INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 04-07-2023

Accepted: 28-09-2023

Published: 27-10-2023

Citation: Pandichelvi V, Vanaja R (2023) A Paradigm for Two Classes of Simultaneous Exponential Diophantine Equations. Indian Journal of Science and Technology 16(40): 3514-3521. https://doi.org/10.17485/IJST/v16i40.1643

* **Corresponding author**.

vanajvicky09@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

A Paradigm for Two Classes of Simultaneous Exponential Diophantine Equations

V Pandichelvi[1], **R Vanaja**[2]*

1 Assistant Professor, PG & Research Department of Mathematics, Urumu Dhanalakshmi College,(Affiliated to Bharathidasan University), Trichy, India
2 Assistant Professor, PG & Research Department of Mathematics, Shrimati Indira Gandhi College, (Affiliated to Bharathidasan University), Trichy, India

Abstract

Objectives: The goal of this article is to find integer solutions to two distinct kinds of simultaneous exponential Diophantine equations in three variables. **Methods:** The system of exponential Diophantine equations is translated into the eminent form of Thue equations, and then their generalised solutions satisfying certain conditions are applied. **Findings:** The finite set of integer solutions for two disparate categories of simultaneous exponential Diophantine equations consisting of three unknowns is scrutinized. In some circumstances, there is no solution in this analysis for both the dissimilar simultaneous Diophantine equations. **Novelty:** The motivation is considered to be two types of simultaneous exponential Diophantine equations are first converted into specific system of Pell equations, then into Thue equations for the possibilities of the sum of the exponents, such as $x+y=1$ or $x+y=2$. If $x+y>3$, then, the equations are transformed into a cubic equation, which is not in the form of Pell equations. So, such cases are discarded for exploring solutions to the necessary equations.

Keywords: Simultaneous Exponential Equations; Simultaneous Pell Equations; Thue Equations; Integer Solutions; Divisibility

1 Introduction

A Diophantine equation with unknowns in exponents is known as an exponential Diophantine equation(1–11). Simultaneous exponential Diophantine equations are a pair of exponential Diophantine equations whose values satisfy both or all of the equations in the collection at the same time. In(12–14), the authors investigate various simultaneous exponential Diophantine equations in the form $a_1{}^x+b_1{}^y=c_1{}^z$, $a_2{}^x+b_2{}^y=c_2{}^z$ where a_1, a_2, a_3, b_1, b_2 and b_3 are integers, by reducing them into simultaneous Pell equations.

A Diophantine equation of the form $f(x,y)=r$, where $f(x,y)$ is an irreducible bivariate form of at least 3 over the rational numbers and r is a non-zero rational number, is said to be the Thue equation.

In this article, two unlike categories of simultaneous exponential Diophantine equations in three unknowns are studied for integer solutions by decoding these equations into the distinguished form of Thue equations and using their general solutions sustaining certain conditions.

1.1 Preliminaries

- **Theorem 1**

Let $a,\ b,\ c$ be three squarefree integers, $a\ >\ 0,\ b\ <\ 0,\ c\ <\ 0$ which are pairwise coprime. Then there exists a nonzero integer solution $(x,\ y,\ z)$ to the Diophantine equation

$$ax^2+by^2+cz^2=0 \tag{1}$$

If and only if all three congruences $t^2\ \equiv\ -ab\ (mod\ c)t^2\ \equiv\ -ac\ (mod\ b)t^2\equiv\ -bc\ (mod\ a)$ are solvable. Furthermore, if a nonzero solution exists, then there exists a nonzero solution $(x_0,\ y_0,z_0)$ of equation (1) satisfying the inequality $max(x_0,\ y_0,z_0)\ \leq\ \sqrt{abc}$.

- **Theorem 2**

Assume that $(x_0,\ y_0,z_0)$ is an integer solution of equation (1) with $z_0\neq\ 0$. Then, all integer solutions (x,y,z) with $z\ \neq\ 0$ of equation (1) are of the form

$$x=\pm\frac{D}{d}\left(-ax_0s^2-2by_0rs+bx_0r^2\right)$$

$$y=\pm\frac{D}{d}\left(ay_0s^2-2ax_0rs-by_0r^2\right)$$

$$z=\pm\frac{D}{d}\left(az_0s^2+bz_0r^2\right)$$

where r and $s\ >\ 0$ are coprime integers, D is a nonzero integer, and d | $2a^2bc{z_0}^3$ is a positive integer.

1.2 Procedure for solving the equations

Deliberate the aforementioned system of two Diophantine equations

$${a_1}^x+{b_1}^y=c_1 \tag{2}$$

$${a_2}^x+{b_2}^y=c_2 \tag{3}$$

in non-negative integers $x,\ y$ and z, where the coefficients are certain integers and none of the coefficients in (3) is zero filling the conditions $a_1b_1<0,\ a_2b_2<0,\ c_1c_2\neq0$ and $a_1c_2-a_2c_1\neq0$. Afterwards, simple algebraic calculations applied in (2) and (3) lead them into

$$(a_1c_2-a_2c_1)x^2+b_1c_2y^2-b_2c_1z^2=0 \tag{4}$$

To achieve the conditions of Legendre's theorem, divide (4) by $gcd\left(a_1c_2-a_2c_1,\ b_1c_2,\ b_2c_1\right)$. Then, it is attained by $a_3x^2+b_3y^2+c_3z^2=0$. Further, if $a_3b_3c_3<0$, then even multiply this equation by(-1). Moreover, let this new-fangled equation be multiplied by $gcd(a_3,b_3)\cdot gcd(a_3,c_3)\cdot gcd(b_3,c_3)$ and let adapt the square complete part of the coefficients into the equivalent variables, relabeling them. Note that

$$aX^2+bY^2+cZ^2=0 \tag{5}$$

where $X,\ Y,\ Z$ is a permutation of $c_x x,\ c_y y,\ c_z z$ with some appropriate positive integers $c_x,\ c_y$ and c_z,

$a > 0,\ b < 0$ and $c < 0$ are pairwise coprime, square-free integers. Obviously, the choice of X is unique, but the parts of Y and Z can be transferred. By the theorem of Legendre, a basic solution $(X_0,\ Y_0,\ Z_0)$ is needed. If (5) is not resolvable, then the system (2), (3) has no solution. Otherwise, let $(X_0,\ Y_0,\ Z_0)$ with $Z_0 \neq 0$ satisfy (5) and possibly $d(2a^2\, bcZ_0{}^3) \le d(2a^2 bcY_0{}^3)$, where $d(\ )$ denotes the number of divisors function. Such a triplet can effortlessly be created by merely pursuit in the intervals $0 \le X_0, Y_0,\ Z_0 \le \sqrt{abc}$.

Now, applying Theorem 2, X, Y and Z can be articulated by

$X = \pm \frac{D}{d}\,(\alpha_1 s^2 + \beta_1 sr + \gamma_1 r^2)$,

$Y = \pm \frac{D}{d}\,(\alpha_2 s^2 + \beta_2 sr + \gamma_2 r^2)$,

$Z = \pm \frac{D}{d}\,(\alpha_3 s^2 + \beta_3 sr + \gamma_3 r^2)$,

where $s > 0$ and r are coprime, D is an arbitrary integer, $d \mid h_d = 2a^2\, bcZ_0{}^3$ is a positive integer and $\beta_3 = 0$. Consequently,

$X = \pm \frac{D}{c_x d}\,(\alpha_{i1} s^2 + \beta_{i1} sr + \gamma_{i1} r^2)$,

$Y = \pm \frac{D}{c_y d}\,(\alpha_{i2} s^2 + \beta_{i2} sr + \gamma_{i2} r^2)$,

$Z = \pm \frac{D}{c_z d}\,(\alpha_{i3} s^2 + \beta_{i3} sr + \gamma_{i3} r^2)$,

where $i_1,\ i_2$ and i_3 are permutation of the subscripts $1,\ 2$ and 3 of $\alpha,\ \beta$ and γ.

These consequences can be functional to return with $x,\ y$ and z, for instance, to (2),

$a_1\left(\frac{D}{c_x d}\,(\alpha_{i1} s^2 + \beta_{i1} sr + \gamma_{i1} r^2)\right)^2 + b_1\left(\frac{D}{c_y d}\,(\alpha_{i2} s^2 + \beta_{i2} sr + \gamma_{i2} r^2)\right)^2 = c_1,$

which implies

$$a_1 c_y{}^2\left(\alpha_{i1} s^2 + \beta_{i1} sr + \gamma_{i1} r^2\right)^2 + b_1 c_x{}^2\left(\alpha_{i2} s^2 + \beta_{i2} sr + \gamma_{i2} r^2\right)^2 = c_1 c_x{}^2 c_y{}^2\left(\frac{D}{d}\right)^2$$

Denote the left-hand side by $T_1(s,\ r)$. Simplify the latest equation by the greatest common divisor of $c_1 c_x{}^2 c_y{}^2$ and the coefficients of T_1. Hence, $T\ (s,\ r) = c_4(d/D)^2$. On the right-hand side, let c_0 be the square-free part of c_4. Thus, there exists a positive integer $c_6$such that $c_4 = c_0 c_6{}^2$. Then the above equation is equal to

$$T(s,r) = c_0\left(\frac{c_6 d}{D}\right)^2 \tag{6}$$

Since T (s, r) is specified, $0 < d$ is a divisor of $h_d = 2a^2\, bcZ_0{}^3$ and $j = \frac{c_6 d}{D}$ must be an integer, it is observed that (6) means finitely many Thue equations of order 4. Suppose that $(s_j,\ r_j\ )$ is a solution of $T\ (s,\ r) = c_0 j^2$ for some eligible j. We reject $(s_j,\ r_j)$ if $s_j \le 0$ or $\gcd(s_j, r_j) > 1$,, otherwise

$X = \pm \frac{c_6}{c_x j}\,(\alpha_{i1} s_j{}^2 + \beta_{i1} s_j r_j + \gamma_{i1} r_j{}^2)$,

$Y = \pm \frac{c_6}{c_y j}\,(\alpha_{i2} s_j{}^2 + \beta_{i2} s_j r_j + \gamma_{i2} r_j{}^2)$,

$Z = \pm \frac{c_6}{c_z d}\,(\alpha_{i3} s_j{}^2 + \beta_{i3} s_j r_j + \gamma_{i3} r_j{}^2)$.

If all $x,\ y$ and z are non-negative integers, then a solution of the system (2), (3) is obtained.

2 Progression of Inspection

The simultaneous exponential equations for trying whether the solution occurs or not in integers are booked in the form as

$$\left.\begin{array}{l} a_1^x + b_1^y = c_1^z \\ a_2^x + b_2^y = c_2^z \end{array}\right\} \tag{7}$$

where $a_1, a_2, a_3, b_1, b_2\ ,\ c_1, c_2 \in Z$

The solutions in integers for this system of equations are scrutinized for the ensuing two cases.

$$(i) x + y = 1 (ii) x + y = 2$$

All the options for x and y in integers for the overhead two cases are displayed below.

(i) $x = 0, y = 1$and $x = 1, y = 0$

(ii) $x=0, y=2, x=1, y=1$ and $x=0, y=2$.

The above five cases, as well as a brief description of how to investigate them, are discussed in sections 2.1 and 2.2 for particular choices of the base values of the concerned equations.

Section 2.1

Consider the succeeding system of exponential equations–

$$\left.\begin{array}{l}(m^2+5)^x+(m^2-3)^y=7z^2\\(n^2+7)^x+(n^2+3)^y=5z^2\end{array}\right\} \tag{8}$$

where $m,n\in Z$, the set of all integers.

Case (i): Suppose $x=0, y=1$

Then, (2) is condensed into

$$\left.\begin{array}{l}7z^2-m^2=-2\\5z^2-n^2=4\end{array}\right\} \tag{9}$$

By effectuating some common algebraic appraisements, the system of equations (9) is renewed into the following single equation

$$19z^2-2m^2-n^2=0 \tag{10}$$

Elect $a=19,\ b=\ -2,\ c=\ -1$.

The lowest integer solution to (4) is searched out by $(z_0,m_0,n_0)=(1,3,1)$

Then, all conceivable integer triples (z,m,n) where $n\neq 0$ in (10) are provided by

$$\left.\begin{array}{l}z=\pm\frac{D}{d}\left(-az_0s^2-2bm_0rs+bz_0r^2\right)=\pm\frac{D}{d}\left(-19s^2+12rs-2r^2\right)\\m=\pm\frac{D}{d}\left(am_0s^2-2az_0rs-bm_0r^2\right)=\pm\frac{D}{d}\left(57s^2-38rs+6r^2\right)\\n=\pm\frac{D}{d}\left(an_0s^2+bn_0r^2\right)=\pm\frac{D}{d}\left(19s^2-2r^2\right)\end{array}\right\} \tag{11}$$

where r and $s>0$ are coprime integers, $D\in Z-(0\}$ and $d/2a^2bcz_0{}^3=1444$ is a natural number.

Therefore, the altered form of (10), named Thue equation, can be inscribed as

$$361s^4+294r^2s^2+4r^4-570rs^3-60r^3s=\frac{d^2}{D^2} \tag{12}$$

for some positive integers $j=\frac{d}{D}/1444$.

The suitable values of j are $j=1,\ 38$ when (s_j,r_j) satisfies the conditions that $s_j>0$ and $gcd\,(s_j,r_j)=1$,

The corresponding triplets $(j,s_j,r_j)=\{(1,1,3),\ (38,6,19)\}$ deliver eight coprime integer solutions $(z,\ m,n\ )$ as follows. $(-1,3,1),(-1,3,-1),(-1,-3,1),(-1,-3,-1),(1,3,1),(1,3,-1),(1,-3,1)$ and $(1,-3,-1)$

Hence, the necessary solutions (m,n,x,y,z) to (2) where $x,\ y, and\ \ z$ belong to the set of all integers are $(3,1,0,1,-1)$, $(3,-1,0,1,-1),(-3,1,0,1,-1)$, $(-3,-1,0,1,-1),(3,1,0,1,1)$, $(-3,1,0,1,1),(3,-1,0,1,1)$ and $(-3,-1,0,1,1)$.

Case (ii): Suppose $x=1, y=0$

These choices of x and y lead to (1) in the simultaneous equations as $7z^2-m^2=6$

$$\left.\begin{array}{l}7z^2-m^2=6\\5z^2-n^2=8\end{array}\right\} \tag{13}$$

After undertaking a few calculations, (13) is converted into

$$M^2-13z^2-3n^2=0 \text{ where } M=2m \tag{14}$$

Put $(a,b,c)=(1,-13,-3)$ As in case (i$(m_0,z_0,n_0)=(4,1,1)$ to (14), all its probable integer solutions (m,z,n) are demonstrated by

$$\left.\begin{array}{l}m=\pm\frac{D}{d}\left(-2s^2+13rs-26r^2\right)\\z=\pm\frac{D}{d}\left(s^2-8rs+13r^2\right)\\n=\pm\frac{D}{d}\left(s^2-13r^2\right)\end{array}\right\} \tag{15}$$

Where r and s are positive coprime integers, D is a non-zero integer, and $d/2a^2bc{z_0}^3 = 78$ is a positive integer.

By means of (15), the alternative form of (8) is noted by

$$s^4 + 119r^2s^2 + 169r^4 - 20rs^3 - 260r^3s = 2\frac{d^2}{D^2} \tag{16}$$

for some natural number $j = \frac{d}{D}/78$. It is keenly observed that there is no value of j when the pair (s_j, r_j) satisfies the double conditions $s_j > 0$ and $gcd\,(s_j, r_j) = 1$.

Hence, it is concluded that (8) does not have an integer solution.

Case (iii): Assume that $x = 0,\ y = 2$. The classification of (8) diminishes to

$$\left.\begin{array}{l} 1 + \left(m^2 - 3\right)^2 = 7z^2 \\ 1 + \left(n^2 + 3\right)^2 = 5z^2 \end{array}\right\} \tag{17}$$

which is expressed in the simplest form for our convenience,

$$\left.\begin{array}{l} 7z^2 - U^2 = 1 \\ 5z^2 - V^2 = 1 \end{array}\right\} \tag{18}$$

where

$$U = m^2 - 3, V = n^2 + 3 \tag{19}$$

After some enterprising mathematical calculations, (18) is rehabilitated into a single equation

$$U^2 - 2z^2 - V^2 = 0 \tag{20}$$

Take the coefficients of (U^2, z^2, V^2) in (14) as $a = 1,\ b = -2,\ c = \ -1$.

As the explanations given in case (i), all possible values of the triplet (U, z, V) are attained by

$$\left.\begin{array}{l} U = \pm\dfrac{D}{d}\left(-s^2 - 2r^2\right) \\ z = \pm\dfrac{D}{d}(-2rs) \\ V = \pm\dfrac{D}{d}\left(s^2 - 2r^2\right) \end{array}\right\} \tag{21}$$

r and s are coprime integers none other than zero, D is any non-zero integer, and $d/4$ is a positive integer.

Employing (21) in (20), the reduced equation is evaluated by

$$-s^4 + 28r^2s^2 - 4r^4 = \frac{d^2}{D^2}$$

for some positive integer $j = \frac{d}{D}/4$. Here also, there are no values of j such that the ordered pair (s_j, r_j) satisfies the desired conditions.

Hence, (8) has no integer solutions.

Case (iv): Suppose $x = 1, y = 1$as follows

$$\left.\begin{array}{l} 7z^2 - 2m^2 = 2 \\ 5z^2 - 2n^2 = 10 \end{array}\right\} \tag{22}$$

which is abbreviated into the unique equation as

$$5m^2 - 15z^2 - n^2 = 0$$

Assume that $(a, b, c) = (5, -15, -1)$

Since $-ac = \ 5$ is not a square modulo $(b| = 15$, then by Lagrange's criterion for the existence of a non-zero solution to the Diophantine equation, the given simultaneous exponential equations do not have any solution.

Case (v): Presume $x=2, y=0$The compatible form of (8) is stated by

$$\left.\begin{array}{l}\left(m^2+5\right)^2+1=7z^2\\ \left(n^2+7\right)^2+1=5z^2\end{array}\right\} \tag{23}$$

Modify (22) as

$$\left.\begin{array}{l}7z^2-T^2=1\\ 5z^2-W^2=1\end{array}\right\} \tag{24}$$

where

$$T=m^2+5, W=n^2+7 \tag{25}$$

Then, the equivalent form of (18) after executing simple algebraic concepts is given by

$T^2-2z^2-W^2=0$

Following the similar procedure as in case (iii), it is proved that the chosen exponential equations do not have any solution. Hence, the possible solutions to the system of Diophantine equation (8) are evaluated by

$(m,n,x,y,z)=\left\{\begin{array}{c}(3,1,0,1,-1),(3,-1,0,1,-1),(-3,1,0,1,-1),(-3,-1,0,1,-1)\\ (3,1,0,1,1),(-3,1,0,1,1),(3,-1,0,1,1),(-3,-1,0,1,1)\end{array}\right\}.$

Section 2.2: Consider the simultaneous exponential equations for exploration as

$$\left.\begin{array}{l}\left(m^2+1\right)^x+\left(m^2+3\right)^y=5z^2\\ \left(n^2+5\right)^x+\left(n^2+7\right)^y=11z^2\end{array}\right\} \tag{26}$$

where $m,n\in Z$. The existence of solutions for various choices of x and y is given below.

Case (i): Assume $x=0, y=1$. These options diminish (26in the subsequent double equations

$$\left.\begin{array}{l}5z^2-m^2=4\\ 11z^2-n^2=8\end{array}\right\} \tag{27}$$

After smearing simple Mathematical operations, (27) is altered into the resulting single equation

$$n^2-2m^2-z^2=0 \tag{28}$$

Let us select $a=1,\ b=\ -1,\ c=\ -2.$

Using the basic solution $(n_0,m_0,z_0)=(1,0,1)$ of (22), its general solution is conquered by

$$\left.\begin{array}{l}n=\pm\frac{D}{d}\left(-s^2-2r^2\right)\\ m=\pm\frac{D}{d}(-2rs)\\ z=\pm\frac{D}{d}\left(s^2-2r^2\right)\end{array}\right\} \tag{29}$$

where

r and $s>0$ and $gcd\,(r,s)=1D\in Z-\{0\}$ and $d/2a^2bcz_0{}^3=4$ is a positive integer.

Retaining (29) in (28), the corresponding Thue equation is presented by

$-s^4+28r^2s^2-4r^4=\frac{d^2}{D^2}$

for some non-negative integers, $j=\frac{d}{D}/4$. When the pair (s_j,r_j) meets the required conditions, j seems to have no value. Consequently, there is no solution to the needed simultaneous exponential equations (26).

Case (ii): Take $x=1, y=0$

Then, the simplified form of (26) is

$$\left.\begin{array}{l} 5z^2-m^2=2 \\ 11z^2-n^2=6 \end{array}\right\} \tag{30}$$

which can be transformed into

$$3m^2-4z^2-n^2=0 \tag{31}$$

Retain $a=3,\ b=-4,\ c=\ -1$

But $-bc=\ -4$ is not a square modulo $|a|=3$ and $-ac=\ 3$ is not a square modulo $|b|=4$.

Then, as in case (iv) of Section 3.1, the specified couple of exponential equations do not have any integer solutions.

Case (iii): Suppose $x=0, y=2$

Thus, (20) is developed into

$$\left.\begin{array}{l} 5z^2-A^2=1 \\ 11z^2-B^2=1 \end{array}\right\} \tag{32}$$

where

$$A=m^2+3,\ B=n^2+7$$

Now, (32) can be revised as

$$B^2-6z^2-A^2=0 \tag{33}$$

The equivalent values of (B,z,A) are calculated by

$$\left.\begin{array}{l} B=\pm\frac{D}{d}\left(-s^2-6r^2\right) \\ z=\pm\frac{D}{d}(-2rs) \\ A=\pm\frac{D}{d}\left(s^2-6r^2\right) \end{array}\right\} \tag{34}$$

$r,\ s$, and D satisfy the conditions as in the previous cases, and $d/2a^2bcz_0{}^3=12$ is a positive integer.

Utilising (34) in (33), it is detected that

$-s^4+32r^2s^2-36r^4=\frac{d^2}{D^2}$

for some $j=\frac{d}{D}/12$. There is no value of j when the solution (s_j,r_j) satisfies the essential conditions as mentioned in earlier cases.

Thus, the system of equations (26) does not possess any solution in integers.

Case (iv): Choose $x=1, y=1$

$$\left.\begin{array}{l} 5z^2-2m^2=4 \\ 11z^2-2n^2=12 \end{array}\right\} \tag{35}$$

Now, (35) is changed into the ensuing equation

$$3m^2-2z^2-n^2=0 \tag{36}$$

The generalized solutions to (35) are estimated by

$$\left.\begin{array}{l} m=\pm\frac{D}{d}\left(-az_0s^2-2bm_0rs+bz_0r^2\right)=\pm\frac{D}{d}\left(-3s^2+4rs-2r^2\right) \\ z=\pm\frac{D}{d}\left(am_0s^2-2az_0rs-bm_0r^2\right)=\pm\frac{D}{d}\left(3s^2-6rs+2r^2\right) \\ n=\pm\frac{D}{d}\left(an_0s^2+bn_0r^2\right)=\pm\frac{D}{d}\left(3s^2-2r^2\right) \end{array}\right\} \tag{37}$$

where r and $s>0$ are relatively prime integers, $D\in Z-\{0\}$ and $d/2a^2bcz_0{}^3=36$ is a natural number.

Interpreting (36) in (37), the two parametric equations are received by

$27s^4+184r^2s^2+12r^4-132rs^3-88r^3s=-\left(2\frac{d}{D}\right)^2$

for some $j=\frac{d}{D}/36$. There is no value of j together with the assumed values of (s_j,r_j).

Therefore, in this case, there is no solution to (20).

Case (v): Suppose $x=2,\ y=0$

These choices of (x,y) lead (26) into

$$\left.\begin{array}{l}5z^2-C^2=1\\11z^2-D^2=1\end{array}\right\}\qquad(38)$$

where

$$C=m^2+1,\ D=n^2+5.$$

Proceeding as in case (iii), it is concluded that the concurrent exponential equations do not have any integer solutions.

Hence, it is proven that the simultaneous exponential equation (26) has no integer solution.

3 Conclusion

In this study, the integer solutions of two different kinds of Simultaneous Exponential Diophantine equations in three variables are investigated by applying standard solutions of the Thue equations. Similarly, one can pursue integer solutions to similar types of simultaneous exponential Diophantine equations by using general solutions of systems of Pell equations.

References

1) Somanath M, Raja K, Kannan J, Nivetha. Exponential Diophantine Equation in Three Unknowns. *Advances and Applications in Mathematical Sciences*. 2020;19(11):1113–1118. Available from: https://www.mililink.com/upload/article/1907508386aams_vol_1911_sep_2020_a4_p1113-1118__manju_somanath_and_s._nivetha.pdf.
2) Aggarwal S, Sharma SD, Vyas A. On the existence of solution of Diophantine equation 181^x+199^y=z^2. *International Journal of Latest Technology in Engineering*. 2020;9(8):85–86. Available from: https://www.ijltemas.in/DigitalLibrary/Vol.9Issue8/85-86.pdf.
3) Aggarwal S, Sharma N. On the non-linear Diophantine equation 379^x+397^y=z^2. *Open Journal of Mathematical Sciences*. 2020;4(1):397–399. Available from: https://pisrt.org/psrpress/j/oms/2020/1/43/on-the-non-linear-diophantine-equation.pdf.
4) Aggarwal S, Sharma SD, Chauhan R. On the non-linear Diophantine equation 331^x+349^y=z^2. *Journal of Advanced Research in Applied Mathematics and Statistics*. 2020;5(3&4):6–8. Available from: http://thejournalshouse.com/index.php/Journal-Maths-Stats/article/view/4.
5) Pandichelvi V, Sandhya P. Explorations of Solution for an Exponential Diophantine Equation p^x+[(p+1)]^y=z^2". *Turkish Journal of Computer and Mathematics Education*. 2021;12(1):659–662. Available from: https://doi.org/10.17762/turcomat.v12i1S.1957.
6) Pandichelvi V, Vanaja R. Novel Approach of Existence of Solutions to the Exponential Equation (3m^2+3)^x+[(7m^2+1)]^y=z^2. . Available from: https://doi.org/10.17762/turcomat.v12i1S.1861.
7) Pandichelvi V, Saranya S. Frustrating Solutions for two Exponential Diophantine Equations p^a+(p+3)^b-1=c^2 and [(p+1)]^a+p^b+1=c^2. *Natural Science Edition*. 2021;17(5):147–156. Available from: https://www.xisdxjxsu.asia/V17I5-13.pdf.
8) Aggarwal S. On the exponential Diophantine equation (2^((2m+1))-1)+13^y=z^2. 2021. Available from: https://pisrt.org/psrpress/j/easl/2021/1/on-the-exponential-diophantine-equation.pdf.
9) Pandichelvi V, Sandhya P. Investigation of Solutions to an Exponential Diophantine Equation [p_1]^x+[p_2]^y+[p_3]^z=M^2 for prime triplets (p_1,p_2,p_3 . *International Journal of Scientific Research and Engineering Development*. 2022;5(2):22–31. Available from: http://www.ijsred.com/volume5-issue2-part1.html#.
10) B U, Pandichelvi V. Perceiving Solutions for an Exponential Diophantine Equation linking Safe and Sophie Germain Prim es q^x+p^y=z^2. 2022. Available from: https://doi.org/10.58250/jnanabha.2022.52219.
11) Pandichelvi V, Vanaja R. Inspecting Integer Solutions for an Exponential Diophantine Equation (p)^x+[(p+2)]^y=z^2. . Available from: https://www.mililink.com/upload/article/167889248aams_vol_218_june_2022_a43_p4693-4701_v._pandichelvi_and_vanaja.pdf.
12) Aslanyan V, Kirby J, Mantova V. A Geometric Approach to Some Systems of Exponential Equations. *International Mathematics Research Notices*. 2022;2023(5):4046–4081. Available from: https://doi.org/10.1093/imrn/rnab340.
13) Dou C, Luo J. Complete solutions of the simultaneous Pell's equations (a^2+2)x^2-y^2=2 and[x]^2-bz^2=1 . *AIMS Mathematics*. 2023;8(8):19353–19373. Available from: https://doi.org/10.3934/math.2023987.
14) Borah PB, Dutta M. On Two Classes of Exponential Diophantine Equations. *Communications in Mathematics and Applications*. 2022;13(1):137–145. Available from: https://doi.org/10.26713/cma.v13i1.1676.