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RESEARCH ARTICLE

Construction of Special dio — triples

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Abstract

Objectives: Diophantine m-tuples are sets of positive integers with the property that the product of any two elements in the set increased by n is a square. The objective of this paper is to analyse the extendibility of pairs of elements to dio — triples for the various choices of n. **Method:** The problem of the occurrence of Diophantine triples is examined by various Mathematicians. There is no universal method to find the extension of dio — triples with specific properties. Diophantine equations form a very major part of research in Number theory. One among the important equations is the Pell's equation for which the general solutions can be derived. In this paper, the concept of Pell's equations is implemented to construct Diophantine triples. **Findings:** In this paper, we have studied the extendibility of (ϕ,τ) of elements in a commutative ring R with the property that $\phi\tau+\phi+\tau$ added with quadratic and bi-quadratic polynomials, yields a perfect square**.** The elements (ϕ,τ) are chosen as linear polynomials, polygonal numbers and centered polygonal numbers and are examined to form triples using different properties. **Novelty:** In this paper, we give some new examples of pairs of polynomial, polygonal and centered polygonal numbers and check for its extendibility as a triple. The examples illustrate various theoretical properties and construction for linear and quadratic polynomials. The Tables 1, 2 and 3 looks to be simple, but they require tremendous calculations and simplifications.

MSC Classification Number: 11D09, 11D99

Keywords: Special dio triples; Diophantine triples; Diophantine equations; Pellian Equation; Polygonal numbers; Centered polygonal numbers

1 Introduction

A set of positive integers $(a_1, a_2, \cdots a_m)$ is said to have the property $D(\lambda)$, $\lambda \in Z-(0\}$, if $a_i a_j + \lambda$ is a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuple with property $D(\lambda)$ [(1)]. The study of Diophantine m-tuple can be traced back to the third century AD, when the Greek Mathematician Diophantus discovered that $\left(\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right)$ is a set of four rationals which has the Diophantine property $D(\lambda)$ [(2)].

Fermat obtained the first quadruple $(1,3,8,120)$ with the property $D(1)$. The case $\lambda \neq 1$ have been studied by several mathematicians, for example, $(1,2,5)$ is a $D(-1)$ triple. If λ is of the form $4k+2$, then there does not exist a Diophantine quadruple with the above property[(3–7)]. The special dio — triple differs from the dio — triple as it is constructed where the product of any two members of the triple with the addition of their sum and increased by the given property is a perfect square.

2 Methodology

A special dio — triple (ϕ,τ,χ) is constructed from a pair (ϕ,τ) which satisfy the property $D(\lambda)$, where ϕ,τ and λ are polynomials in a single variable Θ. Solving the system of equations $\phi\chi+\phi+\chi+\lambda=E^2$; $\tau\chi+\tau+\chi+\lambda=F^2$ and on applying the transformations $E=\rho+(\phi+1)\varepsilon$ and $F=\rho+(\tau+1)\varepsilon$, the Pellian equation $\rho^2-(\phi+1)(\tau+1)\varepsilon^2=\lambda-1$ is generated, where $(\phi+1)(\tau+1)$ is not a perfect square and $\lambda-1\neq 0$. For the particular choice of $\varepsilon=1$ and for various choices of (ϕ,τ), we induce special dio — triples (ϕ,τ,χ) in the following sections.

3 Results and Discussion

3.1 Choice — I

For the different pairs of linear polynomial, we investigate for its extendibility as a special dio — triple satisfying the suitable property.

- **Case 1**

Let $\phi=4\Theta+3$ and $\tau=6\Theta+7$ be two linear polynomials to be extended to a special dio — triple with property $D\left(\Theta^2+4\Theta+5\right)$. Solving the Pellian equation, we obtain the values of E and F as $E=9\Theta+10$ and $F=11\Theta+14$ and thus obtain the special dio — triple $(4\Theta+3, 6\Theta+7, 20\Theta+23)$.

- **Case 2**

The pair (ϕ,τ) represented by a polynomial $(10\Theta+9, 4\Theta+11)$ is to be developed as a special dio — triple with property $D\left(9\Theta^2-6\Theta+2\right)$. By solving the Pellian equation yields $E=17\Theta+21$ and $F=11\Theta+23$. For either values of E or F we get unique χ as $\chi=28\Theta+43$ which leads to a triplet $(10\Theta+9, 4\Theta+11, 28\Theta+43)$. Table 1 represents various special dio — triples with the corresponding properties.

3.2 Choice — II

In this choice, we consider the different pairs of polygonal numbers of rank Θ which can be extended to special dio — triple with the property considered.

Let ϕ be the icosioctagonal number of rank Θ and τ be the triacontagonal number of rank Θ. We extend them to a special dio — triple (ϕ,τ,χ) by obtaining the values of E and F as $E=27\Theta^2-25\Theta+1$ and $F=28\Theta^2-26\Theta+1$ on solving the Pellian equation which is mentioned above. From the values of E or F we attain the special dio — triple $(13\Theta^2-12\Theta, 14\Theta^2-13\Theta, 55\Theta^2-51\Theta+1)$ with the property $D(14\Theta^4-27\Theta^3-14\Theta^2+25\Theta)$. Refer Table 2 for this choice.

3.3 Choice-III

In this choice, we search for the extendibility of the different pairs of centered polygonal numbers of rank Θ with the relevant property.

Taking $\phi=7\Theta^2+7\Theta+1$ and $\tau=8\Theta^2+8\Theta+1$ which are centered tetradecagonal and centered hexadecagonal numbers of rank Θ respectively, we attempt to extend this pair to a special dio — triple with the property $D(1+2\Theta+6\Theta^2-7\Theta^4)$. The value of χ is obtained as $\chi=29\Theta^2+31\Theta+7$ from the values of E and F which are given by $E=14\Theta^2+15\Theta+4$ and $F=15\Theta^2+16\Theta+4$. Table 3 may be referred for this choice.

Table 1. Special dio-triples (ϕ,τ,χ) and its D(λ)

Dio –Triples (ϕ,τ,χ)	D(λ)
$(8\Theta+5, 4\Theta+3, 24\Theta+19)$	$D\left(4\Theta^2+4\Theta+2\right)$
$(13\Theta+2, 4\Theta+11, 33\Theta+26)$	$D\left(12\Theta^2-72\Theta+1\right)$

Continued on next page

Table 1 continued

$(11\Theta+2, 9\Theta+3, 40\Theta+14)$	$D\left(\Theta^2+9\Theta+5\right)$
$(8\Theta+1, 4\Theta+13, 24\Theta+27)$	$D\left(4\Theta^2-48\Theta+9\right)$
$(9\Theta+5, 7\Theta+3, 32\Theta+19)$	$D\left(\Theta^2+2\Theta+2\right)$
$(3\Theta+5, 6\Theta+7, 19\Theta+27)$	$D\left(7\Theta^2+10\Theta+2\right)$
$(5\Theta+4, 7\Theta+3, 24\Theta+18)$	$D\left(\Theta^2+5\Theta+6\right)$

Table 2. Polygonal numbers and Special dio-triples with the property $D(\lambda)$

Polygonal numbers of rank Θ	**Dio-Triples (ϕ, τ, χ)**	$D(\lambda)$
Hexagonal & Octagonal	$(2\Theta^2-\Theta, 3\Theta^2-2\Theta,\ 11\Theta^2-5\Theta+1)$	$D(3\Theta^4+\Theta^3-6\Theta^2+3\Theta)$.
Decagonal & Dodecagonal	$(4\Theta^2-3\Theta, 5\Theta^2-4\Theta,\ 19\Theta^2-13\Theta+1)$	$D(5\Theta^4+\Theta^3-12\Theta^2+7\Theta)$
Icosagonal & Icosidigonal	$(9\Theta^2-8\Theta, 10\Theta^2-9\Theta,\ 39\Theta^2-35\Theta+1)$	$D\left(10\Theta^4-19\Theta^3-10\Theta^2+17\Theta\right)$
Icosagonal & Icositetragonal	$(9\Theta^2-8\Theta, 11\Theta^2-10\Theta,\ 40\Theta^2-36\Theta+1)$	$D\left(\Theta^4-2\Theta^3-19\Theta^2+18\Theta\right)$

Table 3. Centered polygonal numbers and dio-triples (ϕ, τ, χ) with the property $D(\lambda)$

Centered polygonal numbers of rank Θ	**Dio-Triples (ϕ, τ, χ)**	$D(\lambda)$
Centered octadecagonal & Centered icosagonal	$(9\Theta^2+9\Theta+1,\ 10\Theta^2+10\Theta+1, 39\Theta^2+39\Theta+7)$	$D(1+2\Theta+12\Theta^2+20\Theta^3+10\Theta^4)$
Centered square & Centered hexagonal	$(2\Theta^2+2\Theta+1,\ 3\Theta^2+3\Theta+1, 11\Theta^2+9\Theta+7)$	$D(3\Theta^4-2\Theta+1)$
Centered icosihexagonal & Centered icosioctagonal	$(13\Theta^2+13\Theta+1, 14\Theta^2+14\Theta+1, 55\Theta^2+53\Theta+7)$	$D(1-2\Theta-11\Theta^2+14\Theta^4)$

4 Conclusion

Formation of triples and quadruples has been a great interest for research using various properties and relations. Triples and quadruples can be generated for rational, irrational, Gaussian integers etc. In this paper, we have constructed the special dio — triples involving linear polynomials, polygonal numbers and Centered polygonal numbers. Researchers may attempt for other special dio — triples for different numbers with suitable properties.

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