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A Novel Approach on Plithogenic Interval Valued Neutrosophic Hypersoft Sets and its Application in Decision Making

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Abstract

Objectives: In problem solving process, we have advanced the study of plithogenic interval valued neutrosophic hypersoft set, to analyse with all the appendages and traits under consideration for getting the better accuracy for the multi criterion decision making environment. **Methods:** Based on the combination of hypersoft sets, plithogenic sets and neutrosophic fuzzy sets, a plithogenic interval valued neutrosophic hypersoft set has been proposed. **Findings:** The t_{norm} , t_{conorm} , accuracy function and plithogenic interval valued hypersoft set-TOPSIS algorithm has been proposed. To validate the above findings, it has been compared with the Fuzzy-TOPSIS for two different environments with different weightage. The results were quite interesting and it exactly matches. **Novelty:** By the concept of plithogenic interval valued neutrosophic hypersoft set, the results can be viewed with more accuracy for the linguistic variables rather than the crisp values. An illustrative example for multi criterion decision-making environment is solved by using the proposed method by plithogenic interval valued hypersoft set TOPSIS and it has been compared with the Fuzzy-TOPSIS.

Keywords: Hypersoft Set; Plithogenic Hypersoft Set; Neutrosophic Set; Interval Valued Neutrosophic Set; Interval Valued Neutrosophic Hypersoft Set

1 Introduction

It is always necessary to rationalise and deal with uncertainties, ambiguities, and hazy data using a very dependable mathematical method. This has been urged the researchers to come with a different mathematical idea, to compete with the real time problems that we are facing in our day today life. Zadeh, in 1965 introduced the fuzzy concept, a finest idea to deal with the uncertainties which is concerned about the membership

degree in terms of crisp values which evolve in the interval $[0,1]$. In certain situations, this was not satisfying the need for that particular environment, then came the fuzzy interval valued set, where the degrees of membership lie in the interval $[0,1]$. Then the intuitionistic fuzzy set (IFS) was introduced to be dealt with non-membership degree also, which was proposed by Atanassov. In 2020 Atanassov⁽¹⁾ proposed interval valued intuitionistic fuzzy sets. Eventually, IFS do not meet the indeterminate information, which is very common in every walk of life, this made the introduction of Neutrosophic fuzzy set (NFS) by Smarandache in 1998, which incorporates the truthiness, indeterminacy and falsity. The parameterized concept was not met by the previous concepts, Molodtsov in 1999 introduced soft sets, later Zhao⁽²⁾ reframed in terms of single valued neutrosophic hypersoft set. The traits must be further separated into traits in the majority of our real-life multi-criteria decision-making (MCDM) scenarios in order to make a better judgement. This was overturned by Smarandache's⁽³⁾ to explain the plithogenic set in terms of probability and statistics with some illustrative examples in 2021. In 2019, Basset⁽⁴⁾ proposed a hybrid plithogenic decision making algorithm with quality function development for selecting supply chain sustainability metrics with the plithogenic aggregation operations. Basset⁽⁵⁾ in 2021, initiated the plithogenic concept for rough numbers to increase the accuracy of the results and also solved the MCDM by best worst method (BMW). Nivetha⁽⁶⁾ in 2021 debuted a MCDM approach on dual system of decision making which has been validated with the COVID-19 pandemic situation by using frequency matrix multi attributes decision making technique for plithogenic hypersoft set (PHSS). Priyadarshini^{(7), (8)} in 2020 emphasised on plithogenic neutrosophic sets and plithogenic cubic sets. Entropy measures on plithogenic sets were initiated by Quek⁽⁹⁾ in 2020 and shown that they are generalisation structures that may be used to solve a wide range of practical issues in MCDM environments. In 2022, Ahmad⁽¹⁰⁾ developed mathematical modelling and artificial intelligence-based decision making for COVID-19 suspects backed by novel distance and similarity measures on plithogenic hypersoft sets. Wang⁽¹¹⁾ in 2023, proposed a VIKOR method for plithogenic probabilistic linguistic MAGDM and applications to sustainable supply chain financial risk evaluation.

In this article, a novelty advancement of plithogenic interval valued neutrosophic hypersoft set (PIVNHSS) has been proposed by combining the plithogenic hypersoft set, plithogenic neutrosophic set and interval valued neutrosophic hypersoft set. To validate this approach t_{norm}, t_{conorm} , accuracy function and PIVNHSS-TOPSIS is proposed in order to obtain a more exact solution to the MCDM issues. Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), is a pioneer method in solving a MCDM. The trait may have values in a MCDM context, and each trait value has a neutrosophic interval valued degree of appendages for each alternative. This has been verified by the two different sets of alternatives and also it has been compared with the fuzzy TOPSIS. The results were quite inspiring in this regard. This proposed PIVNHSS can be widely used in many industrial sectors where the traits will have a contradiction degree defined between each trait value and the dominant trait value which deals with uncertainty and imprecision which are best suited for process control, Quality control, Fault Diagnosis, Decision Support Systems and Optimization. This approach has filled the research gap between plithogenic neutrosophic hypersoft set and interval valued hypersoft set, which makes all the traits and appendages to inter-relate with each other so that all the aspects under the multi criterion decision making were taken into consideration. In this regard, the computational process becomes time consuming and the results were quite precise and accurate.

Preliminaries

Some of the fundamental definitions for this study are included in this section.

Let us consider \tilde{U} be the non-void universal set. $P(\tilde{U})$ be the power set of \tilde{U} , whereas $X \subseteq \tilde{U}$ be a finite set of alternatives. Let \tilde{A} be a finite set with n unique traits given by $\tilde{A} = (a_1, a_2, \dots, a_n)$ where $n \geq 1$. a_1, a_2, \dots, a_n are the trait values belonging to the sets A_1, A_2, \dots, A_n respectively, where $A_i \cap A_j$, for $i \neq j$ and $i, j \in 1, 2, \dots, n$.

Definition 1.1. (12)

Let \tilde{U} be the universal set and λ be the set of traits concerning \tilde{U} . Let $P(\tilde{U})$ be the power set of \tilde{U} and $\tilde{A} \subseteq \lambda$. A pair (ξ, \tilde{A}) is called a soft set over \tilde{U} and it is represented as $\xi : \tilde{A} \rightarrow P(\tilde{U})$.

It can also be represented as:

$$(\xi, \tilde{A}) = \{\xi(e) \in P(\tilde{U}) : e \in \lambda, \xi(e) = \varphi \text{ if } e \notin \tilde{A}\}.$$

Definition 1.2. (13)

Let \tilde{U} be the universal of discourse and $P(\tilde{U})$ be the power set of \tilde{U} . Consider $m = (m_1, m_2, \dots, m_n); n \geq 1$, be a set of traits and set M_i a set of corresponding sub-traits of m_i respectively with $M_i \cup M_j = \varphi$ for $n \geq 1$ for each $i, j \in 1, 2, \dots, n$ and $i \neq j$. Assume $M_i \times M_i \times \dots \times M_i = (a_{1f} \times a_{2g} \times \dots \times a_{nh})$ be a collection of multi-attributes, where $1 \leq f \leq \alpha$, $1 \leq g \leq \beta$ and $1 \leq h \leq \gamma$ where $\alpha, \beta, \gamma \in N$. I be a collection of all interval-valued neutrosophic subsets over \tilde{U} . Then the pair $(M_1 \times M_2 \times \dots \times M_n =)$ is said to be IVNHSS over \tilde{U} and its representation is given by $\xi : M_1 \times M_2 \times \dots \times M_n = \rightarrow I$.

It is also defined as $(\xi, \check{A}) = \left\{ \check{a}, \xi_{\check{A}}(\check{a}) : \check{a} \in \check{A}, \xi_{\check{A}}(\check{a}) \in \check{N} \right\}$ whereas

$$\xi_{\check{A}}(\check{a}) = \left\{ \langle \sigma, \mathbb{I}_{\xi(\check{a})}(\sigma), \xi(\check{a})(\sigma), \mathcal{F}_{\xi(\check{a})}(\sigma) \rangle : \sigma \in \check{U} \right\}$$

$$\mathbb{I}_{\xi(\check{a})}(\sigma), \xi(\check{a})(\sigma), \mathcal{F}_{\xi(\check{a})}(\sigma)$$

represents the interval valued truth, indeterminacy and falsity grades of the attributes such as

$$\begin{aligned} \mathbb{T}_{\xi(\check{a})}(\sigma) &= [\underline{\mathbb{T}}_{\xi(\check{a})}(\sigma), \overline{\mathbb{T}}_{\xi(\check{a})}(\sigma)], \\ \mathbb{I}_{\xi(\check{a})}(\sigma) &= [\underline{\mathbb{I}}_{\xi(\check{a})}(\sigma), \overline{\mathbb{I}}_{\xi(\check{a})}(\sigma)], \\ \mathcal{F}_{\xi(\check{a})}(\sigma) &= [\underline{\mathcal{F}}_{\xi(\check{a})}(\sigma), \overline{\mathcal{F}}_{\xi(\check{a})}(\sigma)] \end{aligned}$$

where

$$\underline{\mathbb{T}}_{\mathbb{F}(\check{a}_p)}(\sigma), \overline{\mathbb{T}}_{\mathbb{F}(\check{a}_p)}(\sigma), \underline{\mathbb{I}}_{\mathbb{F}(\check{a}_p)}(\sigma), \overline{\mathbb{I}}_{\mathbb{F}(\check{a}_p)}(\sigma), \underline{\mathcal{F}}_{\mathbb{F}(\check{a}_p)}(\sigma), \overline{\mathcal{F}}_{\mathbb{F}(\check{a}_p)}(\sigma) \in [0, 1] \text{ and}$$

$$0 \leq \overline{\mathbb{T}}_{\mathbb{F}(\check{a}_p)}(\sigma) + \overline{\mathbb{I}}_{\mathbb{F}(\check{a}_p)}(\sigma) + \overline{\mathcal{F}}_{\mathbb{F}(\check{a}_p)}(\sigma) \leq 3$$

Definition 1.3. ^{(13), (14)}

Let us consider $X \subseteq \check{U}$ and $\Upsilon = \check{A}_1 \times \check{A}_2 \times \dots \times \check{A}_n$, where $n \geq 1$ and \check{A}_i is, where $n \geq 1$ and i is the set of all trait values of the trait $a_i, i = 1, 2, \dots, n$. Each trait value possesses a corresponding appendage degree $\tau : X \times C \rightarrow P\langle [0, 1]^j \rangle, \forall x \in X$, such that $\tau(x, \eta) \in [0, 1]^j$ and $P\langle [0, 1]^j \rangle$ is the power set of $[0, 1]^j$, where $j = 1, 2, 3$ are for fuzzy, intuitionistic fuzzy and neutrosophic degree of appendage respectively.

Moreover, a function defined by describes the degree of disagreement between any two trait values for the same characteristic $\check{c} : \check{A}_i \times \check{A}_i \rightarrow P\langle [0, 1]^j \rangle, 1 \leq i \leq n, j = 1, 2, 3$

For any two attribute values η_1 and η_2 of the same trait, it is denoted by $\check{c}(\eta_1, \eta_2)$ and satisfies the following conditions:

- (i) $\check{c}(\eta_1, \eta_1) = 0$
- (ii) $\check{c}(\eta_1, \eta_2) = \check{c}(\eta_2, \eta_1)$.

Eventually, $(X, \check{A}, \Upsilon, \tau, \check{c})$ is the plithogenic hypersoft set. For an n-tuple $(\eta_1, \eta_2, \dots, \eta_n) \in \Upsilon, \eta_i \in \breve{A}_i, 1 \leq i \leq n$, a plithogenic hypersoft set $\mathcal{F} : \Upsilon \rightarrow \mathcal{P}(\check{U})$

mathematically written as $\mathcal{F} \langle [n_1, n_2, \dots, n_n] \rangle = \{x, (\tau_x(n_1), \tau_x(n_2), \dots, \tau_x(n_n)), x \in X\}$.

Remark 1. ^{(15), (16)}

“Plithogenic hypersoft set is a generalization of crisp hypersoft set, fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, and neutrosophic hypersoft set”.

2 Methodology

2.1 Plithogenic Interval Valued Neutrosophic Hypersoft Set

Let T be a PIVNHSS and s is a trait value. The degree of conflict $\check{c}(s_\tau, s) = \check{c}_0 \in [0, 1]$ between the trait element and the dominant trait element. According to some criteria, the two experts X and Y assign different interval valued neutrosophic fuzzy degrees of trait elements of x to the set T.

$$\begin{aligned} \tau_X^{IV}(s) &= [(x_1, \bar{x}_1), (x_2, \bar{x}_2), (x_3, \bar{x}_3)] \in [0, 1] \\ \tau_Y^{IV}(s) &= [(y_1, \bar{y}_1), (y_2, \bar{y}_2), (y_3, \bar{y}_3)] \in [0, 1] \end{aligned}$$

The PIVNHSS aggregation operators (Union, Intersection) are the linear combination of the fuzzy t_{norm} and fuzzy t_{conorm} symbolized by \vee and \wedge respectively and the contradiction degree $\check{c}(s_\tau, s) = \check{c}_0 \in [0, 1]$

$$x \vee_k y = (1 - \check{c}_0) [x \vee_I y] + \check{c}_0 [x \wedge_I y] \tag{1}$$

$$x \wedge_k y = (1 - \check{c}_0) [x \wedge_I y] + \check{c}_0 [x \vee_I y] \tag{2}$$

2.2 Interval Valued neutrosophic fuzzy Union with Plithogenic

$$\begin{aligned}
 &[(x_1, \bar{x}_1), (x_2, \bar{x}_2), (x_3, \bar{x}_3)] \vee_k [(y_1, \bar{y}_1), (y_2, \bar{y}_2), (y_3, \bar{y}_3)] = \\
 &\left[(x_1 \vee_k y_1, \bar{x}_1 \vee_k \bar{y}_1), \left(\frac{1}{2} (x_2 \wedge_k y_2 + x_2 \vee_k y_2), \frac{1}{2} (\bar{x}_2 \wedge_k \bar{y}_2 + \bar{x}_2 \vee_k \bar{y}_2) \right), \right. \\
 &\left. (x_3 \wedge_k y_3, \bar{x}_3 \wedge_k \bar{y}_3) \right] \\
 &= \\
 &\left[\begin{aligned} &((1 - \check{c}_0) [x_1 \vee_l y_1] + \check{c}_0 [x_1 \wedge_l y_1], (1 - \check{c}_0) [\bar{x}_1 \vee_l \bar{y}_1] + \check{c}_0 [\bar{x}_1 \wedge_l \bar{y}_1]) \\ &\left[\frac{1}{2} ((1 - \check{c}_0) [x_2 \wedge_l y_2] + \check{c}_0 [x_2 \vee_l y_2] + (1 - \check{c}_0) [x_2 \vee_l y_2] + \check{c}_0 [x_2 \wedge_l y_2]) \right] \\ &\frac{1}{2} ((1 - \check{c}_0) [\bar{x}_2 \wedge_l \bar{y}_2] + \check{c}_0 [\bar{x}_2 \vee_l \bar{y}_2] + (1 - \check{c}_0) [\bar{x}_2 \vee_l \bar{y}_2] + \check{c}_0 [\bar{x}_2 \wedge_l \bar{y}_2]) \\ &((1 - \check{c}_0) [x_3 \wedge_l y_3] + \check{c}_0 [x_3 \vee_l y_3], (1 - \check{c}_0) [\bar{x}_3 \wedge_l \bar{y}_3] + \check{c}_0 [\bar{x}_3 \vee_l \bar{y}_3]) \end{aligned} \right] \tag{3}
 \end{aligned}$$

2.3 Interval Valued neutrosophic fuzzy Intersection with Plithogenic

$$\begin{aligned}
 &[(x_1, \bar{x}_1), (x_2, \bar{x}_2), (x_3, \bar{x}_3)] \wedge_k [(y_1, \bar{y}_1), (y_2, \bar{y}_2), (y_3, \bar{y}_3)] = \\
 &\left[(x_1 \wedge_k y_1, \bar{x}_1 \wedge_k \bar{y}_1), \left(\frac{1}{2} (x_2 \vee_k y_2 + x_2 \wedge_k y_2), \frac{1}{2} (\bar{x}_2 \vee_k \bar{y}_2 + \bar{x}_2 \wedge_k \bar{y}_2) \right), \right. \\
 &\left. (x_3 \vee_k y_3, \bar{x}_3 \vee_k \bar{y}_3) \right] \\
 &= \\
 &\left[\begin{aligned} &((1 - \check{c}_0) [x_1 \wedge_l y_1] + \check{c}_0 [x_1 \vee_l y_1], (1 - \check{c}_0) [\bar{x}_1 \wedge_l \bar{y}_1] + \check{c}_0 [\bar{x}_1 \vee_l \bar{y}_1]) \\ &\left[\frac{1}{2} ((1 - \check{c}_0) [x_2 \vee_l y_2] + \check{c}_0 [x_2 \wedge_l y_2] + (1 - \check{c}_0) [x_2 \wedge_l y_2] + \check{c}_0 [x_2 \vee_l y_2]) \right] \\ &\frac{1}{2} ((1 - \check{c}_0) [\bar{x}_2 \vee_l \bar{y}_2] + \check{c}_0 [\bar{x}_2 \wedge_l \bar{y}_2] + (1 - \check{c}_0) [\bar{x}_2 \wedge_l \bar{y}_2] + \check{c}_0 [\bar{x}_2 \vee_l \bar{y}_2]) \\ &((1 - \check{c}_0) [x_3 \vee_l y_3] + \check{c}_0 [x_3 \wedge_l y_3], (1 - \check{c}_0) [\bar{x}_3 \vee_l \bar{y}_3] + \check{c}_0 [\bar{x}_3 \wedge_l \bar{y}_3]) \end{aligned} \right] \tag{4}
 \end{aligned}$$

3 Results and Discussion

3.1 Application of Plithogenic Interval Valued Neutrosophic Hypersoft Set

To apply the PIVNHSS fuzzy operations, we consider four bike riders and their feedbacks to test the accuracy of the wholesome performance of the newly introduced bike. There may be different views from different riders, that might lead to uncertainties. In order to overcome this issue, the proposed PIVNHSS fuzzy operations with the given appendage will give the higher level of accuracy.

The expert values between “Riders” and “Feedback” $Rider = R_1, R_2, R_3, R_4$ and $Feedback = F_1, F_2, F_3, F_4$. The cardinal of R_4 is $|R_4| = 4 * 4 = 16$. The cartesian product of Riders and their feedbacks are represented by

$$\begin{matrix}
 \text{Rider} \times \text{Feedback} = & \begin{bmatrix} (R_1, F_1) & (R_1, F_2) & (R_1, F_3) & (R_1, F_4) \\ (R_2, F_1) & (R_2, F_2) & (R_2, F_3) & (R_2, F_4) \\ (R_3, F_1) & (R_3, F_2) & (R_3, F_3) & (R_3, F_4) \\ (R_4, F_1) & (R_4, F_2) & (R_4, F_3) & (R_4, F_4) \end{bmatrix}
 \end{matrix}$$

Suppose the dominant value of trait “Rider” be “ R_1 ” and of trait “Feedback” be “ F_1 ”. The disagreement fuzzy degrees of their respective traits are:

$$\begin{aligned}
 \check{c}(R_1, R_1) &= 0, \check{c}(R_1, R_2) = \frac{1}{4}, \check{c}(R_1, R_3) = \frac{2}{4}, \check{c}(R_1, R_4) = \frac{3}{4}, \check{c}(F_1, F_1) = 0, \check{c}(F_1, F_2) = \frac{1}{4}, \\
 \check{c}(F_1, F_3) &= \frac{2}{4}, \check{c}(F_1, F_4) = \frac{3}{4}.
 \end{aligned}$$

Let $\tau_X(x, s_i)$ be the appendage degree of the trait value (A.V) s_i of the element x to the set X and $\tau_Y(x, s_i)$ be the appendage degree of the trait value s_i of the element x to the set Y . Then the trait s_i and its degree of disagreement (D.D) depends on s_τ be $\check{c}(s_\tau, s_i) = \check{c}_i$.

Considering the fuzzy

$$t_{\text{norm}} : x \vee_I y = x + y - xy \tag{5}$$

The fuzzy

$$t_{\text{conorm}} : x \wedge_I y = xy \tag{6}$$

$$\tau_X : \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4\} \rightarrow [0, 1]$$

Table 1. Appendage degree of each trait according to X expert

D.D	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
A. V	\mathcal{R}_1	\mathcal{R}_2	\mathcal{R}_3	\mathcal{R}_4	\mathcal{F}_1	\mathcal{F}_2	\mathcal{F}_3	\mathcal{F}_4
IVNFD	[(.4, .6), (.3, .5), (.4, .9)]	[(.1, .5), (.6, .9), (.3, .5)]	[(.0, .3), (.2, .4), (.5, .8)]	[(.1, .3), (.4, .7), (.2, .7)]	[(.2, .7), (.3, .7), (.4, .6)]	[(.3, .5), (.5, .9), (.4, .7)]	[(.1, .9), (.5, .6), (.3, .8)]	[(.2, .4), (.4, .6), (.3, .7)]

$$\tau_Y : \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4\} \rightarrow [0, 1]$$

Table 2. Appendage degree of each trait according to Y expert

D.D	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
A.V	\mathcal{R}_1	\mathcal{R}_2	\mathcal{R}_3	\mathcal{R}_4	\mathcal{F}_1	\mathcal{F}_2	\mathcal{F}_3	\mathcal{F}_4
IVNFD	[(.3, .7), (.2, .5), (.6, .8)]	[(.3, .5), (.2, .5), (.6, .8)]	[(.0, .2), (.4, .7), (.6, .8)]	[(.2, .5), (.4, .7), (.6, .9)]	[(.7, .9), (.1, .5), (.2, .7)]	[(.6, .8), (.4, .5), (.3, .6)]	[(.3, .7), (.5, .6), (.2, .7)]	[(.1, .5), (.3, .7), (.5, .9)]

Table 3. Interval valued Neutrosophic fuzzy set union and intersection with Plithogenic

D.D	A.V	Expert X	Expert Y	Union	Intersection
0	\mathcal{R}_1	[(.4, .6), (.3, .5), (.4, .9)]	[(.3, .7), (.2, .5), (.6, .8)]	[(.58, .88), (.25, .5), (.24, .72)]	[(.12, .42), (.25, .5), (.76, .98)]
$\frac{1}{4}$	\mathcal{R}_2	[(.1, .5), (.6, .9), (.3, .5)]	[(.3, .5), (.2, .5), (.6, .8)]	[(.26, .63), (.4, .7), (.32, .53)]	[(.16, .38), (.4, .7), (.59, .78)]
$\frac{2}{4}$	\mathcal{R}_3	[(.0, .3), (.2, .4), (.5, .8)]	[(.0, .2), (.4, .7), (.6, .8)]	[(.0, .25), (.3, .55), (.55, .8)]	[(.0, .25), (.3, .55), (.55, .8)]
$\frac{3}{4}$	\mathcal{R}_4	[(.1, .3), (.4, .7), (.2, .7)]	[(.2, .5), (.4, .7), (.6, .9)]	[(.09, .28), (.4, .7), (.54, .89)]	[(.22, .53), (.4, .7), (.26, .72)]
0	\mathcal{F}_1	[(.2, .7), (.3, .7), (.4, .6)]	[(.7, .9), (.1, .5), (.2, .7)]	[(.76, .97), (.2, .6), (.08, .42)]	[(.14, .63), (.2, .6), (.52, .88)]
$\frac{1}{4}$	\mathcal{F}_2	[(.3, .5), (.5, .9), (.4, .7)]	[(.6, .8), (.4, .5), (.3, .6)]	[(.59, .78), (.45, .7), (.24, .54)]	[(.32, .53), (.45, .7), (.47, .77)]
$\frac{2}{4}$	\mathcal{F}_3	[(.1, .9), (.5, .6), (.3, .8)]	[(.3, .7), (.5, .6), (.2, .7)]	[(.2, .8), (.5, .6), (.25, .75)]	[(.2, .8), (.5, .6), (.25, .75)]
$\frac{3}{4}$	\mathcal{F}_4	[(.2, .4), (.4, .6), (.3, .7)]	[(.1, .5), (.3, .7), (.5, .9)]	[(.09, .33), (.35, .65), (.53, .89)]	[(.22, .58), (.35, .65), (.28, .72)]

3.2 The proposed PIVNHSS – TOPSIS Method to solve a MCDM Problem

In this section, a novel MCDM strategy is built using the PIVNHSS concept in order to solve it using the TOPSIS method. An eminent candidate has to be selected and appointed as the Dean for Research in an educational institution. As this is the most esteemed and dynamic designation, to take the institution to next level to compete with the world-wide institutions. In this case a proper and fair scrutiny is needed. In order to meet this, an PIVNHSS values are dealt by the TOPSIS method.

3.3 Proposed PIVNHSS – TOPSIS Algorithm

Consider \tilde{U} be a non-void universal set. Let $X \subseteq \tilde{U}$ be the set of alternatives under consideration where $X = \{x_1, x_2, \dots, x_m\}$. Let $Y = A_1 \times A_2 \times \dots \times A_n, n \geq 1$ and A_i is the set of all trait values of the trait $a_i, i = 1, 2, \dots, n$. The trait value $a_i, i = 1, 2, \dots, n$. Each attribute value η has a corresponding appendage degree $\xi(x, \eta)$ of a member $x \in X$, with respect to some given conditions. We are in need to select the best alternative out of the alternative set X . This has been explained by the following steps:

Step 1: An ordered tuple $(n_1, n_2, \dots, n_m) \in Y$ has to be chosen, based on this a matrix of order $m \times n$, its elements represent the degree of appurtenance for each trait value η with regard to each trait $x \in X$ in the interval valued neutrosophic space.

Step 2: The newly developed PIVNHSS accuracy function

$$A_{piv} = \frac{\underline{T}_\eta + \overline{T}_\eta + \underline{I}_\eta + \overline{I}_\eta + \underline{F}_\eta + \overline{F}_\eta}{6} + \frac{\underline{T}_{\xi\eta} + \overline{T}_{\xi\eta} + \underline{I}_{\xi\eta} + \overline{I}_{\xi\eta} + \underline{F}_{\xi\eta} + \overline{F}_{\xi\eta}}{6} \times \check{c}(\eta, \xi\eta)$$

where

$$[\underline{T}_\eta, \overline{T}_\eta], [\underline{I}_\eta, \overline{I}_\eta], [\underline{F}_\eta, \overline{F}_\eta]$$

represent the interval valued truthiness, indeterminacy, falsity degrees of appendage of the trait value η to the set X and

$$[\underline{T}_{\xi\eta}, \overline{T}_{\xi\eta}], [\underline{I}_{\xi\eta}, \overline{I}_{\xi\eta}], [\underline{F}_{\xi\eta}, \overline{F}_{\xi\eta}]$$

represent the interval valued truthiness, indeterminacy, falsity degrees of corresponding dominant trait value, whereas $\check{c}(\eta, \xi\eta)$ denotes the fuzzy degree of disagreement between an trait value η and its corresponding dominant trait value $\xi\eta$. This gives the plithogenic interval valued neutrosophic hypersoft(PIVNHS) accuracy matrix.

Step 3: To get the PIVNHS decision matrix, transpose the PIVNHS accuracy matrix. $P = [a_{ij}]_{m \times n}$ of traits against the criteria.

Step 4: A PIVNHS normalized decision matrix $Q = (q_{ij})_{m \times n}$ is constructed, it compares the effectiveness of several options and whose components are determined as follows:

$$q_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^m r_{ij}^2}}, j = 1, 2, \dots, n.$$

Step 5: A PIVNHS weighted normalized decision matrix, where $S = [s_{ij}]_{m \times n} = Q\hat{w}_n$ where $\hat{w}_n = [w_1, w_2, \dots, w_n]$ is a row matrix of weights w_p that have been allocated to the criteria $a_p, p = 1, 2, \dots, n$. Depending on their significance during the decision-making process, selection criteria are given with varying weights by the decision maker.

Step 6: Analyse the plithogenic interval valued positive ideal solution

$$S^+ = \left\{ \max_i (s_{ij}) \text{ if } a_j \in \text{beneficial criteria}, \min_i (s_{ij}) \text{ if } a_j \in \text{cost criteria}, i, j = 1, 2, \dots, n \right\}$$

$$S^- = \left(\min_i (s_{ij}) \text{ if } a_j \in \text{beneficial criteria}, \max_i (s_{ij}) \text{ if } a_j \in \text{cost criteria}, i, j = 1, 2, \dots, n \right)$$

Step 7: Estimate PIVNHS U_i^+ and PIVNHS negative distance U_i^- of each of the alternatives from S^+ and S^- .

$$U_i^+ = \sqrt{\sum_{j=1}^n (s_{ij} - s_j^+)^2}, i = 1, 2, \dots, m.$$

$$U_i^- = \sqrt{\sum_{j=1}^n (s_{ij} - s_j^-)^2}, i = 1, 2, \dots, m.$$

Step 8: Relative closeness co-efficient $R_i = \frac{U_i^-}{(U_i^+ + U_i^-)}, i = 1, 2, \dots, m$.

Step 9: Ranking is done based on the calculated value from R_i . The best trait is the highest value of R_i and the worst trait will be the least value calculated from R_i .

3.4 An Illustrative Example

Let $\tilde{U} = \{H_1, H_2, \dots, H_{10}\}$ be a universe containing ten promising candidates. An educational institution has to promote a person as a dean for academic research, the selection will be based on the candidates on consideration (alternatives) which are contained in $X \subseteq \tilde{U}$ which is mentioned as $X = (H_1, H_2, H_3, H_4)$. The traits of the candidates belong to the set $\tilde{A} = \{a_1, a_2, a_3, a_4\}$, were a_1 = Experience in Research, a_2 = Number of Research Articles Published in International Journals, a_3 = Qualification, a_4 = Age of the candidate. The trait values of a_1, a_2, a_3, a_4 are contained in the sets A_1, A_2, A_3, A_4 which are listed below:

$$A_1 = \{> 10 \text{ years } (\alpha_1), 5 \text{ to } 10 \text{ years } (\alpha_2), < 5 \text{ years } (\alpha_3)\}$$

$$A_2 = (2 \text{ to } 4 (\beta_1), 5 \text{ to } 7 (\beta_2), 8 \text{ to } 10 (\beta_3), 11 \text{ to } 13 (\beta_4), 14 \text{ to } 16 (\beta_5), 17 \text{ to } 19 (\beta_6))$$

$$A_3 = (Post Doctrate (\gamma_1), Doctrate (\gamma_2))$$

$$A_4 = \{ > 45 \text{ years } (\lambda_1), 35 \text{ to } 45 \text{ years } (\lambda_2), < 35 \text{ years } (\lambda_3) \}$$

The dominant trait values of a_1, a_2, a_3, a_4 are chosen to be $\alpha_1, \beta_1, \gamma_1, \lambda_1$ respectively, the single-valued fuzzy degree of contradiction between the dominant attribute value and all other attribute values are given below.

$$c_F(\alpha_1, \alpha_2) = \frac{1}{3}, c_F(\alpha_1, \alpha_3) = \frac{2}{3},$$

$$c_F(\beta_1, \beta_2) = \frac{1}{6}, c_F(\beta_1, \beta_3) = \frac{2}{6}$$

$$c_F(\beta_1, \beta_4) = \frac{3}{6}, c_F(\beta_1, \beta_5) = \frac{4}{6},$$

$$c_F(\beta_1, \beta_6) = \frac{5}{6}$$

$$c_F(\gamma_1, \gamma_2) = \frac{1}{2},$$

$$c_F(\lambda_1, \lambda_2) = \frac{1}{3}, c_F(\lambda_1, \lambda_3) = \frac{2}{3}.$$

A comparative study of IVPNHSS based on PIVNHSS-TOPSIS along with fuzzy TOPSIS is discussed below. The alternatives under consideration are $X = (H_1, H_2, H_3, H_4)$ which are contained in the set $X \in \tilde{U}$. The neutrosophic degree of appendage of each trait value corresponding to each alternative H_1, H_2, H_3, H_4 are listed below in Table 4.

Table 4. Interval Valued neutrosophic values for the appendages and traits

S. No	Variables	H ₁	H ₂	H ₃	H ₄
1	α_1	[(.4,.6), (.3,.5), (.4, .9)]	[(.1,.5), (.6,.9), (.3,.5)]	[(.0,.3), (.2,.4), (.5,.8)]	[(.1,.3), (.4,.7), (.2,.7)]
2	α_2	[(.6,.8), (.2,.4), (.3,.7)]	[(.4,.7), (.6,.9), (.1,.4)]	[(.1,.5), (.5,.6), (.3,.6)]	[(.3,.6), (.1,.6), (.2,.5)]
3	α_3	[(.1,.3), (.3,.7), (.6,.8)]	[(.2,.4), (.3,.6), (.5,.8)]	[(.2,.5), (.4,.6), (.1,.3)]	[(.5,.8), (.2,.5), (.4,.7)]
4	β_1	[(.3,.7), (.2,.5), (.6,.8)]	[(.3,.5), (.2,.5), (.6,.8)]	[(.0,.3), (.4,.7), (.6,.8)]	[(.2,.5), (.4,.7), (.6,.9)]
5	β_2	[(.4,.7), (.1,.4), (.3,.6)]	[(.1,.6), (.4,.6), (.2,.7)]	[(.4,.7), (.4,.5), (.7,.9)]	[(.0,.4), (.3,.5), (.6,.9)]
6	β_3	[(.3,.6), (.2,.7), (.5,.8)]	[(.3,.4), (.7,.9), (.1,.4)]	[(.1,.3), (.3,.6), (.5,.8)]	[(.1,.5), (.3,.4), (.2,.7)]
7	β_4	[(.2,.5), (.3,.7), (.1,.4)]	[(.7,.9), (.1,.4), (.2,.7)]	[(.3,.6), (.4,.7), (.5,.7)]	[(.4,.5), (.1,.4), (.5,.9)]
8	β_5	[(.4,.7), (.3,.6), (.2,.5)]	[(.2,.6), (.1,.5), (.3,.5)]	[(.0,.3), (.4,.7), (.5,.7)]	[(.1,.7), (.3,.7), (.2,.5)]
9	β_6	[(.4,.6), (.5,.7), (.1,.4)]	[(.6,.8), (.1,.5), (.3,.6)]	[(.3,.6), (.5,.9), (.2,.6)]	[(.2,.4), (.3,.7), (.1,.6)]
10	γ_1	[(.2,.7), (.3,.7), (.4,.6)]	[(.3,.7), (.5,.6), (.2,.7)]	[(.2,.5), (.0,.3), (.5,.8)]	[(.0,.3), (.3,.7), (.2,.5)]
11	γ_2	[(.3,.5), (.5,.9), (.4,.7)]	[(.0,.3), (.4,.7), (.5,.8)]	[(.3,.7), (.1,.7), (.5,.8)]	[(.3,.7), (.0,.3), (.1,.5)]
12	λ_1	[(.1,.9), (.5,.6), (.3,.8)]	[(.1,.5), (.3,.7), (.5,.9)]	[(.2,.5), (.3,.6), (.0,.3)]	[(.3,.5), (.6,.7), (.7,.9)]
13	λ_2	[(.3,.7), (.5,.7), (.6,.8)]	[(.2,.5), (.0,.6), (.3,.7)]	[(.1,.5), (.3,.7), (.2,.4)]	[(.2,.6), (.7,.9), (.3,.5)]
14	λ_3	[(.2,.4), (.4,.6), (.3,.7)]	[(.3,.5), (.5,.6), (.1,.7)]	[(.3,.5), (.1,.3), (.5,.7)]	[(.1,.5), (.0,.4), (.5,.8)]

Let $C = A_1 \times A_2 \times A_3 \times A_4$ and consider an element $(\alpha_2, \beta_1, \gamma_2, \lambda_1) \in C$. By applying the accuracy function of PIVNHSS, we obtain the plithogenic interval valued accuracy matrix is given below:

$$M = \begin{bmatrix} 0.6722 & 0.6778 & 0.5555 & 0.5166 \\ 0.5167 & 0.4833 & 0.4667 & 0.5500 \\ 0.7917 & 0.7000 & 0.7084 & 0.4834 \\ 0.5333 & 0.5000 & 0.3167 & 0.6167 \end{bmatrix}$$

A weighted normalized matrix $W_4 = [0.36, 0.22, 0.28, 0.14]$

$$S = \begin{bmatrix} 0.1985 & 0.1125 & 0.1629 & 0.0741 \\ 0.2001 & 0.1052 & 0.1440 & 0.0695 \\ 0.1640 & 0.1016 & 0.1457 & 0.0440 \\ 0.1525 & 0.1198 & 0.0995 & 0.0857 \end{bmatrix}$$

Table 5. Ranking table for the alternatives by PIVNHSS-TOPSIS

Alt/Criteria	U^+	U^-	R_i	Rank
H ₁	0.0479	0.0710	0.5972	1
H ₂	0.0556	0.0515	0.4805	4
H ₃	0.0499	0.0587	0.5404	2
H ₄	0.0634	0.0658	0.5093	3

The PIVNHSS’s positive ideal solution S^+ and PIVNHSS’s negative ideal solution S^- are determined as follows:

$$S^+ = (0.1525, 0.1198, 0.1629, 0.0857), S^- = (0.2001, 0.1016, 0.0995, 0.0440)$$

It is evident that H_1 is the most suitable candidate for the post of Dean for Research Studies.

3.5 Fuzzy-TOPSIS Method

The averaging operator for the alternatives and the traits are used for PIVNFS numbers in order to get its decision matrix.

$$P = \begin{bmatrix} 0.5000 & 0.5167 & 0.5500 & 0.5333 \\ 0.5167 & 0.4833 & 0.4500 & 0.5000 \\ 0.4333 & 0.4667 & 0.5167 & 0.3167 \\ 0.3833 & 0.5500 & 0.3167 & 0.6167 \end{bmatrix}$$

Using the decision matrix with fuzzy TOPSIS with the same weightage, the PIVNHSS positive distance U^+ the PIVNHSS negative distance U^- , closeness coefficient and the ranking are summarized in the table below.

Table 6. Ranking table for the alternatives by Fuzzy-TOPSIS

Alt / Criteria	U^+	U^-	R_i	Rank
H ₁	0.0475	0.0772	0.6189	1
H ₂	0.0639	0.0475	0.4266	4
H ₃	0.0505	0.0682	0.5748	2
H ₄	0.0700	0.0691	0.4970	3

Table 7. Relative Comparative analysis of ranking by PIVNHSS-TOPSIS and Fuzzy-TOPSIS

(PIVNHSS-TOPSIS)	R_i (Fuzzy-TOPSIS)	Rank
0.5972	0.6189	1
0.4805	0.4266	4
0.5404	0.5748	2
0.5093	0.4970	3

4 Conclusion

In this research work, the novelty of PIVNHSS has been established by the proposed t_{norm}, t_{conorm} and the score function in order to enhance the accuracy level of the results in most vague, uncertain and complex situations. In most of the real time situations, the given traits have to be further sub-divided to another set of traits in order to analyse each alternative with every other trait which has taken for consideration to give the detailed analysis to reach a fair decision. In this aspect PIVNHSS-TOPSIS method is proposed and it has been validated with Fuzzy TOPSIS to check the results are matching by taking a MCDM situation consisting of ten promising candidates, with different traits and alternatives. A very interesting results has been arrived which has been re-checked with two different sets of values in selecting the best candidate by PIVNHSS-TOPSIS ranking hierarchy $H_1 > H_3 > H_4 > H_2$. In this proposed method both the ranking were matching even though the closeness coefficient values differ by very meagre values. This study helps us to consider all the traits under different levels of appendage degrees which is the advantage of this proposed method.

The potential of this study (PIVNHSS) provides us with a new perspective to assess the real-time MCDM in a very precise manner using the suggested t_{norm}, t_{conorm} and score function, all without neglecting any small considerations. If two or more

alternatives arrive at the same ranking hierarchy, then an additional trait can be added for consideration. This study can be further extended to deal with different sets of traits and alternatives under consideration for Quadri-partitioned and their aggregation operators.

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