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* **Corresponding author**.

kalaivanistatistics1994@gmail.com

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Robust Weighted Support Vector Regression Approach for Predictive Modeling

R Muthukrishnan[1], S Kalaivani[2]*

1 Professor, Department of Statistics, Bharathiar University, Coimbatore, 641 046, Tamil Nadu, India
2 Research Scholar, Department of Statistics, Bharathiar University, Coimbatore, 641 046, Tamil Nadu, India

Abstract

Objectives: To improve the robustness of Support Vector Regression (SVR), Robust Weighted Support Vector Regression (RWSVR) procedure has been proposed by using Hampel's weight function. It overcomes the drawback by adding weight to each sample observations. **Methods:** Regression analysis plays a vital role in many areas of science. In this article, the classical regression method, robust regression methods, Support Vector Regression and proposed Robust Weighted Support Vector Regression method are presented. A type of Support Vector Machine (SVM) known as Support Vector Regression (SVR) supports both linear and non-linear regression. This work mainly focuses on Support Vector Regression method. The datasets which are used for this study is StarsCYG, Carbonation and Prostate. Simulation study of three cases are considered with sample size 50,100 and 500 with the contamination levels of 0%, 5%, 10% and 20%. **Findings:** The most commonly used classical procedure is Least Squares, which is less efficient and very sensitive when the data contains outliers. This work mainly focuses on increasing the accuracy of Proposed method. It overcomes the drawback by adding weight to each sample observations. The efficiency of the proposed method has been observed with the existing regression methods such as Least Squares (LS), Robust Linear Model (RLM) and Support Vector Regression (SVR) by computing various error measures such as Mean Absolute Error (MAE), Median Absolute Error (MDAE), Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE). The proposed RWSVR method is used for the researchers whenever the data contains outliers. **Novelty:** For getting more accuracy over the conventional SVR, robust Hampel's weight function is proposed. Newly developed function has a higher mapping power than the commonly used SVR function. The experimental results were carried out on real datasets and simulation study. It shows that the proposed RWSVR is significantly more robust than other regression methods.

Keywords: Least Squares; Robust Regression; Supervised Learning; Support Vector Regression; Hampel's Weight Function

1 Introduction

The statistical learning theory provides a framework for machine learning in the statistics-related fields. It is started, when the first generation computers become capable of conducting multidimensional analysis of real-life problems [1]. The statistical learning theory was developed to overcome the curse of dimensionality. The goal for statistical learning is to find a function, representing the systematic information of the response from its predictors, which will be used for prediction and inference. It asserts bound on the error of a prediction function.

Support vector machine (SVM) is currently a popular topic in the statistical learning area. SVM was first introduced in Conference on computational learning theory (COLT). SVM is one of the most successful algorithms for classification and regression without any distributional assumption [2]. The SVM framework has become a popular toolbox for addressing real world applications that involve non-linear classification and regression tasks. The successful application of the kernel trick in SVMs has flashed a new type of techniques for addressing nonlinear task (classification or regression), called as kernel based methods. Support vector machines are a classification or regression tool used for optimally predicting the class membership or real value of unseen outputs that are generated or characterized by one or more inputs, by means of looking at some available training input-output pairs, then building a model based on the observed input-output relations [3]. SVMs are learning machines that employ the structural risk minimization principle to achieve good generalisation on a constrained set of learning patterns.

Robust Support vector regression in primal with asymmetric Huber loss by Balasundaram. S and Yogendra meena (2019) [4]. Application of Support Vector Regression for Modeling Low Flow Time Series by Bibhuti Bhusan Sahoo et al (2019) [5]. Linear Regression Supporting Vector Machine and Hybrid LOG Filter-Based Image Restoration by Khalandar Basha. D and Venkateswarlu. T (2020) [6]. Significance SVR for Image Denoising was given by Bing Sun and Xiaofeng Liu (2021) [7]. SVR model for seasonal time series data by Hanifah Muthiah et al (2021) [8]. Robust regression using support vector regressions by Mostafa Sabzekar (2021) [9]. A New Support Vector Regression Model for Equipment Health Diagnosis was studied by Qinming Liu et al (2021) [10]. Projection wavelet weighted twin support vector regression by Wang et al (2021) [11]. Evolution of Support Vector Machine and Regression Modeling by Raquel Rodriguez Perez (2022) [12]. An overview on twin support vector regression was discussed by Huajuan Huang et al (2022) [13]. Short-term forecasting of COVID-19 using support vector regression by Claris Shoko and Caston Sigauke (2023) [14]. Adaptive Weighted Least Squares Support Vector Regression Based on Genetic Algorithm was studied by Maosheng Wei et al., (2023) [15].

One class of SVM methods is the Support Vector Regression (SVR), which is established as a robust technique for constructing data-driven and non-linear empirical regression models. The concept of SVR has been applied to various fields by the researchers. This paper is organized as follows. Section 2 gives a brief introduction to the methodology of regression procedures. Section 3 presents empirical comparisons demonstrating the advantages of the RWSVR procedure by real datasets and simulation study in results and discussion. Finally, Section 4 provides a conclusion.

2 Methodology

Regression analysis is one of the most extensively used statistical techniques, which is used in almost all fields such as engineering, medical science, commerce, marketing research, social science, etc.

In regression, there are two types of variables, independent variables (or predictor variables or explanatory variables or regressor variables) and dependent variable (or response variable). The method of the Least Squares, Robust linear Model, Support Vector Regression and Robust Weighted Support Vector Regression procedures has been discussed in this section.

2.1 Least Squares Method (LS)

The least squares (LS) estimator of regression parameter is obtained by minimizing the sum of squares of deviations between the actual and predicted values of the response variable. The simple linear regression model is defined as

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{1}$$

where y is the dependent variable, x is an independent variable, β_0 is a intercept, β_1 is a slope variable and ε is an error term.

The standard multiple regression model in matrix notation is given as

$$Y = X\,\beta + \,\varepsilon \tag{2}$$

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, X = \begin{pmatrix} x_{11} & \dots & x_{1k} \\ x_{21} & \dots & x_{2k} \\ \vdots & \vdots & \vdots \\ x_{1k} & \dots & x_{nk} \end{pmatrix}, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

where $Y = (y_1, y_2, \ldots, y_n)'$ is a $n\,x$ 1 vector of n observations, X is a n x k matrix of n observations on each of the k explanatory variables, $\beta = (\beta_0, \beta_1, \ldots, \beta_k)'$ is a $k\,x$ 1 vector of regression coefficients and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)'$ is a $n\,x$ 1 vector of random error components.

2.2 Robust Linear Model (RLM)

The ordinary least squares estimates for linear regression are optimal when all of the regression assumptions are valid. Least squares regression may perform poorly when some of these presumptions are false. By requiring fewer strict assumptions than least squares regression, robust regression techniques offer an alternative. These techniques make an effort to reduce the impact of extreme cases in order to better fit the majority of the data. Outliers influence is diminished by robust regression, which causes their residuals to grow and become more noticeable.

Consider a dataset set of size n such that

$$y_i \;=\; x_i{}^T\,\beta + \,\varepsilon_i \tag{3}$$

$$\varepsilon_i\,\,(\beta) = \,y_i - \,x_i{}^T\,\beta \tag{4}$$

where $i = (1, 2, 3, \ldots, n)$.

The M-estimator class of estimators is the robust regression technique used in this study. M-estimators aim to reduce the sum of a selected function acting on the residuals ρ (.). M-estimators can be defined by

$$\widehat{\beta_M} = argmin_\beta \sum_{i=1}^{n} \rho\,\left(\varepsilon_i\,(\beta)\right) \tag{5}$$

Since ρ (.) is related to the likelihood function for an appropriate assumed residual distribution, the M stands for ”maximum likelihood”.

2.3 Support Vector Regression

The SVR (Support Vector Regression) or is a supervised learning technique that can, given one dependent variable and one or more independent variables, find a separating hyperplane that has a maximum deviation value equal $\mathscr{E}$ to the boundary hyperplanes[16]. This property makes the SVR to be less sensitive to outliers than the quadratic loss function used in the linear regressions[10]. The margin, which is the distance between two boundary hyperplanes ($-\varepsilon$ and $+\varepsilon$) that are parallel to the separating hyperplane, can be maximised using support vectors, which are data points that show where the separating hyperplane is located[17].

The SVR can be formulated as quadratic programming problem, in special, the dual formulation of the problem is indicated because it reduces the number of constraints and allows the application of the Kernel Trick to solve non-linear problems.

Suppose the training data points given are $\{(x_1,y_1),\ldots,(x_l,y_l)\} \subset X\, X\, R$ where χ denotes the space of the training set. In ε - SV regression, the main aim is to find a function $f(x)$ which has at most ε deviation from the actual values of the response variables y_i for all the training data, and at the same instance is as flat as possible.

The function f can be defined as

$$f(x) = \langle W, X\rangle \text{ with } w \in X, b \in \mathbb{R} \tag{6}$$

where $\langle .,.\rangle$denotes the dot product in X. The flatness can be obtained by minimizing the norm that is, $\|w\| = \langle w,w\rangle$. This problem can be inscribed as a convex optimization problem given by

$$\begin{gathered}\text{minimize } \frac{1}{2}\|w\|^2 \\ \text{subject to } \left\{\begin{array}{c} y_i - \langle w,x_i\rangle - b \leq \varepsilon \\ \langle w,x_i\rangle + b - y_i \leq \varepsilon \end{array}\right\}\end{gathered} \tag{7}$$

In SV machines the soft margin loss function, similarly slack variables $\xi_i,\ \xi_i{}^*$ are introduced to deal with otherwise infeasible constraints of the optimization problem.

Vapnik formulated as

$$\begin{gathered}\text{minimize } \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \\ \text{subject to } \left\{\begin{array}{c} y_i - \langle w,x_i\rangle - b \leq \varepsilon + \xi_i \\ \langle w,x_i\rangle + b - y_i \leq \varepsilon + \xi_i{}^* \\ \xi_i, \xi_i{}^* \geq 0 \end{array}\right\}\end{gathered} \tag{8}$$

The constant $C > 0$ determines the trade-off between the flatness of $f(x)$ and the extent up to which deviations larger than $\varepsilon$are tolerated.

The main aim is to construct a Lagrange function from the objective function and the corresponding constrains, by involving the dual set of variables can be written by,

$$\text{maximize } \left\{\begin{array}{l} -\frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i - \alpha_i{}^*)(\alpha_j - \alpha_j{}^*)\langle x_i, x_j\rangle \\ -\varepsilon\sum_{i=1}^{l}(\alpha_i - \alpha_i{}^*) + \sum_{i=1}^{l} y_i(\alpha_i - \alpha_i{}^*) \end{array}\right. \tag{9}$$

subject to $\sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0$ and $\alpha_i, \alpha_i{}^* \in [0,C]$

Using the equation, the dual variables are removed and which can be reconstructed as

$$w = \sum_{i=1}^{l}(\alpha_i - \alpha_i{}^*)x_i \tag{10}$$

Thus

$$f(x) = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)\langle x_i, x\rangle + b \tag{11}$$

This function depicts the so-called Support Vector (SV) Expansion.

Support vector machines and other models employing the kernel trick do not scale well to large numbers of training samples or large numbers of features in the input space, several approximations to the RBF kernel have been introduced. The commonly used kernel function is Radial Basis Function kernel (RBF) and it is expressed as follows

$$k(x_i, x) = \exp\left(-\gamma\left\|(x_i - x)^2\right\|\right) \tag{12}$$

$\left\|(x_i - x)^2\right\|$ may be recognized as the squared Euclidean distance between the two feature vectors. The optimal hyper parameters in RBF kernel is $(C,\ \gamma \, and \, \varepsilon)$, where C is cost of constraints in the Lagrange formulation, ε is the insensitive-loss function and γ is the kernel parameter. The greater number of support vectors explains the model to be more efficient.

2.4 Robust Weighted Support Vector Regression (RWSVR)

In this paper, a robust weight-based support vector regression is proposed to address the problem of outliers in Support Vector Regression (SVR). When performing a sum, integral, or average, a weight function is a mathematical tool that is used to give some observations in the same set more "weight" or influence on the outcome than other observations in the same set. Hampel's three part Redescending M-estimator was proposed by Hampel. Hampel's local method entails building an estimator with a predetermined influence function, which in turn establishes an estimation procedure's sensitivity to significant outliers, rounding off, and other qualitative robustness characteristics.

M-estimators are a broad class of extremum estimators for which the objective function is a sample average. Both non-linear least squares and maximum likelihood estimation are special cases of M-estimators. The definition of M-estimators was motivated by robust statistics, which contributed new types of M-estimators. The statistical procedure of evaluating an M-estimator on a data set is called M-estimation. Maximum Likelihood Estimators (MLE) are thus a special case of M-estimators. With suitable rescaling, M-estimators are special cases of extremum estimators.

The function ρ, or its derivative, ψ, can be chosen in such a way to provide the estimator desirable properties (in terms of bias and efficiency) when the data are truly from the assumed distribution, and 'not bad' behaviour when the data are generated from a model that is, in some sense, close to the assumed distribution.

The Hampel's three part Redescending M-estimator has three tuning constants a, b and c. . Its objective function $\rho(r)$ is given by

$$\rho(r)=\begin{cases}\frac{1}{2}r^2 & ;\text{ if } |r|<a\\ a|r|-\frac{1}{2}a^2 & ;\text{ if } a\le|r|<b\\ a\frac{c|r|-\frac{1}{2}r^2}{c-b}-\frac{7a^2}{6} & ;\text{ if } b\le|r|\le c\\ a(b+c-a) & ;\text{ otherwise}\end{cases} \tag{13}$$

Its score function (r) is given by

$$\Psi(\text{r})=\begin{cases} r & ;\text{ if } |r|<a\\ a\,\text{sign}\,r & ;\text{ if } a\le|r|<b\\ a\frac{c\,\text{sign}\,r-r}{c-b} & ;\text{ if } b\le|r|\le c\\ 0 & ;\text{ otherwise}\end{cases}$$

Its weight function $w(r)=\frac{\psi(r)}{r}$ is given by,

$$w(r)=\begin{cases} 1 & ;\text{ if } |r|<a\\ \frac{a}{|r|} & ;\text{ if } a\le|r|<b\\ a\frac{\frac{c}{|r|}-1}{c-b} & ;b\le|r|\le c\\ 0 & ;\text{ otherwise}\end{cases} \tag{15}$$

where a,b,c are positive constants and $0<a\le b<c<\infty$ and r are the residuals scaled over Median Absolute Deviation, MAD. The graph for Hampel's Influence function and weight function is given in Figures 1 and 2.

Steps of the proposed RWSVR are as follows,

Step 1: Calculate the coefficients and residuals for robust linear model using Hampel based weight function.

Step 2: The values of the residuals are transformed and used to constrain the weights as $w_i = {r_i}^2$. These weights will down weightening the extreme observations

Step 3: Obtain the number of support vectors and coefficients by combining the class weights w_i using Support vector regression based RBF kernel.

Step 4: If the class weight w_i is given, the kernel element will be changed as

$$K_w\ (x_i,\ x)=exp(-\gamma\left(\sum_{i=1}^{l}(w_i\ (x_{ik}-x_k))^2\right) \tag{16}$$

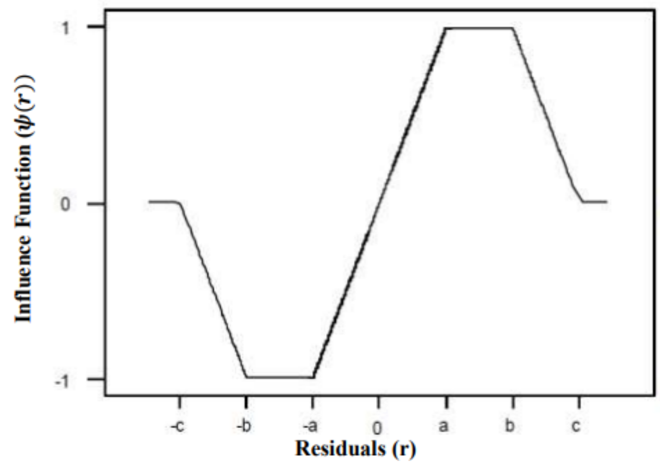

Fig 1. Graph of Hampel's three part Weight Function

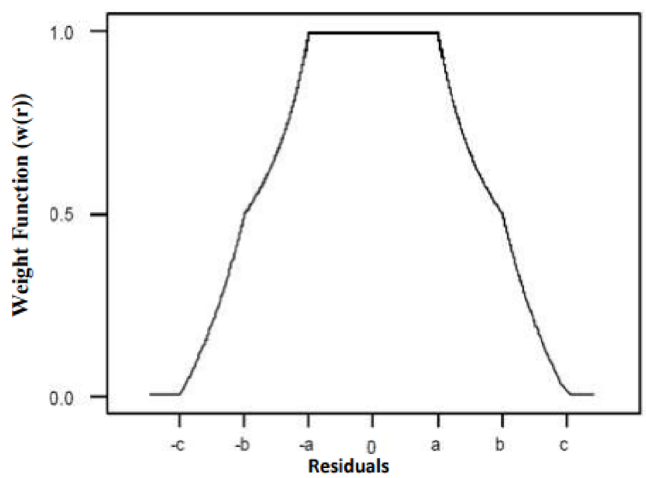

Fig 2. Graph of Hampel's three part Influence Function

The model can be further modified as

$$f(x) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) \ K_w \ (x_i, x) + b \tag{17}$$

This proposed model is said to be Robust weighted based support vector Regression (RWSVR) and it is applied for real and simulation study.

3 Results and Discussion

For the purpose of illustrating the performance of Robust Weighted Support Vector Regression over linear, SVR and robust regression models, the real and simulation study has been performed. The most commonly used classical procedure is the Least Squares, which is less efficient and very sensitive when the data contains outliers. This work mainly focuses on increasing the accuracy of Proposed method. It overcomes the drawback by adding weight to each sample observations. The efficiency of the proposed method has been observed with the existing regression methods such as Least Squares (LS), Robust Linear Model (RLM) and Support Vector Regression (SVR) by computing various error measures such as Mean Absolute Error (MAE), Median Absolute Error (MDAE), Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE). The proposed RWSVR method is used for the researchers whenever the data contains outliers.

3.1 Real Data

This section presents the experimental results being carried out under real datasets having one, two and more than two predictors by considering cases of three different real datasets. The efficiency of LS, RLM, SVR and RWSVR procedures has been studied by computing the error measures under with and without extreme observations.

Case 1: starsCYG dataset - For this study, data taken from the R package robustbase, which contains 47 stars in the Hertzsprung-Russell Diagram of the Star Cluster CYG OB1. It has one predictor variable, logarithm of the effective temperature at the surface of the star (log.Te) and a response variable, logarithm of its light intensity (log.light). In that dataset, 9 outliers are identified using cook's distance.

Case 2: Carbonation dataset - It contains 12 observations. The first variable is the Temperature, which is a predictor variable and the second variable Pressure is another predictor variable and Carbonation, which is a response variable. In this dataset, one outlier is appeared using cook's distance.

Case 3: Prostate dataset - The dataset has 97 observations each having 8 independent variables namely lweight (log of prostate weight), age, lbph (log of benign prostatic hyperplasia amount), svi (seminal vesicle invasion), lcp (log of capsular penetration), gleason (Gleason score), pgg45 (percentage Gleason scores 4 or 5), lpsa (log of prostate specific antigen) and one dependent variable lcavol (log of cancer volume). This dataset contains 9 outliers when it is checked using cook's distance.

The error measures are computed for the datasets under with and without outliers and summarized in Table 1. From the result, it is clear that the RWSVR performs well in all the three cases under with and without outliers over other regression procedures.

Table 1. Computed Error Measures under various procedures for real datasets

	Case 1 (StarsCYG)				**Case 2 (Carbonation)**				**Case 3 (Prostate)**			
Method	**MAE**	**MAPE**	**MDAE**	**RMSE**	**MAE**	**MAPE**	**MDAE**	**RMSE**	**MAE**	**MAPE**	**MDAE**	**RMSE**
LM	0.478 (0.264)	0.098 (0.054)	0.478 (0.240)	0.552 (0.309)	0.682 (0.601)	0.152 (0.149)	0.571 (0.547)	0.823 (0.722)	0.542 (0.450)	2.314 (2.217)	0.517 (0.403)	0.667 (0.557)
RLM	0.477 (0.264)	0.098 (0.054)	0.474 (0.239)	0.553 (0.309)	0.677 (0.599)	0.15 (0.148)	0.573 (0.538)	0.823 (0.722)	0.541 (0.449)	2.340 (2.229)	0.509 (0.387)	0.667 (0.557)
SVR	0.280 (0.222)	0.057 (0.045)	0.223 (0.176)	0.354 (0.282)	0.55 (0.498)	0.102 (0.102)	0.557 (0.494)	0.684 (0.546)	0.405 (0.370)	1.877 (1.831)	0.234 (0.194)	0.563 (0.503)
RWSVR	0.230 (0.202)	0.046 (0.041)	0.167 (0.147)	0.306 (0.273)	0.518 (0.498)	0.090 (0.098)	0.555 (0.494)	0.680 (0.531)	0.390 (0.353)	1.861 (1.734)	0.217 (0.176)	0.548 (0.486)

(.)without outliers

3.2 Simulation study

This section deals with the results of the simulation environment. The efficiency of LS, RLM, SVR and RWSVR procedures have been studied by computing the error measures.

Here, the simulation study has been performed by considering three cases with the sample of sizes 50, 100 and 500. The details are briefly described as follows:

Case 1: Let, X ~ N(μ, σ), where μ = 30 and σ = 0.5,

The regression model is

$$Y = 10 + 2X + error \tag{18}$$

Error distribution follows normal with μ = 0 and σ = 1.

The simulated model is contaminated with N(μ, σ), where μ = 30 and σ = 1.01 of 0%, 5%, 10% and 20% levels.

Case 2 : X ~ N(μ, $\sum$), where, μ= (μ_1, μ_2, μ_3) = (0,0,0) and the covariance matrix $\sum = I_3$

The regression model is

$$Y = 10 + 3X_1 + 5\,X_2 + 8\,X_3 + error \tag{19}$$

Error distribution follows normal distribution with μ= (μ_1, μ_2, μ_3) = (0,0,0) and the covariance matrix $\sum = 5 * I_3$

The simulated model is contaminated with N(μ, $\sum$), μ= (μ_1, μ_2, μ_3) = (0,0,0) and the covariance matrix $\sum = 1.01 * I_3$ of 0%, 5%, 10% and 20% levels.

Case 3 : X ~ N($\mu, \sum$), where, μ= ($\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$) = (0,0,0,0,0,0) and the covariance matrix $\sum = I_6$
The regression model is

$$Y = 10 + 5\,X_1 + 4\ X_2 + 3\,X_3 + 8\,X_4 + X_5 + 6\,X_6 + error \tag{20}$$

Error distribution follows normal distribution with μ= ($\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$) = (0,0,0,0,0,0) and the covariance matrix $\sum = 5 * I_6$.

The simulated model is contaminated with N($\mu, \sum$), where, μ= ($\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$) = (0,0,0,0,0,0) and the covariance matrix $\sum = 1.01\ I_6$ of 0%, 5%, 10% and 20% levels.

Here, the contamination levels of 0%, 5%, 10% and 20% are considered for checking the efficiencies of proposed method over the existing methods and thus obtained results are summarized in Table 2 given in appendix. From the study, it is clear that the proposed method is more robust for all sample sizes.

4 Conclusion

A novel robust procedure namely, Robust Weighted Support Vector Regression (RWSVR) is proposed to deal with the outlier sensitivity problem in Support Vector Regression (SVR). It improves robustness by utilizing the Hampel weight function in the support vector regression. Traditional regression models performs bad when the cause of outliers. To overcome this drawback, RWSVR concept has been proposed. The robust Hampel's weight function is extended into kernel space to get more accuracy over the traditional SVR. Newly developed kernel function has a higher mapping power than the commonly used linear, polynomial and RBF kernel functions. Experimental results have shown that the proposed RWSVR can reduce the effect of outliers and yield higher accuracy rate than standard SVR does when the data set is contaminated by outliers. This study concluded that the proposed RWSVR is more applicable in almost all areas of statistical learning, specifically when building prediction models with/without contamination in the data. Also, this can be applied almost all the fields of statistical learning in the context of construction of prediction models. Future study is that, may consider depth function as kernel and apply in the contest of pattern recognition.

A Appendix

Table 2. Simulation study

Method	n	C	Case 1				Case 2				Case 3			
			MAE	MAPE	MDAE	RMSE	MAE	MAPE	MDAE	RMSE	MAE	MAPE	MDAE	RMSE
LS	50	0.00	1.197	0.017	1.028	1.492	3.819	1.772	3.240	4.764	3.665	0.041	3.132	4.582
		0.05	1.182	0.017	1.013	1.474	3.837	1.825	3.224	4.784	3.738	0.042	3.205	4.651
		0.10	1.182	0.017	1.013	1.474	3.837	1.825	3.224	4.784	3.738	0.042	3.205	4.651
		0.20	1.182	0.017	1.013	1.474	3.837	2.400	3.223	4.785	3.738	0.042	3.205	4.651
	100	0.00	1.193	0.017	1.016	1.490	3.920	1.628	3.323	4.895	3.885	0.044	3.286	4.859
		0.05	1.183	0.017	1.014	1.478	3.899	1.960	3.306	4.876	3.841	0.043	3.263	4.793
		0.10	1.183	0.017	1.014	1.478	3.899	1.944	3.306	4.876	3.841	0.043	3.263	4.793
		0.20	1.183	0.017	1.014	1.478	3.899	1.940	3.306	4.876	3.841	0.043	3.263	4.793
	500	0.00	1.196	0.017	1.017	1.496	3.983	2.822	3.377	4.985	3.971	0.045	3.362	4.972
		0.05	1.193	0.017	1.011	1.494	3.981	3.162	3.372	4.988	3.971	0.045	3.362	4.972
		0.10	1.193	0.017	1.011	1.494	3.981	3.164	3.372	4.988	3.966	0.045	3.336	4.980
		0.20	1.193	0.017	1.011	1.494	3.981	3.176	3.372	4.988	3.966	0.045	3.336	4.980
RLM	50	0.00	1.194	0.017	1.021	1.494	3.799	1.770	3.182	4.776	3.629	0.041	3.031	4.603
		0.05	1.179	0.017	1.003	1.476	3.815	1.842	3.164	4.796	3.693	0.041	3.065	4.677
		0.10	1.179	0.017	1.003	1.476	3.815	1.842	3.164	4.796	3.693	0.041	3.064	4.677
		0.20	1.179	0.017	1.003	1.476	3.815	2.419	3.164	4.796	3.693	0.041	3.064	4.677
	100	0.00	1.192	0.017	1.014	1.491	3.910	1.628	3.307	4.901	3.866	0.043	3.235	4.869
		0.05	1.181	0.017	1.013	1.479	3.888	1.938	3.288	4.881	3.823	0.043	3.214	4.802
		0.10	1.181	0.017	1.013	1.479	3.888	1.922	3.287	4.881	3.823	0.043	3.214	4.802
		0.20	1.181	0.017	1.013	1.479	3.888	1.918	3.287	4.881	3.823	0.043	3.214	4.802
	500	0.00	1.196	0.017	1.016	1.496	3.981	2.807	3.373	4.986	3.967	0.045	3.353	4.974
		0.05	1.193	0.017	1.011	1.494	3.978	3.175	3.367	4.989	3.967	0.045	3.353	4.974

Continued on next page

Table 2 continued

		0.10	1.193	0.017	1.011	1.494	3.978	3.177	3.367	4.989	3.963	0.045	3.332	4.982
		0.20	1.193	0.017	1.011	1.494	3.978	3.189	3.367	4.989	3.963	0.045	3.332	4.982
	50	0.00	0.998	0.014	0.680	1.362	3.101	1.158	1.623	4.364	2.897	0.033	1.386	4.199
		0.05	1.011	0.014	0.718	1.375	3.081	1.303	1.551	4.374	2.899	0.033	1.421	4.123
		0.10	1.011	0.014	0.720	1.375	3.081	1.303	1.551	4.374	2.899	0.033	1.421	4.124
		0.20	1.011	0.014	0.721	1.375	3.081	1.393	1.553	4.374	2.900	0.033	1.421	4.124
SVR	100	0.00	1.069	0.015	0.810	1.423	3.306	1.159	1.947	4.547	3.085	0.035	1.473	4.357
		0.05	1.060	0.015	0.792	1.416	3.276	1.487	1.947	4.533	3.044	0.035	1.466	4.283
		0.10	1.060	0.015	0.792	1.416	3.276	1.484	1.946	4.533	3.044	0.035	1.467	4.283
		0.20	1.061	0.015	0.792	1.416	3.276	1.482	1.944	4.533	3.044	0.035	1.466	4.283
	500	0.00	1.158	0.017	0.957	1.479	3.690	2.685	2.846	4.818	3.441	0.039	2.310	4.582
		0.05	1.154	0.017	0.952	1.476	3.683	3.108	2.817	4.828	3.441	0.039	2.310	4.582
		0.10	1.154	0.017	0.952	1.476	3.683	3.109	2.816	4.828	3.454	0.039	2.320	4.596
		0.20	1.154	0.017	0.952	1.476	3.683	3.129	2.814	4.828	3.454	0.039	2.321	4.597
	50	0.00	0.991	0.014	0.673	1.358	3.100	1.144	1.596	4.347	2.855	0.033	1.360	4.159
		0.05	1.000	0.014	0.710	1.359	3.071	1.236	1.544	4.363	2.876	0.033	1.411	4.103
		0.10	1.001	0.014	0.710	1.359	3.071	1.236	1.544	4.363	2.876	0.033	1.411	4.104
		0.20	1.000	0.014	0.709	1.359	3.071	1.326	1.474	4.364	2.877	0.033	1.412	4.104
RWSVR	100	0.00	1.066	0.015	0.804	1.422	3.325	1.145	1.030	4.547	3.077	0.035	1.473	4.353
		0.05	1.058	0.015	0.778	1.411	3.260	1.477	1.945	4.516	3.043	0.035	1.456	4.282
		0.10	1.058	0.015	0.776	1.411	3.260	1.475	1.939	4.516	3.035	0.035	1.457	4.277
		0.20	1.058	0.015	0.775	1.411	3.260	1.473	1.940	4.516	3.043	0.035	1.465	4.282
	500	0.00	1.157	0.017	0.957	1.478	3.688	2.676	2.839	4.818	3.440	0.039	2.309	4.581
		0.05	1.154	0.017	0.944	1.476	3.671	3.070	2.812	4.822	3.441	0.039	2.309	4.582
		0.10	1.154	0.017	0.951	1.476	3.681	3.072	2.811	4.822	3.443	0.039	2.300	4.587
		0.20	1.154	0.017	0.951	1.476	3.681	3.094	2.801	4.802	3.443	0.039	2.300	4.587

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