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Cordial Labeling of Subdivision of Central Edge of Bistar Graph and Spider Graph

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Abstract

Objectives: To analyse cordial labelling of subdivision of central edge of bistar graph and Spider graph. **Methods:** Cordial labeling is defined as a function $g : V(\theta) \to \{0,1\}$ in which each edge ab is assigned the label $|g(a) - g(b)|$ with the conditions $\left|v_g(0) - v_g(1)\right| \leq 1$ and $\left|e_g(0) - e_g(1)\right| \leq 1$ 1 where $v_g(0)$ and $v_g(1)$ signify the number of vertices with 0's and 1's, similarly $e_g(0)$ *and* $e_g(1)$ signify the number of edges with 0's and 1's. **Findings:** In this paper, it is proved that subdivision of central edge of Bistar graph and spider graph with n spokes admit cordial labeling. **Novelty:** We have subdivided the central edge of the bistar graph with a new vertex w and analyzed for cordial labeling. We have also proved spider graph with n spokes admit cordial labeling

Keywords: Star Graph; Bistar Graph; Subdivision; Spider Graph; Cordial Labeling

1 Introduction

When certain criteria are met, graph labelling is the process of giving labels—which are represented by integers—to vertices, edges, or both. Cordial labeling was introduced by Cahit[1] in 1987 and it was found to be a less effective variation of graceful and harmonious labeling. In[2] Devakirubanithi, et.al established graphs such as uniform sub-divided shell bow graph, uniform sub-divided shell flower graph, one point union of multiple sub-divided shell graph, sub-divided shell graph with star graphs coupled to the apex and path vertices are cordial. In[3] Pariksha Gupta, et.al proved that Cordial labeling pattern for star of bistar graph. In[4] Ashraf Elrokh, et.al introduced some new results on Cordial labeling, total Cordial labeling and present necessary and sufficient conditions for Cordial labeling, total Cordial labeling for Corona Product of paths and Second Order of Lemniscate Graphs. In this paper, we prove that subdivision of central edge of Bistar graph and spider graph with n spokes admit cordial labeling.

Definition 1 . Cordial labeling[5] is defined as a function $g : V(\theta) \to \{0,1\}$ in which each edge ab is assigned the label $|g(a) - g(b)|$ with the conditions $\left|v_g(0) - v_g(1)\right| \leq 1$ and $\left|e_g(0) - e_g(1)\right| \leq 11$ where $v_g(0)$ and $v_g(1)$ signify the number of vertices with 0's and 1's, similarly $e_g(0)$ *and* $e_g(1)$ signify the number of edges with 0's and 1's.

Definition 2. The subdivision of Bistar graph $< K_{1,n}, K_{1,m} : w >$ is obtained by joining the centre v and v' of the star graph $K_{1,n}$ and $K_{1,m}$ to a new vertex w.

Definition 3. The Spider graph $S_{n,2}$ is obtained by attaching a pendent edge to each vertex of the star graph.

Theorem 4. Subdivision of central edge of Bistar graph $< K_{1,n}, K_{1,m} : w >$ is cordial.

Proof: Let $G = < K_{1,n}\ ,\ \ K_{1,m}\ : w >$ be the subdivision of the central edge of the bistar graph where n and m are the vertices of the two different star graphs. Let the vertices of $< K_{1,n}\ ,\ \ K_{1,m}\ : w >$ be labeled as a_i where $i\ =\ 0,\ 1,\ 2,\ 3,\ ...,n,n+1,n+2,\ ...,\ n+m+1$. Fix the new vertex w as $a_0^{'} = 0$

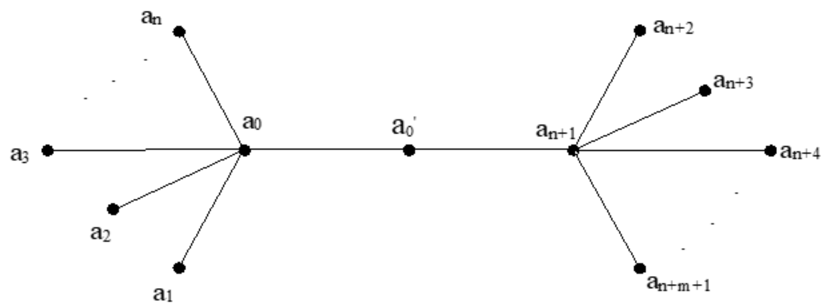

Fig 1. Generalized subdivision of central edge of the bistar graph

The graph $(< K_{1,n}\ ,\ \ K_{1,m}\ : w >)$ contains $n+m+3$ vertices and $n+m+\ 2$ edges.

Case 1 : When n is even and m is odd

Define the vertex labeling as follows

$$f\left(a_i\right) = \begin{cases} 0, \text{ if } i\ \equiv 1,3(\bmod 4) \\ 1, \text{ if } i\ \equiv 0,2(\bmod 4) \end{cases}$$

The number of vertices marked with 0 and 1 is defined as follows:

$V_f(0) = \left[\frac{n+m+3}{2}\right]$

$V_f(1) = \left[\frac{n+m+3}{2}\right]$

The number of edges marked with 0 and 1 is defined as follows:

$e_f(0) = \ \left[\frac{n+m+2}{2}\right]$

$e_f(1) = \ \left[\frac{n+m+2}{2}\right] + 1$

Case 2 : When n is even and m is even

Define the vertex labeling as follows

$$f\left(a_i\right) = \begin{cases} 0, \text{ if } i\ \equiv 1,3(\bmod 4) \\ 1, \text{ if } i\ \equiv 0,2(\bmod 4) \end{cases}$$

The number of vertices labeled with 0 and 1 is defined as follows:

$V_f(0) = \left[\frac{n+m+3}{2}\right] + 1$

$V_f(1) = \left[\frac{n+m+3}{2}\right]$

The number of edges labeled with 0 and 1 is defined as follows

$e_f(0) = \ \left[\frac{n+m+2}{2}\right]$

$e_f(1) = \ \left[\frac{n+m+2}{2}\right]$

Case 3 : When n is odd and m is odd

Define the vertex labeling as follows

$$f\left(a_i\right) = \begin{cases} 0, \text{ if } i\ \equiv 0,3(\bmod 4) \\ 1, \text{ if } i\ \equiv 1,2(\bmod 4) \end{cases}$$

The number of vertices marked with 0 and 1 is defined as follows:

$V_f(0) = \left[\frac{n+m+3}{2}\right] + 1$
$V_f(1) = \left[\frac{n+m+3}{2}\right]$
The number of edges marked with 0 and 1 is defined as follows:
$V_f(1) = \left[\frac{n+m+3}{2}\right]$
$e_f(0) = \left[\frac{n+m+2}{2}\right]$
From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.
Therefore, Subdivision of the central edge of the bistar graph $< K_{1,n}, \ K_{1,m} : w >$ admits Cordial labeling.
Illustration 5. Case 1.

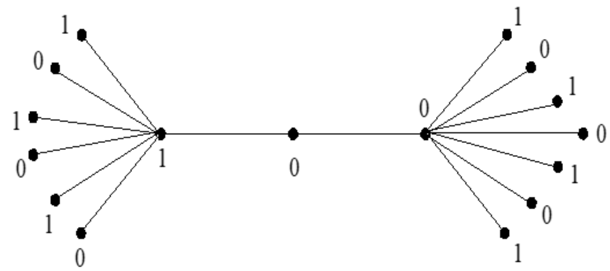

Fig 2. Cordial labeling of $< K_{1,6}, \ K_{1,7} : w >$.

$$Here, \ n = 6, \ m = 7, \ V = 16, \ E = 15, V_f(0) = 8, \ V_f(1) = 8, \ e_f(0) = 7, \ e_f(0) = 8$$

Theorem 6. The Spider graph with n spokes $S_{n,2}$ is cordial when n is even

Proof. Let $S_{n,2}$ be the Spider graph where n is the number of vertices of the star graph. Here, the vertices of the Star graph are denoted by a_i where $i = 1, 2, 3, \ldots, n$ and the vertices joining the star graph to the pendent vertices are denoted by b_j where $j = 1, 2, 3, \ldots, m$. Fix the Central vertex of the Star graph as $a_0 = 0$. The graph $(S_{n,2})$ contains $2n + 1$ vertices and $2n$ edges

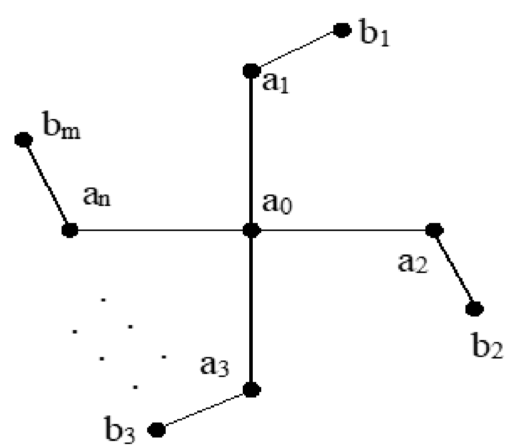

Fig 3. Generalized Spider graph

Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, \text{ if } i \equiv 0, 2 \pmod 4 \\ 1, \text{ if } i \equiv 1, 3 \pmod 4 \end{cases}$$

$$f(b_j) = \begin{cases} 0, \text{ if } j \equiv 0,3(\bmod 4) \\ 1, \text{ if } j \equiv 1,2(\bmod 4) \end{cases}$$

Case 1: if $n \equiv 0(mod4)$

The number of vertices marked with 0 and 1 is defined as follows:

$$V_f(0) = \left[\frac{2n+1}{2}\right] + 1$$

$V_f(1) = \left[\frac{2n+1}{2}\right]$

Case 2: if $n \equiv 2(mod4)$

The number of vertices marked with 0 and 1 is defined as follows:

$$V_f(0) = \left[\frac{2n+1}{2}\right]$$

$V_f(1) = \left[\frac{2n+1}{2}\right] + 1$

For both Case (1) and Case (2), the number of edges marked with 0 and 1 is defined as follows

$e_f(0) = \left[\frac{2n}{2}\right]$

$e_f(1) = \left[\frac{2n}{2}\right]$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)|$.

The Spider graph $S_{n,2}$ admits cordial labeling.

Illustration 7. Case 1.

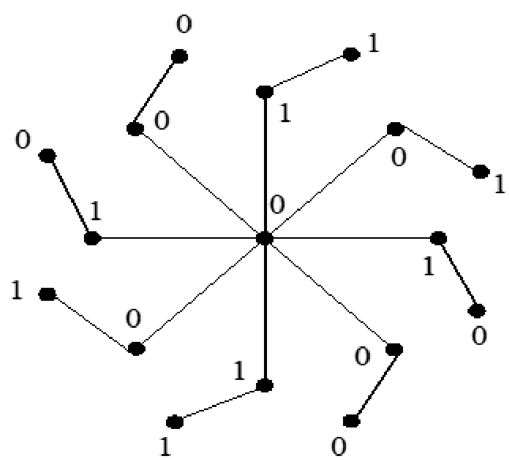

Fig 4. Cordial labeling of $S_{8,2}$

$$Here,\ n = 8,\ V = 17,\ E = 16, V_f(0) = 9,\ V_f(1) = 8,\ e_f(0) = 8,\ e_f(1) = 8$$

2 Conclusion

One of the important areas of research is graph Labeling. We have presented that subdivision of central edge of the bistar graph and Spider graph with n spokes admits cordial labeling. We have seen some illustrations which justify the theorems.

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