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Bending Analysis for Stress and Deflection for Cross-Ply Laminated Composite Plate Using Higher Order Shear Deformation Theory Subjected to Sinusoidal Loading using Finite Element Method

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Abstract

Background: Laminated composite plates find extensive applications across aerospace, military, civil infrastructure, and automotive industries due to their advantageous properties, including superior fatigue resistance, high strength and stiffness resulting in reduced weight, and customizable fiber orientation, materials, and stacking patterns. Consequently, it becomes imperative to conduct analyses of deflections and stress levels. **Objectives:** In this current research, a mathematical formulation based on the higher-order deformation theory (HSDT) is employed, featuring 12 degrees of freedom per node. The focus of the analysis involves studying stress and deflection in two-layer cross-ply laminate plates. The obtained results are then compared with established findings in existing literature. Furthermore, a parametric investigation is conducted, exploring different side-to-thickness ratios at values of a/h equal to 4, 5, 10, 20, 40, 50, and 100. **Methodology:** In this research, the finite element analysis was performed with HSDT using MATLAB software. A two-layer laminate with material properties from category 2 was employed. The deflection and in-plane stresses were subsequently compared to analytical solutions provided by Kant & Manjunath, Pandya & Kant, and Pagano. **Findings:** The non-dimensional deflection values for a two-layer cross-ply configuration under sinusoidal loading conditions are 0.2133 for an a/h ratio of 4 and 0.1220 for an a/h ratio of 10. Additionally, the non-dimensional in-plane stress values, when considering an a/h ratio of 10, are as follows: σ_x at the top fiber is 0.7202, σ_y at the top fiber is 0.0857, and τ_{xy} at the top fiber is -0.0543. Conversely, σ_x at the bottom fiber is -0.0879, σ_y at the bottom fiber is -0.7206, and τ_{xy} at the bottom fiber is 0.0546. Detailed results are presented in tables, and they exhibit a high degree of agreement with established standard results. **Novelty:**

In the current analysis, the mathematical formulation incorporates 12 degrees of freedom per node, while a 9-noded rectangular element is employed for finite element meshing. A mesh size of 11 x 11 is utilized for the plate element. It's worth noting that these specific configurations have yielded highly accurate results, with errors of less than 10%.

Keywords: Displacement; Stresses; Higher Order Shear Deformation Theory; Laminated Composite Plates; Finite Element Method

1 Introduction

In classical plate theory (CLPT), the influence of transverse shear is neglected. In the first-order shear deformation theory (FSDT), transverse shear effects are considered but with a linear variation. However, in higher-order shear deformation theory, the incorporation of an appropriate displacement model adjustment leads to improved accuracy in stress resultants. An investigation into the application of the trigonometric shear deformation theory for analyzing the displacements and stress patterns in a cross-ply laminated beam under varying loads, as conducted by Tupe and Dahake[(1)]. The investigation of buckling analysis in a laminated composite skew plate was carried out by Mishra, Kumar, Samui and Roshni. They employed a C0 finite element (FE) model based on the higher-order shear deformation theory (HSDT) in combination with minimax probability machine regression (MPMR) and multivariate adaptive regression spline[(2)]. Komal Navale and Dr. Pise have introduced theories that posit a non-linear variation of transverse shear strain throughout the plate's thickness, resulting in transverse shear stress-free surfaces at both the top and bottom of the plate[(3)]. Research conducted by Thai, Ferreira and Nguyen-Xuan, involves an investigation employing a nonlocal strain gradient meshfree plate method that integrates the nonlocal strain gradient theory (NSGT), higher-order shear deformation theory (HSDT), and meshfree technique. This study focuses on conducting bending and free vibration analyses of laminated composite and sandwich nanoplates[(4)]. Dhuria, Grover and Goyal have undertaken a study in which they have formulated a novel higher-order hyperbolic shear deformation theory for analyzing the mechanical behavior of cross-ply and angle-ply multilayered plates. The proposed theory incorporates the secant hyperbolic function of the thickness coordinate within the displacement field, leading to non-linear displacement distributions while ensuring that both the upper and lower surfaces of the plates exhibit zero shear stresses[(5)]. Attia Bachiri, Ahmed Amine Daikh, and Abdelouahed Tounsi have introduced a mathematical model founded on an innovative higher shear deformation plate theory. This model serves to explore the thermo-elastic behavior of cross-ply laminated plates reinforced with carbon nanotubes (CNTRC) when subjected to thermal loading. Their study examines both functionally graded distributions (FG) and uniform distribution (UD) of carbon nanotube reinforcement material[(6)]. The research carried out by Naik and Sayyad, showcases a thermal and hygrothermal stress analysis of composite layered and sandwich plates with one dimension extending infinitely and being simply supported at the edges. This analysis is based on a novel fifth-order theory[(7)]. Chaubey, Kumar, Fic S, Barnat-Hunek D and Sadowska have conducted a study focusing on the hygrothermal analysis, which encompasses the influence of temperature and moisture loadings, in laminated composite skew conoids with reasonable depth and thickness. To address the hygrothermal challenges associated with these laminated composite skew conoids, the study incorporates a cubic variation in the displacement field and takes into account the cross-curvature effects of the shell[(8)]. Kumar R and Kumar have conducted research investigating the flexural behavior of laminated composite plates containing porosity, employing an enhanced third-order shear deformation theory. This theory guarantees the continuity of transverse shear stress at each layer interface while also maintaining zero transverse shear stresses at the top and bottom surfaces of the plate. Furthermore, they have developed a C0 finite-element (FE) formulation based on this theory[(9)]. Suresh Kumar, Dharma Raju and Vijaya Kumar Reddy conducted research to examine the behavior of composite structures and their failure mechanisms. They employed an analytical approach to investigate the free vibration characteristics of various laminated composite plates, utilizing a higher-order shear displacement model incorporating a zig-zag function[(10)]. Masoud Kazemi has introduced findings related to the buckling, of a bimodular laminated plate using a modified higher-order plate theory that incorporates seven kinematic variables. This study includes a comparison of the buckling coefficients derived from this theory with those obtained from the Mindlin plate theory and higher-order plate theory. Additionally, a comprehensive parametric analysis is conducted to examine how parameters such as bi-modularity, thickness ratio, aspect ratio, and lamination scheme influence the critical buckling load response[(11)]. Tarun Kant and Manjunatha employed an asymmetric C0 finite element model for their research[(12)]. Pandya and Kant conducted a finite element analysis of laminated composite plates employing a Higher-Order Displacement Model[(13)]. Pagano has provided exact solutions for rectangular bidirectional composites and sandwich plates[(14)].

- **Research gap:** The literature referenced does not contain any instances of employing a higher degree of freedom in nodes for finite element formulation or using higher-noded elements with smaller mesh sizes in the analysis.
- **Problem statement:** In the current investigation, the mathematical formulation incorporates 12 degrees of freedom per node, while a 9-noded rectangular element is utilized for meshing in the finite element formulations. The plate element employs a mesh size of 11 x 11.

In the current research, an analysis of stress in a simply supported cross-ply composite is conducted. This analysis utilizes finite element formulations founded on a higher-order shear deformation theory that incorporates twelve degrees of freedom. The governing differential equations are numerically solved using the Finite Element Method (FEM). The current theory is employed to determine the variation of displacement and stresses throughout the thickness of the laminate. In this research, an analysis is conducted on a square plate that is simply supported, considering sinusoidal loading conditions. A MATLAB program is employed to obtain finite element solutions for displacements and stresses. Solutions are derived for laminates with a cross-ply configuration, each having varying values of side-to-thickness ratios.

2 Methodology

The mathematical formulation for the displacement model, derived in accordance with the higher-order shear deformation theory, is presented in the following section, and it is expressed as:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) + z\theta_z(x,y) + z^2 w_0^*(x,y) + z^3\theta_z^*(x,y) \end{aligned} \tag{1}$$

Wherein, the terms are defined as follows:

Table 1. Defining the terms of displacement model

u_0, v_0, w_0	In-plane and transverse displacement of a point (x,y) on the midplane
$\theta_x, \theta_y, \theta_z$	Rotations of the normal to mid plane about x,y and z axes
$u_0^*, v_0^*, w_0^*, \theta_x^*, \theta_y^*, \theta_z^*$	Corresponding higher order shear deformation terms defined at the midplane.

The strain-displacement relations:

$$\begin{aligned} &\varepsilon_x = \varepsilon_{x_0} + z\kappa_x + z^2\varepsilon_{x_0}^* + z^3\kappa_x^*, \varepsilon_y = \varepsilon_{y_0} + z\kappa_y + z^2\varepsilon_{y_0}^* + z^3\kappa_y^*, \quad \varepsilon_z = \varepsilon_{z_0} + z\kappa_z^* + z^2\varepsilon_{z_0}^* \\ &\gamma_{xy} = \varepsilon_{xy} + z\kappa_{xy} + z^2\varepsilon_{xy}^* + z^3\kappa_{xy}^*, \gamma_{yz} = \phi_y + z\kappa_{yz} + z^2\phi_y^* + z^3\kappa_{yz}^*, \gamma_{xz} = \phi_x + zkk_{xz} + z^2\phi_x^* + z^3\kappa_{xz}^* \end{aligned} \tag{2}$$

The strain expressions mentioned above can be expressed in matrix form as follows:

$$\varepsilon_{MB}^{L}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{x_0}\\ \varepsilon_{y_0}\\ \varepsilon_{z_0}\\ \gamma_{xy}\end{Bmatrix}+z\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_z\\ \kappa_{xy}\end{Bmatrix}+z^2\begin{Bmatrix}\varepsilon_{x_0}^{*}\\ \varepsilon_{y_0}^{*}\\ \varepsilon_{z_0}^{*}\\ \varepsilon_{xy}^{*}\end{Bmatrix}+z^3\begin{Bmatrix}\kappa_x^{*}\\ \kappa_y^{*}\\ 0\\ \kappa_{xy}^{*}\end{Bmatrix}$$
$$\varepsilon_{S}^{L}=\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\phi_y\\ \phi_x\end{Bmatrix}+z\begin{Bmatrix}\kappa_{yz}\\ \kappa_{xz}\end{Bmatrix}+z^2\begin{Bmatrix}\phi_y^{*}\\ \phi_x^{*}\end{Bmatrix}+z^3\begin{Bmatrix}\kappa_{yz}^{*}\\ \kappa_{xz}^{*}\end{Bmatrix} \tag{3}$$

Constitutive relations concerning the laminate axes are derived in the following manner:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}^{L}=\left[\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&Q_{14}&0&0\\ Q_{21}&Q_{22}&Q_{23}&Q_{24}&0&0\\ Q_{31}&Q_{32}&Q_{33}&Q_{34}&0&0\\ Q_{41}&Q_{42}&Q_{43}&Q_{44}&0&0\\ 0&0&0&0&Q_{55}&Q_{56}\\ 0&0&0&0&Q_{65}&Q_{66}\end{bmatrix}^{L}\right]^{L}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^{L} \tag{4}$$

Components of stress resultants:

$$\begin{aligned}\sigma_x&=Q_{11}\varepsilon_x+Q_{12}\varepsilon_y+Q_{13}\varepsilon_z++Q_{14}\gamma_{xy}\\ \sigma_y&=Q_{12}\varepsilon_x+Q_{22}\varepsilon_y+Q_{23}\varepsilon_z+Q_{24}\gamma_{xy}\\ \sigma_z&=Q_{13}\varepsilon_x+Q_{32}\varepsilon_y+Q_{33}\varepsilon_z+Q_{34}\gamma_{xy}\\ \tau_{xy}&=Q_{41}\varepsilon_x+Q_{42}\varepsilon_y+Q_{43}\varepsilon_z+Q_{44}\gamma_{xy}\\ \tau_{yz}&=Q_{55}\gamma_{yz}+Q_{56}\gamma_{xz}\\ \tau_{xz}&=Q_{65}\gamma_{yz}+Q_{66}\gamma_{xz}\end{aligned} \tag{5}$$

3 Results and Discussion

In this section, the results obtained from our newly developed HSDT12 model, which employs 12 degrees of freedom per node, are initially compared with the previously established standard results obtained from analytical solutions published by Kant & Manjunath, Pandya & Kant, and Pagano. The comparison of results is presented in Tables 2 and 3.

Table 2. (Non-Dimensional) Maximum Deflection for a simply supported Un-symmetric cross ply (0/90) square plate under Sinusoidal Load (Material 2) - SS1 Boundary Conditions

Source	a/h	w	Error (w)
Present (HSDT12)	4	0.2133	-
Kant & Manjunath [12]	4	0.2007	5.87 %
Pandya and Kant [13]	4	0.2020	5.30 %
Present (HSDT12)	10	0.1220	-
Kant & Manjunath [12]	10	0.1220	0.0 %
Pandya and Kant [13]	10	0.1220	0.0 %

Table 3. (Non-Dimensional) Maximum stresses for a simply supported Un-symmetric cross ply (0/90) square plate under Sinusoidal Load (Material 2) - SS1 Boundary Conditions

Source	a/h	z/h	σx	Error % (σx)	σy	Error % (σy)	τxy	Error % (τxy)
Present (HSDT12)	4	0.5	0.7626	-	0.0945	-	-0.0529	-
Kant & Manjunath [12]	4	0.5	0.8225	7.86 %	0.1081	14.44 %	-0.0578	9.25 %
Pandya and Kant [13]	4	0.5	0.8000	4.91 %	0.1038	9.88 %	-0.0579	9.44 %
Pagano [14]	4	0.5	0.7807	2.38 %	0.0955	1.10 %	-0.0591	11.70 %

Continued on next page

Table 3 continued

Present (HSDT12)	4	-0.5	-0.1089	-	-0.8031	-	0.0582	-
Kant & Manjunath [12]	4	-0.5	-0.0957	12.12 %	-0.7781	3.11 %	0.0581	0.30 %
Pandya and Kant [13]	4	-0.5	-0.1037	4.77 %	-0.8000	0.38 %	0.0579	0.59%
Pagano [14]	4	-0.5	-0.1098	0.83 %	-0.8417	4.81 %	0.0588	0.95 %
Present (HSDT12)	10	0.5	0.7202	-	0.0857	-	-0.0546	-
Kant & Manjunath [12]	10	0.5	0.7379	2.45 %	0.0884	3.19 %	-0.0536	1.80 %
Pandya and Kant [13]	10	0.5	0.7367	2.29 %	0.0884	3.19 %	-0.0537	1.64 %
Pagano [14]	10	0.5	0.7300	1.36 %	-	-	-0.0538	1.46 %
Present (HSDT12)	10	-0.5	-0.0879	-	-0.7206	-	0.0543	-
Kant & Manjunath [12]	10	-0.5	-0.0874	0.58 %	-0.7368	2.25 %	0.0538	0.99 %

The study employs the following conditions: The Finite Element Method is applied, utilizing the Higher Order Shear Deformation Theory (HSDT). A square plate with a/b ratio of 1 is subjected to simply supported boundary conditions. MATLAB coding is used to obtain the Finite Element (FE) results, and these values are subsequently validated against established standard values found in the literature.

The material properties employed in the examples are as follows:

Material 1:

E1 =40 Gpa

E2=E3 =1 Gpa

G12 = G13 = 0.6 Gpa

G23 = 0.5 Gpa

ϑ 12 = ϑ 13 = ϑ 23 = 0.25 Gpa

Support condition = Simply supported

Loading = Sinusoidal applied Transverse direction.

Material 2:

E1 =25 Gpa

E2=E3 =1 Gpa

G12 = G13 = 0.5 Gpa

G23 = 0.2 Gpa

ϑ 12 = ϑ 13 = ϑ 23 = 0.25 Gpa

Support condition = Simply supported

Loading = Sinusoidal applied Transverse direction.

The results presented in both the table and plots are derived using a non-dimensional form.

The applied boundary condition is SS1, as depicted below (Figure 1):

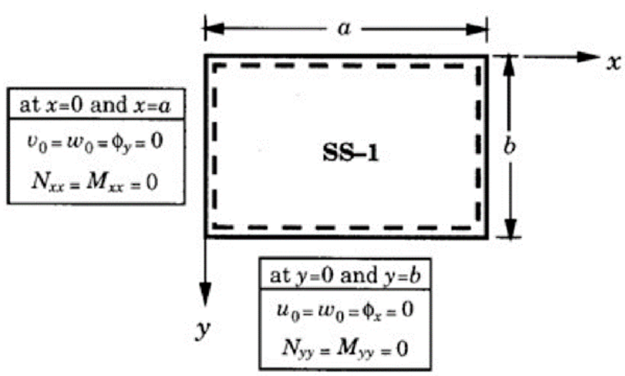

Fig 1. SS1 Boundary Condition

Example 1:

In this example, a square plate with an unsymmetric cross-ply (0/90) orientation subjected to sinusoidal loading (Material 2) and placed under SS1 boundary conditions is being analyzed. A comparison of the (non-dimensional) maximum deflection values is presented in Table 2.

It's important to observe that the precision of the current HSDT 12 model improves as the a/h ratio varies between 4 and 10. Furthermore, the table indicates that the percentage of errors decreases as the a/h ratio ranges from 4 to 10, and these errors become insignificant when the thickness ratio a/h $\geq$ 10.

In the same example, Table 3 displays a comparison of (non-dimensional) stress values. It is noteworthy that the values predicted by the current HSDT12 model align with those from Kant & Manjunath, Pandya & Kant, and Pagano. Specifically, for a/h=4, the percentage difference is less than 10% when compared to Kant & Manjunath and Pandya & Kant, while it falls within the range of 10% to 15% for Pagano.

In the following section, tabular and graphical representations of the variations in deflection and stresses across the thickness of 2-layer cross-ply laminates under sinusoidal loading have been provided.

Table 4. Throughthickness variation of transverse displacement (w) (Non-Dimensional) for asimply supported Un-symmetric cross ply 2 layered (0/90) square plate subjectedto Sinusoidal Load (Material 2) -SS1 Boundary condition for various a/h ratios

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.1950	0.1642	0.1212	0.1101	0.1073	0.1069	0.1065
-0.375	0.1969	0.1655	0.1215	0.1102	0.1073	0.1070	0.1065
-0.250	0.1989	0.1666	0.1217	0.1102	0.1073	0.1070	0.1065
-0.120	0.2010	0.1677	0.1219	0.1102	0.1073	0.1070	0.1065
0.000	0.2033	0.1686	0.1220	0.1103	0.1073	0.1070	0.1065
0.000	0.2033	0.1686	0.1220	0.1103	0.1073	0.1070	0.1065
0.125	0.2056	0.1696	0.1220	0.1103	0.1073	0.1070	0.1065
0.250	0.2081	0.1704	0.1220	0.1102	0.1073	0.1070	0.1065
0.375	0.2106	0.1711	0.1219	0.1102	0.1073	0.1070	0.1065
0.500	0.2133	0.1718	0.1217	0.1101	0.1073	0.1069	0.1065

Table 5. Throughthickness variation of normal stress (σ_x) (non-dimensional) for asimply supported Un-symmetric cross ply 2 layered (0/90) square plate subjectedto Sinusoidal Load (Material 2) -SS1 Boundary condition for various a/h ratios

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	-0.7626	-0.7460	-0.7202	-0.7122	-0.7100	-0.7098	-0.7094
-0.375	-0.3491	-0.3617	-0.3764	-0.3792	-0.3799	-0.3799	-0.3800
-0.250	-0.0231	-0.0333	-0.0466	-0.0498	-0.0506	-0.0507	-0.0508
-0.120	0.2517	0.2621	0.2749	0.2775	0.2781	0.2782	0.2783
0.000	0.5114	0.5479	0.5939	0.6042	0.6067	0.6070	0.6074
0.000	0.0219	0.0206	0.0187	0.0182	0.0180	0.0180	0.0180
0.125	0.0376	0.0367	0.0351	0.0346	0.0345	0.0345	0.0344
0.250	0.0561	0.0544	0.0519	0.0511	0.0509	0.0509	0.0509
0.375	0.0792	0.0751	0.0694	0.0678	0.0674	0.0674	0.0673
0.500	0.1089	0.1001	0.0879	0.0848	0.0840	0.0839	0.0838

Table 6. Throughthickness variation of normal stress (σ_y) (non-Dimensional) for asimply supported Un-symmetric cross ply 2 layered (0/90) square plate subjectedto Sinusoidal Load (Material 2) -SS1 Boundary condition for various a/h ratios

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	-0.0945	-0.0910	-0.0857	-0.0842	-0.0839	-0.0838	-0.0838
-0.375	-0.0662	-0.0667	-0.0672	-0.0673	-0.0673	-0.0673	-0.0673
-0.250	-0.0440	-0.0464	-0.0497	-0.0506	-0.0508	-0.0508	-0.0509
-0.120	-0.0258	-0.0287	-0.0329	-0.0340	-0.0343	-0.0344	-0.0344
0.000	-0.0099	-0.0126	-0.0166	-0.0176	-0.0179	-0.0179	-0.0180
0.000	-0.5605	-0.5741	-0.5974	-0.6048	-0.6068	-0.6071	-0.6074
0.125	-0.2845	-0.2801	-0.2777	-0.2781	-0.2782	-0.2783	-0.2783

Continued on next page

Table 6 continued

0.250	0.0121	0.0262	0.0448	0.0493	0.0505	0.0506	0.0508
0.375	0.3633	0.3671	0.3756	0.3789	0.3798	0.3799	0.3800
0.500	0.8031	0.7646	0.7206	0.7119	0.7099	0.7097	0.7094

Table 7. Throughthickness variation of shear stress (τ_{xy}) (non-Dimensional) for a simply supported Un-symmetric cross ply2 layered (0/90) square plate subjected to Sinusoidal Load (Material 2) -SS1 Boundary condition for various a/h ratios

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.0529	0.0538	0.0546	0.0546	0.0546	0.0547	0.0547
-0.375	0.0393	0.0399	0.0407	0.0409	0.0410	0.0410	0.0410
-0.250	0.0261	0.0264	0.0270	0.0272	0.0273	0.0273	0.0273
-0.120	0.0134	0.0133	0.0135	0.0136	0.0136	0.0136	0.0136
0.000	0.0010	0.0005	0.0000	0.0000	-0.0001	-0.0001	-0.0001
0.000	0.0010	0.0005	0.0000	0.0000	-0.0001	-0.0001	-0.0001
0.125	-0.0112	-0.0123	-0.0134	-0.0137	-0.0137	-0.0137	-0.0138
0.250	-0.0234	-0.0251	-0.0269	-0.0273	-0.0274	-0.0274	-0.0274
0.375	-0.0357	-0.0380	-0.0405	-0.0410	-0.0411	-0.0411	-0.0411
0.500	-0.0482	-0.0512	-0.0543	-0.0547	-0.0547	-0.0548	-0.0548

In the section below, tabular representations of through-thickness variations in deflection and stresses for 4-layer cross-ply laminates under sinusoidal loading can be found. This encompasses different ply orientations, namely [0/90/90/0] and [0/90/0/90], at both a/h = 10 and a/h = 20.

Table 8. Effectof Ply orientation on displacement (w) and normal stress (σ_x), for asimply supported Unsymmetrical cross ply 4 Layered square plate (Material 1) -SS1Boundary condition, Sinusoidal Loading

	W, a/h =10		W, a/h =20			σx, a/h =10		σx, a/h =20	
z/h	**0/90/90/0**	**0/90/0/90**	**0/90/90/0**	**0/90/0/90**	**z/h**	**0/90/90/0**	**0/90/0/90**	**0/90/90/0**	**0/90/0/90**
-0.500	0.043	0.0456	0.0319	0.0365	-0.500	-0.5624	-0.5211	-0.5543	-0.5065
-0.438	0.0431	0.0456	0.0319	0.0365	-0.438	-0.4643	-0.4220	-0.4775	-0.4200
-0.375	0.0431	0.0457	0.0319	0.0365	-0.375	-0.3774	-0.3297	-0.4038	-0.3352
-0.313	0.0431	0.0457	0.032	0.0365	-0.313	-0.3002	-0.2431	-0.3326	-0.2518
-0.250	0.0432	0.0458	0.032	0.0365	-0.250	-0.2311	-0.1609	-0.2636	-0.1695
-0.250	0.0432	0.0458	0.032	0.0365	-0.250	-0.0075	-0.0066	-0.0083	-0.0072
-0.188	0.0432	0.0458	0.032	0.0365	-0.188	-0.0052	-0.0039	-0.0061	-0.0046
-0.125	0.0433	0.0458	0.032	0.0365	-0.125	-0.0030	-0.0012	-0.0040	-0.0020
-0.063	0.0433	0.0459	0.032	0.0365	-0.063	-0.0009	0.0014	-0.0018	0.0006
0.000	0.0433	0.0459	0.032	0.0365	0.000	0.0011	0.0039	0.0003	0.0031
0.000	0.0433	0.0459	0.032	0.0365	0.000	0.0011	0.1492	0.0003	0.1547
0.063	0.0433	0.0459	0.032	0.0365	0.063	0.0032	0.2282	0.0024	0.2361
0.125	0.0434	0.046	0.032	0.0365	0.125	0.0053	0.3103	0.0045	0.3183
0.188	0.0434	0.046	0.032	0.0365	0.188	0.0075	0.3967	0.0067	0.4016
0.250	0.0434	0.046	0.032	0.0365	0.250	0.0098	0.4888	0.0089	0.4862
0.250	0.0434	0.046	0.032	0.0365	0.250	0.2274	0.0150	0.2629	0.0136
0.313	0.0434	0.046	0.032	0.0365	0.313	0.2973	0.0182	0.3320	0.0163
0.375	0.0435	0.046	0.032	0.0365	0.375	0.3753	0.0216	0.4032	0.0191
0.438	0.0435	0.046	0.032	0.0365	0.438	0.4632	0.0253	0.4771	0.0219
0.500	0.0435	0.046	0.032	0.0365	0.500	0.5626	0.0294	0.5539	0.0249

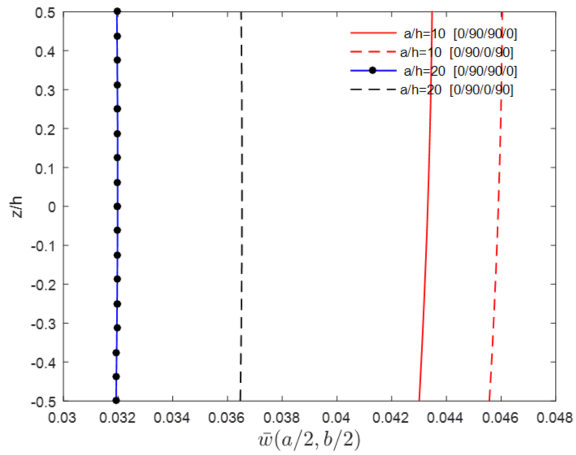

Fig 2. Effect of Ply orientation on displacement w, for asimply supported Un-symmetric cross ply 4 Layered square plate -SS1 Boundary condition, -Sinusoidal Loading

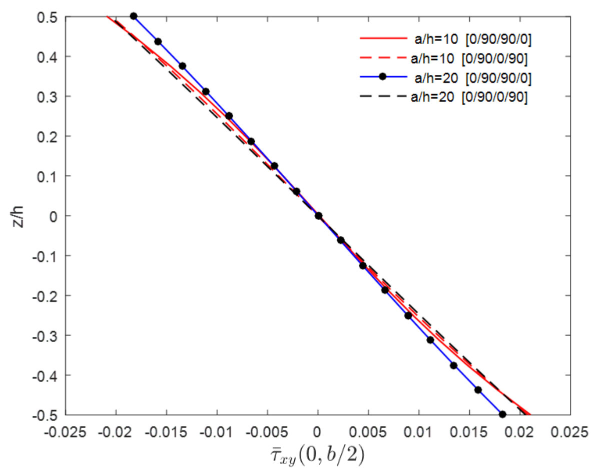

Fig 3. Effect of Ply orientation on shear stress (τ_{xy}),for a simply supported Unsymmetrical cross ply 4 Layered square plate -SS1Boundary condition, -Sinusoidal Loading.

4 Conclusion

In this study, the analysis of laminated composite plates is carried out utilizing the finite element method, based on the Higher Order Shear Deformation Theory (HSDT12). To achieve this, a MATLAB code has been developed, and analyses have been conducted for various layer configurations, orientations, and a/h ratios under sinusoidal load. The results obtained are then subjected to verification against established values from existing literature, revealing a high degree of consistency with the values reported in the literature.

In the current analysis, a mathematical formulation that includes 12 degrees of freedom per node has been utilized, and a 9-noded rectangular element has been employed for meshing in finite element formulations. The plate element has been assigned a mesh size of 11 x 11. All of the aforementioned particulars have produced results of high accuracy, with errors below 10%.

When comparing the (non-dimensional) displacement values, it's important to observe that the accuracy of the current HSDT 12 model improves as the a/h value ranges from 4 to 10.

It is additionally observed from Table 1 that the percentage of errors decreases as the a/h value varies from 4 to 10, and these errors become insignificant when the thickness ratio a/h is greater than or equal to 10.

Table 2 represents the comparison of (non-dimensional) stress values. The values predicted by the current HSDT12 model are noted to align with those of Kant & Manjunath, Pandya & Kant, and Pagano. For Kant & Manjunath and Pandya & Kant, the percentage difference remains below 10% at a/h=4, while for Pagano, it falls within the range of 10% to 15% at the same a/h value. Moreover, the investigation can be extended by augmenting the degrees of freedom per node, expanding the node count per element, and increasing the element's mesh size.

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