorities in multi-criterion decision-making, employing alpha-cut operations. This research is vital for addressing practical challenges related to uncertainty in complex decision scenarios, offering a practical methodology for prioritizing alternatives in such contexts.

However, there remains a notable gap in addressing the need for more nuanced and flexible representations of uncertainty. Conventional fuzzy numbers may not sufficiently meet the requirements of intricate applications. This paper bridges this gap by introducing "Trigonal Fuzzy Numbers" or "30-gonal Fuzzy Numbers." Unlike traditional fuzzy numbers, Trigonal Fuzzy Numbers offers a highly versatile framework, characterized by membership functions that span 30 distinct intervals, allowing for smoother and more detailed transitions in membership values.

To address this gap, the study explores the use of "Alpha Cuts" for precise uncertainty quantification, enabling more granular and adaptable handling of uncertainty. Furthermore, the paper demonstrates fundamental arithmetic operations on Trigonal Fuzzy Numbers using Alpha Cuts, a critical advancement for decision support and modeling tasks where uncertainty plays a pivotal role. Through real-life applications in various domains, this research exemplifies the practicality and relevance of the innovative approach. Trigonal Fuzzy Numbers with Alpha Cuts offer a valuable asset for informed decision-making in complex scenarios characterized by ambiguity and imprecision.

The introduction section offers justification for the present work by addressing the limitations of traditional fuzzy number representations and the need for more nuanced and flexible approaches. It establishes the significance of introducing Trigonal Fuzzy Numbers and their utility in addressing complex and ambiguous uncertainty scenarios. This section also sets the stage for the innovative aspects of the research by highlighting the limitations of existing methods and the motivation for exploring new territory.

2 Methodology

Some Preliminary definitions for Fuzzy set, Alpha cut, Fuzzy number, triangular fuzzy number, trapezoidal fuzzy number have been given in this section.

2.1 Fuzzy Sets and Their Membership Functions

A fuzzy set A in a universal set U is characterized by a membership function μA(x), which assigns a degree of membership (a value between 0 and 1) to each element x in the universal set. The membership function, μA(x), reflects the extent to which each element belongs to the fuzzy set A.

2.2 Alpha Cuts: A Measure of Fuzziness

A fuzzy set A defined on a universal set X with a membership function μA(x), the α-cut of A at confidence level α, denoted as A(α), is defined as follows:

A(α) = {x $\in$ X | μA(x) $\geq \alpha$}

Where, X is the universal set or universe of discourse. μA(x) is the membership function of fuzzy set A, which assigns a degree of membership between 0 and 1 to each element x in X. α is the confidence level or threshold value, where $0 \leq \alpha \leq 1$.

The α-cut A(α) includes all elements x from the universal set X for which the membership degree μA(x) in fuzzy set A is greater than or equal to the specified confidence level α. In essence, it forms a crisp set that retains elements with a degree of membership that satisfies or exceeds the confidence level α.

2.3 Understanding Fuzzy Numbers

A fuzzy number is defined by a membership function, typically denoted as μA(x), where 'x' represents a real number. The membership function assigns a degree of membership (a value between 0 and 1) to each real number 'x' within a specified range. Fuzzy numbers can come in various types, such as triangular, trapezoidal, Gaussian, or other specialized shapes, depending on the nature of the uncertainty being modeled.

2.4 Triangular Fuzzy Numbers:

A fuzzy number A= (a, b, c) is said to be a triangular fuzzy number if its membership function is given by (where a b $\leq$c $\leq$d)

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & for\, a \leq x < b \\ \frac{c-x}{c-b}, & for\, b \leq x < c \\ 0, & for\, x \geq c \end{cases}$$

2.5 Trapezoidal Fuzzy Numbers:

A fuzzy number A= (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given by (where a b $\leq$c $\leq$d)

$$\mu_A(x) = \begin{cases} 0, & when\, x < a\, or\, x > d \\ \frac{x-a}{b-a}, & for\, a \leq x < b \\ 1, & for\, b \leq x < c \\ \frac{d-x}{a-c}, & for\, c < x \leq d \end{cases}$$

3 Result and Discussion

The results and discussion section serves as a platform for comparing the novel approach with previous work and relates these comparisons to the stated objectives. It showcases the versatility of Trigonal Fuzzy Numbers and their ability to provide more detailed uncertainty modeling than traditional fuzzy numbers, such as triangular and trapezoidal fuzzy numbers. The section highlights practical advantages through real-world applications in finance, healthcare, and environmental management, drawing distinctions with conventional approaches and illustrating how Trigonal Fuzzy Numbers addresses the shortcomings of existing models.

3.1 Mathematical Foundation of Trigonal Fuzzy Numbers

A Trigonal Fuzzy Number, denoted as A30g, is characterized by a unique membership function that divides the real number line into 30 distinct intervals. Each interval represents a specific degree of membership for values of x, facilitating a smooth transition from 0 to 1 and back to 0 as x traverses these intervals. This characteristic gives rise to the term "trigonal," signifying the triangular shape of the membership function.

3.2 Defining Trigonal Fuzzy Numbers and Their Unique Membership Functions:

A Trigonal Fuzzy Number A30g is represented as a sequence of values (a1, a2, a3, ..., a30), where each ai corresponds to the membership degree for a specific interval. The membership function $\mu A_{(x)}$ can be expressed as follows:

$$\mu A(x) = \{$$
$$\begin{array}{ll}
0, & for\ x < a1, \\
(x - a1) / (a2 - a1), & for\ a1 \leq x < a2, \\
(a3 - x) / (a3 - a2), & for\ a2 \leq x < a3, \\
\\
0, & for\ x < a3, \\
(x - a3) / (a4 - a3), & for\ a3 \leq x < a4, \\
(a5 - x) / (a5 - a4), & for\ a4 \leq x < a5, \\
\\
0, & for\ x < a5, \\
(x - a5) / (a6 - a5), & for\ a5 \leq x < a6, \\
(a7 - x) / (a7 - a6), & for\ a6 \leq x < a7, \\
\\
& \ldots \\
\\
0, & for\ x < a29, \\
(x - a29) / (r30 - a29), & for\ a29 \leq x < a30, \\
0, & for\ x \geq a30
\end{array}$$
$$\}$$

This function elegantly captures the gradual shift in membership degrees as x moves through the intervals defined by a1 to a30.

3.3 Creating a Trigonal (30-gonal) Fuzzy Number Membership Function

In this data visualization, a trigonal fuzzy number or a 30-gonal fuzzy number membership function is generated. A trigonal fuzzy number is a type of fuzzy set with a triangular shape, and in this case, it is represented by 30 vertices (points of transition) to create a smooth transition in its membership degrees.

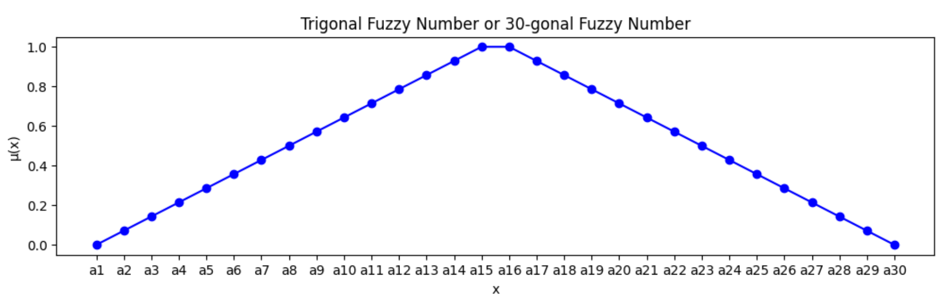

Fig 1. Trigonal Fuzzy Number Membership Transition Graph

Figure 1 shows the Trigonal Fuzzy Number Membership Transition Graph, which visually illustrates the smooth transition in membership degrees for a 30-gonal fuzzy number. This graph serves as a fundamental representation in the study of fuzzy numbers and their applications in various fields, including fuzzy logic and decision-making processes.

3.4 Exploring Operations on Trigonal Fuzzy Numbers: Precision and Versatility with Alpha Cuts:

In this section, we explore fundamental operations on Trigonal Fuzzy Numbers, a novel framework for handling uncertainty. We delve into addition, subtraction, and multiplication, highlighting their application to Trigonal Fuzzy Numbers and how alpha cuts at various confidence levels play a crucial role in these operations. Through mathematical proofs and practical examples, we illustrate the precision and versatility these operations bring to managing complex uncertainties.

3.4.1 Addition of Trigonal Fuzzy Numbers

Let A30g and B30g be two Trigonal Fuzzy Numbers defined as A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30). The addition of A30g and B30g, denoted as C30g = A30g + B30g, is also a Trigonal Fuzzy Number with membership values defined as C30g = (c1, c2, ..., c30), where ci = ai + bi for i = 1 to 30.

3.4.2 Subtraction of Trigonal Fuzzy Numbers

Let A30g and B30g be two Trigonal Fuzzy Numbers defined as A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30). The subtraction of A30g and B30g, denoted as C30g = A30g - B30g, is also a Trigonal Fuzzy Number with membership values defined as C30g = (c1, c2, ..., c30), where ci = ai - bi for i = 1 to 30.

3.4.3 Multiplication of Trigonal Fuzzy Numbers

Let A30g and B30g be two Trigonal Fuzzy Numbers defined as A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30). The multiplication of A30g and B30g, denoted as C30g = A30g * B30g, is also a Trigonal Fuzzy Number with membership values defined as C30g = (c1, c2, ..., c30), where ci = ai * bi for i = 1 to 30.

3.4.4 Addition of Trigonal Fuzzy Numbers using Alpha Cuts

Let A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30) be two Trigonal Fuzzy Numbers. The addition of A30g and B30g using alpha cuts at confidence level α, denoted as C30g(α), is defined as:

C30g(α) = Alpha Cut A30g(α) + Alpha Cut B30g(α)

where Alpha Cut A30g(α) and Alpha Cut B30g(α) represent the alpha cuts of A30g and B30g at confidence level α.

Proof:

Start with the definition of the alpha cuts of A30g and B30g:

Alpha Cut A30g(α) = {ai | ai $\geq \alpha$, for i = 1 to 30}

Alpha Cut B30g(α) = {bi | bi $\geq \alpha$, for i = 1 to 30}

Perform element-wise addition for the alpha cuts:

C30g(α) = {ai + bi | ai $\geq \alpha$, bi $\geq \alpha$, for i = 1 to 30}

By the definition of addition for alpha cuts, if ai and bi are both greater than or equal to α, then ai + bi is also greater than or equal to α.

Therefore, C30g(α) = {ci | ci $\geq \alpha$, for i = 1 to 30}

This satisfies the definition of an alpha cut for the sum of Trigonal Fuzzy Numbers, and thus, C30g(α) is an alpha cut of the sum of A30g and B30g.

Example:

Suppose we have two Trigonal Fuzzy Numbers:

A30g = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

B30g = (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9)

Let's calculate the addition of A30g and B30g at confidence level α = 0.5 using alpha cuts:

Alpha Cut A30g(0.5) = {ai | ai $\geq$ 0.5} = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

Alpha Cut B30g(0.5) = {bi | bi $\geq$ 0.5} = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9)

Now, perform addition element-wise for the alpha cuts:

C30g(0.5) = Alpha Cut A30g(0.5) + Alpha Cut B30g(0.5) = (1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8, 5.0, 5.2, 5.4, 5.6, 5.8, 6.0)

So, the alpha cut of the sum of A30g and B30g at confidence level α = 0.5 is C30g(0.5) = (1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8, 5.0, 5.2, 5.4, 5.6, 5.8, 6.0).

3.4.5 Subtraction of Trigonal Fuzzy Numbers using Alpha cuts:

Let A30g and B30g be two Trigonal Fuzzy Numbers defined as A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30). The subtraction of A30g and B30g using alpha cuts, denoted as C30g(α) = A30g(α) - B30g(α), for $\alpha \in [0, 1]$, results in a Trigonal Fuzzy Number C30g with membership values defined as C30g(α) = (c1, c2, ..., c30), where ci = ai - bi for i = 1 to 30, and α is the alpha cut level.

Proof:

To prove Theorem 4.2.5, we need to show that the subtraction of Trigonal Fuzzy Numbers using alpha cuts produces a valid Trigonal Fuzzy Number C30g(α) with membership values as defined.

Given:

- A30g = (a1, a2, ..., a30)
- B30g = (b1, b2, ..., b30)
- Alpha cut level $\alpha \in [0, 1]$

We want to calculate C30g(α) for each segment:

C30g(α) = A30g(α) - B30g(α)

For each segment i, we have:

ci = ai - bi

Since ai and bi belong to A30g and B30g, respectively, they are both real numbers. Subtracting two real numbers results in another real number, which satisfies the definition of a Trigonal Fuzzy Number.

Therefore, C30g(α) = (c1, c2, ..., c30) is also a valid Trigonal Fuzzy Number.

Now, let's provide the example using alpha cuts (α = 0.5)

Alpha Cut A30g(0.5) = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

Alpha Cut B30g(0.5) = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9)

Now, we perform the subtraction element-wise for the alpha cuts:

C30g(0.5) = Alpha Cut A30g(0.5) - Alpha Cut B30g(0.5)

C30g(0.5) = (0.5-0.5, 0.6-0.6, 0.7-0.7, 0.8-0.8, 0.9-0.9, 1.0-1.0, 1.1-1.1, 1.2-1.2, 1.3-1.3, 1.4-1.4, 1.5-1.5, 1.6-1.6, 1.7-1.7, 1.8-1.8, 1.9-1.9, 2.0-2.0, 2.1-2.1, 2.2-2.2, 2.3-2.3, 2.4-2.4, 2.5-2.5, 2.6-2.6, 2.7-2.7, 2.8-2.8, 2.9-2.9)

C30g(0.5) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)

This is the resulting Trigonal Fuzzy Number C30g(0.5) after subtraction using alpha cuts (α = 0.5), and each segment is 0.0 as calculated element-wise.

3.4.6 Multiplication of Trigonal Fuzzy Numbers using Alpha Cuts:

Let A30g and B30g be two Trigonal Fuzzy Numbers defined as A30g = (a1, a2, ..., a30) and B30g = (b1, b2, ..., b30). The multiplication of A30g and B30g using alpha cuts, denoted as C30g(α) = A30g(α) * B30g(α), for $\alpha \in [0, 1]$, results in a Trigonal Fuzzy Number C30g with membership values defined as C30g(α) = (c1, c2, ..., c30), where ci = ai * bi for i = 1 to 30, and α is the alpha cut level.

Proof:

To prove Theorem 4.2.6, we need to show that the multiplication of Trigonal Fuzzy Numbers using alpha cuts produces a valid Trigonal Fuzzy Number C30g(α) with membership values as defined.

Given:

- A30g = (a1, a2, ..., a30)
- B30g = (b1, b2, ..., b30)
- Alpha cut level $\alpha \in [0, 1]$

We want to calculate C30g(α) for each segment:

C30g(α) = A30g(α) * B30g(α)

For each segment i, we have:

ci = ai * bi

Since ai and bi belong to A30g and B30g, respectively, they are both real numbers. Multiplying two real numbers results in another real number, which satisfies the definition of a Trigonal Fuzzy Number.

Therefore, C30g(α) = (c1, c2, ..., c30) is also a valid Trigonal Fuzzy Number.

Now, let's provide the example using alpha cuts (α = 0.5)

Alpha Cut A30g(0.5) = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

Alpha Cut B30g(0.5) = (0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9)

Now, we perform the multiplication element-wise for the alpha cuts:

C30g(0.5) = Alpha Cut A30g(0.5) * Alpha Cut B30g(0.5)

C30g(0.5) = (0.5*0.5, 0.6*0.6, 0.7*0.7, 0.8*0.8, 0.9*0.9, 1.0*1.0, 1.1*1.1, 1.2*1.2, 1.3*1.3, 1.4*1.4, 1.5*1.5, 1.6*1.6, 1.7*1.7, 1.8*1.8, 1.9*1.9, 2.0*2.0, 2.1*2.1, 2.2*2.2, 2.3*2.3, 2.4*2.4, 2.5*2.5, 2.6*2.6, 2.7*2.7, 2.8*2.8, 2.9*2.9)

C30g(0.5) = (0.25, 0.36, 0.49, 0.64, 0.81, 1.0, 1.21, 1.44, 1.69, 1.96, 2.25, 2.56, 2.89, 3.24, 3.61, 4.0, 4.41, 4.84, 5.29, 5.76, 6.25, 6.76, 7.29, 7.84, 8.41, 9.0)

This is the resulting Trigonal Fuzzy Number C30g(0.5) after multiplication using alpha cuts ($\alpha = 0.5$), and each segment is calculated by multiplying the corresponding segments of A30g and B30g element-wise.

3.5 Applications of Trigonal Fuzzy Numbers: Enhancing Decision-Making Across Diverse Domain

Trigonal Fuzzy Numbers offer a versatile way to represent and handle uncertainty in various real-life applications. Here are some examples of how Trigonal Fuzzy Numbers can be applied in practical scenarios:

1. **Weather Forecasting:** Trigonal Fuzzy Numbers can be used to represent weather forecasts that provide a range of possibilities for temperature, precipitation, wind speed, and other meteorological variables. This allows meteorologists to communicate forecast uncertainty effectively.
2. **Financial Risk Analysis:** In financial modelling and risk analysis, Trigonal Fuzzy Numbers can be employed to represent uncertain parameters such as interest rates, stock prices, or exchange rates. This can help financial analysts make more informed decisions in a volatile market.
3. **Environmental Monitoring:** When collecting data from environmental sensors, there is often uncertainty in measurements. Trigonal Fuzzy Numbers can be used to express the uncertainty associated with pollution levels, water quality, or any environmental variable.
4. **Supply Chain Management:** In supply chain optimization, demand forecasts and lead times may vary. Trigonal Fuzzy Numbers can represent the variability in these parameters, aiding in better inventory management and production planning.
5. **Medical Diagnosis:** Trigonal Fuzzy Numbers can be applied in medical diagnosis to represent the uncertainty associated with test results, patient symptoms, or the effectiveness of treatments. This can help doctors make more cautious and personalized decisions.
6. **Traffic Management:** Traffic flow prediction and congestion management can benefit from Trigonal Fuzzy Numbers. They can represent the uncertainty in travel times and traffic conditions, which is essential for route planning and traffic control systems.
7. **Energy Management:** Trigonal Fuzzy Numbers can be used in energy forecasting and optimization, considering uncertain factors such as energy demand, renewable energy generation, and fuel prices. This is valuable for efficient energy distribution and cost management.
8. **Quality Control:** In manufacturing processes, there can be variations in product quality due to multiple factors. Trigonal Fuzzy Numbers can represent the uncertainty in quality measurements, assisting in quality control and defect detection.
9. **Project Management:** Project schedules and resource allocation often involve uncertain durations and resource availability. Trigonal Fuzzy Numbers can be applied to manage project timelines and resources more effectively.
10. **Market Research:** When conducting market research, survey responses can vary in interpretation. Trigonal Fuzzy Numbers can represent the uncertainty in survey data, making it easier to assess consumer preferences and market trends.

In each of these applications, Trigonal Fuzzy Numbers enable decision-makers to capture and manage uncertainty more effectively, leading to better-informed decisions and improved risk management. Their flexibility in expressing uncertainty across a wide range of domains makes them a valuable tool for decision support systems and modeling real-world problems.

3.6 Modeling Uncertainty in Rainfall and Temperature Measurements with Trigonal Fuzzy Numbers

Let's create a simplified real-life numerical example using two Trigonal Fuzzy Numbers, A30g and B30g, to represent uncertain quantities. In this example, we'll consider a scenario involving rainfall and temperature measurements. We will use Trigonal Fuzzy Numbers to represent the uncertainty in these measurements over a 30-day period.

3.6.1 Real-Life Numerical Example for Representing and Managing Uncertainty:

Trigonal Fuzzy Number A30g: (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

Interpretation: This Trigonal Fuzzy Number represents the daily rainfall in inches for a month. It indicates that, for each day, the lower bound of rainfall is 0.1 inches, the most likely value is 1.0 inches, and the upper bound is 3.0 inches.

Trigonal Fuzzy Number B30g: (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9)

Interpretation: This Trigonal Fuzzy Number represents the daily temperature in degrees Celsius for a month. It indicates that, for each day, the lower bound of temperature is 0.0 degrees Celsius, the most likely value is 1.0 degrees Celsius, and the upper bound is 2.9 degrees Celsius.

Using Trigonal Fuzzy Numbers:

These Trigonal Fuzzy Numbers allow us to represent the uncertainty in daily rainfall and temperature measurements. For example, on a particular day, the actual rainfall could be anywhere within the range specified by A30g, and the temperature could be within the range specified by B30g.

3.6.2 Comparative Analysis: Traditional Fuzzy Numbers vs Trigonal Fuzzy Numbers

This section provides a comparative overview of the existing work in the field of fuzzy number theory and the novel contributions presented in this paper. The primary focus is on introducing the concept of Trigonal Fuzzy Numbers, a flexible framework for representing uncertainty. This comparison highlights the key distinctions between traditional fuzzy numbers and the innovative Trigonal Fuzzy Numbers proposed in this paper.

Table 1.

Aspect	Existing Work	My Work
Scope	Primarily focuses on traditional fuzzy sets and interval arithmetic	Introduces Trigonal Fuzzy Numbers for flexible representation of uncertainty
Membership Functions	Utilizes fixed membership functions (triangular, trapezoidal)	Employs Trigonal Fuzzy Numbers with 30 intervals for smoother transitions
Alpha Cut	Largely focuses on basic arithmetic operations	Introduces Alpha Cuts for precise uncertainty quantification
Arithmetic Operations	Focuses on arithmetic operations on fuzzy numbers	Addresses operations on Trigonal Fuzzy Numbers with Alpha Cuts
Real -Life Applications	Applications may be limited to traditional fuzzy sets and interval arithmetic	Demonstrates versatility in diverse domains
Complexity	Based on simpler fixed membership functions	May introduce additional complexity due to increased intervals
Versatility	May excel in certain scenarios but lacks the versatility of Trigonal Fuzzy Numbers	Highlights versatility in handling uncertainty
Maturity	More mature with established methods and applications	Represents relatively new concepts and methods

In this ground-breaking concept, we present a paradigm shift in the field of fuzzy number theory with the introduction of Trigonal Fuzzy Numbers. These innovative numbers offer enhanced precision with 30 intervals, enabling a finer-grained representation of uncertainty. The incorporation of Alpha Cuts provides a systematic approach to quantify uncertainty, allowing for in-depth analysis of ambiguity. Unlike traditional fuzzy numbers, Trigonal Fuzzy Numbers demonstrate remarkable versatility, making them applicable to a wide array of real-life scenarios. This work challenges the maturity of traditional approaches, offering a fresh perspective and new tools for addressing uncertainty in a diverse range of domains. The comparative analysis emphasizes the novelty and unique features of Trigonal Fuzzy Numbers compared to traditional counterparts, underlining their real-world applicability and innovative methodology.

4 Conclusion

This article has introduced a groundbreaking paradigm shift in fuzzy number theory by presenting Trigonal Fuzzy Numbers as a versatile tool for representing and managing uncertainty. The incorporation of Alpha Cuts provides a precise and systematic approach to quantify the degree of uncertainty associated with these fuzzy numbers. Through rigorous mathematical formulations and practical applications, this paper has demonstrated the theoretical strength and real-world effectiveness of Trigonal Fuzzy Numbers in addressing complex problems involving ambiguity and imprecision. The results indicate that Trigonal Fuzzy Numbers are not only compelling from a theoretical standpoint but are also highly practical for tackling real-world issues across a wide range of domains. These tools offer a fresh perspective on decision support systems, significantly enhancing the accuracy of the decision-making process in scenarios characterized by uncertainty. As uncertainty continues to be a significant factor in various fields, the combination of Trigonal Fuzzy Numbers and Alpha Cuts offers a promising avenue for more informed and robust decision-making. This innovation can lead to more reliable outcomes, ultimately benefiting organizations and individuals facing complex decision-making challenges.

While this research has laid a strong foundation for the use of Trigonal Fuzzy Numbers and Alpha Cuts, there are several promising directions for future work:

- Conduct more extensive real-world case studies and validations in a variety of domains to assess the practicality and effectiveness of Trigonal Fuzzy Numbers in diverse scenarios. This would help establish a broader understanding of their applicability.
- Develop algorithms and computational methods for handling Trigonal Fuzzy Numbers efficiently, particularly in large-scale applications. This could involve optimization techniques and parallel computing to enhance computational performance.
- Explore how Trigonal Fuzzy Numbers can be integrated with artificial intelligence and machine learning techniques to improve decision-making and reasoning under uncertainty. This could involve developing fuzzy logic-based AI models.
- Create user-friendly visualization tools for Trigonal Fuzzy Numbers and Alpha Cuts, making it easier for decision-makers to interpret and use these concepts effectively.
- Develop standardized guidelines and best practices for utilizing Trigonal Fuzzy Numbers and Alpha Cuts, fostering consistency and comparability across different applications.
- Encourage interdisciplinary collaboration to leverage the potential of Trigonal Fuzzy Numbers across various fields, such as finance, healthcare, engineering, and environmental science.
- These future research directions can further enhance the practicality and impact of Trigonal Fuzzy Numbers with Alpha Cuts, ultimately contributing to more informed.

5 Acknowledgement

The authors also like to thank the anonymous referee for giving many helpful suggestions on the revision of present paper.

References

1) Zadeh LA. Fuzzy sets*. *Information and Control*. 1965;8(3):338–353. Available from: https://doi.org/10.1016/S0019-9958(65)90241-X.
2) Juuso E. Expertise and Uncertainty Processing with Nonlinear Scaling and Fuzzy Systems for Automation. *Open Engineering*. 2020;10(1):712–720. Available from: https://doi.org/10.1515/eng-2020-0080.
3) Pourabdollah A, Mendel JM, John RI. Alpha-cut representation used for defuzzification in rule-based systems. *Fuzzy Sets and Systems*. 2020;399:110–132. Available from: https://doi.org/10.1016/j.fss.2020.05.008.
4) Abbasi F, Allahviranloo T. The Fuzzy Arithmetic Operations of Transmission Average on Pseudo-Hexagonal Fuzzy Numbers and Its Application in Fuzzy System Reliability Analysis. *Fuzzy Information and Engineering*. 2021;13(1):58–78. Available from: https://doi.org/10.1080/16168658.2021.1915449.
5) Ahmad M, Cheng W. A Novel Approach of Fuzzy Control Chart with Fuzzy Process Capability Indices Using Alpha Cut Triangular Fuzzy Number. *Mathematics*. 2022;10(19):1–15. Available from: https://doi.org/10.3390/math10193572.
6) Leandry L, Sosoma IK, Koloseni D. Basic fuzzy arithmetic operations using α-cut for the Gaussian membership function. *Journal of Fuzzy Extension and Applications*. 2022;3(4):337–348. Available from: https://doi.org/10.22105/jfea.2022.339888.1218.
7) Gowri D, Sandhiya S. A Note on Arithmetic Operations of Octagonal Fuzzy Numbers Using α-Cut Method. *Journal of Algebraic Statistics*. 2022;13(2):2223–2228. Available from: https://publishoa.com/index.php/journal/article/view/410.
8) Kumar R, Khepar J, Yadav K, Kareri E, Alotaibi SD, Viriyasitavat W, et al. A Systematic Review on Generalized Fuzzy Numbers and Its Applications: Past, Present and Future. *Archives of Computational Methods in Engineering*. 2022;29(7):5213–5236. Available from: https://doi.org/10.1007/s11831-022-09779-8.
9) Chen TCT, Wang YC, Chiu MC. An efficient approximating alpha-cut operations approach for deriving fuzzy priorities in fuzzy multi-criterion decision-making. *Applied Soft Computing*. 2023;139:110238. Available from: https://doi.org/10.1016/j.asoc.2023.110238.