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Wave Propagation in a Homogeneous Poroelastic Layer Bounded Between Transversely Isotropic Poroelastic Half-space and an in-homogeneous Elastic Half-space

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Abstract

Objectives: The present work investigates the propagation of love waves in an isotropic poroelastic layer that is bounded by an inhomogeneous orthotropic half-space and a transversely isotropic elastic half-space. The transversely isotropic poroelastic layer and elastic half-space are taken as initially stressed. **Methods:** Using Biot's theory, the equations of motion for poroelastic layers are derived. The dispersion equation for phase velocity has been derived analytically. **Findings:** The effect of initial stresses of both elastic half-space and layer is observed. Also, the dispersion equation is derived for poroelastic layer and elastic layer as particular cases. Numerical work has been carried out and results are presented graphically. **Novelty:** The propagation of Love waves at the boundary between two elastic substances has received a lot of attention. Present study deals with three solid structures, a layer bounded between two half-spaces.

Keywords: Poroelastic; Transversely isotropic; Phase velocity; Wave number; Initially stressed

1 Introduction

The two-phase medium made up of a solid part (skeleton) and a liquid part (pore space) is referred to as porous material. The study of wave propagation in porous media is significant to many different branches of science and engineering. The theory behind the phenomena has received substantial research in a variety of fields, including soil mechanics, earth science, acoustics, geotechnical engineering, ocean engineering, geophysics, and many others. Primary characteristic of porous media is their enormous solid-liquid contact area, which leads to novel diffusion and transport phenomena in the fluid in relation to the micro-geometry of the pore space. Wave propagation through porous rocks is a topic of interest in geophysics because it can reveal details about the fluid content and soil composition.

[1] discussed Love waves in a transversely isotropic poroelastic layer bounded between two distinct viscous fluids.[2] studied radial vibrations in a composite poroelastic cylindrical shell filled with fluids internally and externally. Gravity and Magnetic fields' effect on thermo-microstretch elastic medium with two temperatures is studied by[3] using dual-phase lag model. The effect of buried moving source on dynamic response at an interface of poroelastic layer and a stratified transversely isotropic half-space is studied by[4].[5] Observed the torsional waves on a magneto-poroelastic dissipative layer bounded between two different poroelastic half-spaces where each solid is transversely isotropic and two half-spaces are initially stressed.[5] discussed effect of gravity and initial stress on Love waves at the interface of porous layer and heterogeneous half-space.[6] investigated plane harmonic wave propagation in fluid saturated transversely isotropic solid using potential method. Novel spectral element procedure has been used by[7] in finding the response in multi-layered structures with respect to moving load. A Green's function approach has been adopted by[8] and presented anti-plane waves in the functionally graded structures with respect to external impulsive force.

In the present paper, Love waves propagating on poroelastic layer between transversely isotropic and an inhomogeneous half-spaces are observed. Few particular cases are observed and discussed. Dispersion equations have been developed and discussed for all the cases under suitable conditions. The phase velocity of plane waves is obtained for a particular model. The effect of initial stress of the half-spaces and inhomogeneity parameters of the lower half-space on the propagation of Love waves has been demonstrated through numerical computation of the dispersion equation. It is found that increasing the initial stress in the lower half of space causes a decrease in phase velocity while having little to no impact on the phase velocity in the upper half-space scenario.

2 Methodology

A poroelastic layer with thickness h is considered. It is bounded between transversely poroelastic half-space (upper half-space) and an in-homogeneous elastic half-space (lower half-space) where both are initially stressed. The origin is located at the interface of lower half-space and poroelastic layer. The positive z-axis is taken towards the interior of the lower half space and wave propagates in x-axis direction.

The solutions of equations of motion in the three solids are derived.

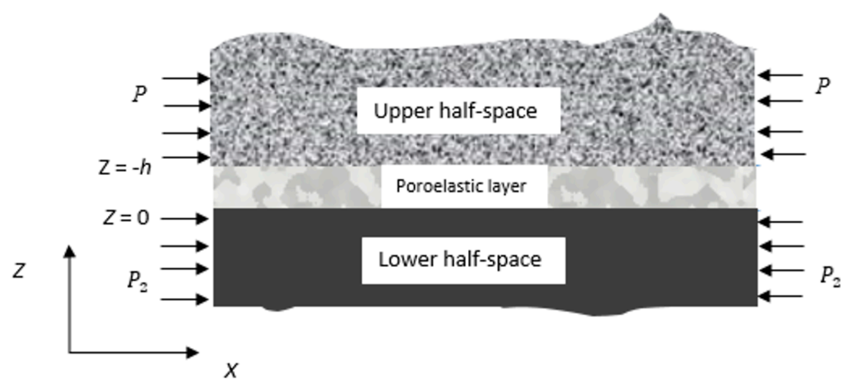

Fig 1. Geometry of the problem

2.1 Solution in upper half-space

In the absence of body forces, the dynamic equations of motion [9] in a transversely isotropic poroelastic material under initial stress P are

$$\begin{aligned}
&\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+\frac{\partial \sigma_{xz}}{\partial z}-P\left(\frac{\partial \omega_z}{\partial y}-\frac{\partial \omega_y}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(e_{11}u_x+e_{12}U_x\right)\\
&\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{yz}}{\partial z}-P\frac{\partial \omega_z}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{11}u_y+e_{12}U_y\right)\\
&\frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}-P\frac{\partial \omega_y}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{11}u_z+e_{12}U_z\right)\\
&\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_x+e_{22}U_x\right)\\
&\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_y+e_{22}U_y\right)\\
&\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_z+e_{22}U_z\right)
\end{aligned} \tag{1}$$

where u,v,w are the displacement components in the solid along x,y,z directions respectively, whereas U,V,W are the components of displacement in the fluid present in the porous solid. The angular components ω_x, ω_y, ω_z are given by

$$\omega_x=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right);\omega_y=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right);\omega_z=\frac{1}{2}\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right) \tag{2}$$

The mass coefficients ρ_{11}, ρ_{12}, ρ_{22} are such that

$\rho_{11}>0,\rho_{12}\leq 0,\rho_{22}>0$

Stress-strain relations in a transversely isotropic poroelastic half space under initial P are presented as

$$\begin{aligned}
&\sigma_{xx}=(A+2N+P)e_{xx}+(A+P)e_{yy}+(F+P)e_{zz}+M\in\\
&\sigma_{yy}=Ae_{xx}+(A+2N)e_{yy}+Fe_{zz}+M\in\\
&\sigma_{zz}=Fe_{xx}+Fe_{yy}+Ce_{zz}+Q\in\\
&\sigma_{xy}=Ne_{xy}\\
&\sigma_{yz}=Le_{yz}\\
&\sigma_{zx}=Le_{zx}\\
&s=Me_{xx}+Me_{yy}+Qe_{zz}+R\in
\end{aligned} \tag{3}$$

Strain components are expressed in terms of displacements

$$\begin{aligned}
&e_{xx}=\frac{\partial u_x}{\partial x},e_{yy}=\frac{\partial u_y}{\partial y},e_{zz}=\frac{\partial u_z}{\partial z}\\
&e_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)\\
&e_{yz}=\frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right)\\
&e_{yz}=\frac{1}{2}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)
\end{aligned} \tag{4}$$

For propagation of love waves displacements along x and z-axes vanishes thus we have

$u_x=0$, $u_z=0$, $u_y=u_y$ (x, z, t) ‘

$U_x=0$, $U_z=0$, $U_y=U_y$ (x, z, t)

Combining the equations (2)-(4) with the above displacements, equation (1) reduces to

$$\begin{aligned}
\left(\frac{N-P}{2}\right)\frac{\partial^2 u_y}{\partial x^2}+\frac{1}{2}\frac{\partial^2 u_y}{\partial z^2}&=\frac{\partial^2}{\partial t^2}\left(e_{11}u_y+e_{12}U_y\right)\\
0&=\frac{\partial^2}{\partial t^2}\left(e_{12}u_y+e_{22}U_y\right)
\end{aligned} \tag{5}$$

Assuming harmonic wave solution in the form

u_y = f(z) $e^{i(kx-t)}$

U_y = g(z) $e^{i(kx-t)}$

where k is a wavenumber, equation (5) yields

$$\frac{L}{2}\frac{d^2 f}{dx^2}-k^2\left(\frac{N-P}{L}\right)=-\omega^2\left(\rho_{11}f+\rho_{12}\,\mathrm{g}\right)$$
$$0=-\omega^2\left(\rho_{12}f+\rho_{22}\,\mathrm{g}\right) \tag{6}$$

Using the solution of the above equations, the displacement u_y in the transversely isotropic poroelastic half-space is attained as

$$u_y=\left(C_1e^{\gamma z}\right)e^{i(kx-wt)} \tag{7}$$

where γ^2 = $k^2\frac{(N-P)}{L}$ - $\frac{2N^2}{LV_1^2}$ and $V_1^2=\left(\frac{N\rho_{22}}{\rho_{11}\rho_{22-}\,\rho_{12}{}^2}\right)$

2.2 Solution in lower half-space

In an inhomogeneous elastic solid with initial stress P the equations of motion [(10)] are given by

$\frac{\partial\tau_{xx}}{\partial x}$ + $\frac{\partial\tau_{xy}}{\partial y}$ + $\frac{\partial\tau_{xz}}{\partial z}$ - $P_2\left(\frac{\partial\Omega_x}{\partial y}\right.$ $-$ $\left.\frac{\partial\Omega_y}{\partial z}\right]$ = $\rho\frac{\partial^2u_x}{\partial t^2}$

$\frac{\partial\tau_{yx}}{\partial x}$ + $\frac{\partial\tau_{yy}}{\partial y}$ + $\frac{\partial\tau_{yz}}{\partial z}$ - $P_2\left[\frac{\partial\Omega_z}{\partial x}\right]$ = $\rho\frac{\partial^2u_y}{\partial t^2}$

$$\frac{\partial\tau_{zx}}{\partial x}+\frac{\partial\tau_{zy}}{\partial y}+\frac{\partial\tau_{zz}}{\partial z}-P_2\left[\frac{\partial\Omega_y}{\partial x}\right]=\rho\frac{\partial^2u_z}{\partial t^2} \tag{8}$$

Where $\tau_{xx},\tau_{xy},\ \tau_{xz},\ \tau_{yx},\tau_{yy},\ \tau_{yz},\tau_{zx},\tau_{zy}$ and τ_{zz} are incremental stresses in the half space u_x, u_y and u_z are displacement components and

Ω_x = $\frac{1}{2}\left[\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right]$

Ω_y = $\frac{1}{2}\left[\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right]$

Ω_z = $\frac{1}{2}\left[\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right]$

For love waves $u_x=0,\ u_z=0$ and $u_y=u_y(x,z,t)$ we get

$$\frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\tau_{yz}}{\partial z}-\frac{P_2}{2}\left[\frac{\partial^2u}{\partial x^2}\right]=\rho\frac{\partial^2u_x}{\partial t^2} \tag{9}$$

The non-zero stress-strain relations are

τ_{yx} = m e_{xy} = $\frac{\mu}{2}\left[\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}\right]$

$$\tau_{yz}=\mu e_{yz}=\frac{\mu}{2}\left[\frac{\partial u_z}{\partial \mathrm{y}}+\frac{\partial u_y}{\partial \mathrm{z}}\right] \tag{10}$$

In-homogeneous of rigidity and density parameters of the half-space are taken as

$\mu=\ \mu_2\left(1+\varepsilon z\right),\ \ \rho=\rho_2(1+\xi\mathrm{z})$

Substituting μ in equation (10),

τ_{yx} =$\mu_2\frac{(1+\in z)}{2}\frac{\partial u_y}{\partial x}$

τ_{yz} = $\mu_2\frac{(1+\in z)}{2}\frac{\partial u_y}{\partial z}$

Using these stresses, the equation of motion (9) reduces to

$$\left(1-\frac{P_2}{\mu_2(1+\varepsilon\mathrm{z})}\right)\frac{\partial^2u_y}{\partial\mathrm{x}^2}+\frac{\partial^2u_y}{\partial\mathrm{z}^2}+\left(\frac{\varepsilon}{1+\varepsilon\mathrm{z}}\right)\frac{\partial u_y}{\partial\mathrm{z}}=2\frac{\rho_2}{\mu_2}\frac{(1+\xi\mathrm{z})}{(1+\varepsilon\mathrm{z})}\frac{\partial^2u_y}{\partial\mathrm{t}^2} \tag{11}$$

Assuming wave solution in the form

$u_y(z) = \varphi(z)e^{i(kx-\omega t)}$
and substituting it in equation (11), we get

$$\frac{d^2\varphi(z)}{dz^2}+\left(\frac{\in}{1+\in z}\right)\frac{d\varphi}{dz}+\left(2\frac{\rho_2}{\mu_2}\frac{(1+z)}{(1+\varepsilon z)}c^2-\left(1-\frac{P_2}{2\ \mu_2\,(1+\ \in z)}\right)\right)k^2\varphi(z)=0 \tag{12}$$

where $c^2=\frac{\omega^2}{k^2}$
Taking g(z) = $\frac{\varphi(z)}{\sqrt{1+\in z}}$ in above equation, we obtain

$$\frac{\mathrm{d}^2\varphi(z)}{\mathrm{d}z^2}+\left(\frac{1}{4}\frac{\varepsilon^2}{(1+\varepsilon Z)^2}-\left(\left(1-\frac{P_2}{\mu'(1+\varepsilon z)}\right)-\frac{2c^2}{c_1^2}\frac{(1+\xi z)}{(1+\varepsilon z)}\right)\mathrm{k}^2\right)\varphi(z)=0 \tag{13}$$

where $c_l^2=\frac{\mu_2}{\rho_2}$
Now defining the variables $\beta^2=1-\frac{P_2}{\mu_2(1+\varepsilon z)}-\frac{2c^2}{c_l^2}\in$ and $s=\frac{2\beta k(1+\varepsilon z)}{\varepsilon}$,
equation (13) can be written as

$$\frac{d^2\varphi(s)}{ds^2}+\left(\frac{R}{2s}+\frac{1}{4s^2}-\frac{1}{4}\right)\varphi(\mathrm{s})=0 \tag{14}$$

where $R=\frac{2\omega^2(\varepsilon-\xi)}{c_l^2\beta\varepsilon^2 k}=\frac{2c^2(\varepsilon-\xi)k}{c_l^2\beta\varepsilon^2}$
The solution of equation (14) is

$$\varphi(s)=D_1W_{\frac{R}{2},0}(s)+D_2W_{-\frac{R}{2},0}(-s) \tag{15}$$

where D_1, D_2 are constants and $W_{\frac{R}{2},0}(s)$ and $W_{-\frac{R}{2},0}(-s)$ are the Whittaker's functions. Since the solution is required in half-space, we must have $u_y(z)\to 0$ as $z\to\infty$ i.e., $\varphi(s)\to 0$ as $s\to\infty$ and hence
$\varnothing(s) = D_2W_{-\frac{R}{2},0}(-s)$
Now, the displacement component $u_y(z)$ can be written as

$$u_y(z)=D_2\left(\frac{W-\frac{R}{2},0}{(-s)}\right)e^{i(\omega t-kx)} \tag{16}$$

Considering linear terms of the Whittaker's functions, equation (16) can be written as

$$u_y(z)=D_2e^{\frac{(1+\varepsilon z)\beta k}{\varepsilon}}\left(\frac{2\beta}{\varepsilon}\right)^{-\frac{R}{2}}(1+\varepsilon z)^{-\left(\frac{R+1}{2}\right)}\left(1-\frac{\varepsilon\left(\frac{R+1}{2}\right)^2}{2\beta k(1+\varepsilon z)}\right) \tag{17}$$

2.3 Solution in poroelastic layer

In the absence of body forces, the governing equations of an isotropic poroelastic solid which is homogeneous are

$$N'\ \nabla^2u+grad[(A'+N')e+Q'\in]=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U)$$

$$grad(Q'e+R'\in)=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U). \tag{18}$$

where $u(u_x,\ u_y,\ u_z)$ and $U(U_x,\ U_y,\ U_z)$ are displacements of the solid and fluid media, respectively, while $A^{'}$, $N^{'}$, $Q^{'}$ and $R^{'}$are all poroelastic constants, and $\rho_{11},\ \rho_{12},\ \rho_{22}$ are the mass coefficients; whereas $e,\ \in$ are solid and fluid dilatations, respectively according to[11]. The stresses S_{ij} and the fluid pressure s of the solid are given by

$$\begin{aligned} S_{xx} &= 2N'e_{xx} + A'e + Q'\varepsilon \\ S_{yy} &= 2N'e_{yy} + A'e + Q'\varepsilon \\ S_{zz} &= 2N'e_{zz} + A'e + Q'\varepsilon \\ S_{xy} &= 2N'e_{xy} \\ S_{yz} &= 2N'e_{yz} \\ S_{zx} &= 2N'e_{zx} \\ s &= Q'e + R'\varepsilon \end{aligned} \tag{19}$$

The displacement components of the solid $(0,\ u_y, 0)$ and fluid $(0,\ U_y, 0)$ are functions of x, y and time t. Now the equation (18) reduces to

$$\begin{aligned} N'\frac{\partial^2 u_y}{\partial x^2} &= \frac{\partial^2}{\partial t^2}\left(\rho'_{11}u_y + \rho'_{12}U_y\right) \\ 0 &= \frac{\partial^2}{\partial t^2}\left(\rho'_{12}u_y + \rho'_{22}U_y\right). \end{aligned} \tag{20}$$

We suppose that the solid and liquid propagation modes u_y and $U_y$are

$$u_y = \phi(z)e^{i(kx-\omega t)}, \quad U_y = \psi(z)e^{i(kx-\omega t)} \tag{21}$$

where t represents time, w the circular frequency, k the wave number and i the complex unity.

Using the equation (21) into (20) yields

$$\begin{aligned} k^2N'\phi &= \omega^2\left(\rho'_{11}\phi + \rho'_{12}\psi\right) \\ 0 &= -\omega^2\left(\rho'_{12}\phi + \rho'_{22}\psi\right) \end{aligned} \tag{22}$$

Solving the two equations of equation (22) for ϕ we obtain,

$$\phi^{''} + \lambda^2\phi = 0, \tag{23}$$

where

$$\lambda^2 = \left(\frac{\omega^2}{\left(V'_2\right)^2} - k^2\right) \text{ and } \left(V'_2\right)^2 = \left(\frac{N'\rho'_{22}}{\rho'_{11}\rho'_{22} - \left(\rho'_{12}\right)^2}\right) \tag{24}$$

On simplification, equation (23) gives

$$\phi(z) = E_1 cos\lambda z + E_2 sin\lambda z, \tag{25}$$

where E_1 and E_2 are constants.

When ϕ (z) from equation (25) is substituted into the first equation of (21), the displacement u_y can be obtained as $u_y = \left(E_1 cos\lambda z + E_2 sin\lambda z\right)e^{i(kx-\omega t)}$.

The current problem's geometry results in the following boundary conditions:

$$\begin{aligned} &\text{at } z = -h, \quad \sigma_{yz} = S_{yz} \\ &\qquad\qquad\qquad u^1_y(z) = u^2_y(z) \\ &\text{at } z = 0, \quad\ \ S_{yz} = \tau_{yz} \\ &\qquad\qquad\qquad u^2_y(z) = u^3_y(z) \end{aligned} \tag{26}$$

Equations represented in boundary conditions (26) are

$$\begin{aligned}
&C_1\gamma L e^{-\gamma h} - E_1 N'\lambda \sin(\lambda h) - E_2 N'\lambda \cos(\lambda h) = 0\\
&C_1 e^{-\gamma h} - E_1 \cos(\lambda h) - E_2 \sin(\lambda h) = 0\\
&E_2 \frac{N'}{2}\lambda - D_2 B = 0\\
&E_1 - D_2 A = 0
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
B &= \frac{\mu}{2}\left(\frac{2\beta}{\varepsilon}\right)^{\frac{-R}{2}} e^{\frac{-\beta k}{\varepsilon}} \left[\frac{\varepsilon^2 (R+1)^2 (R+3)}{16\beta K} - \beta k + \in (R+1)(R-3)\right]\\
A &= e^{-\frac{\beta k}{\varepsilon}} \left(\frac{2\beta}{\varepsilon}\right)^{\frac{-R}{2}} \left(1 - \frac{\varepsilon}{2\beta K}\left(\frac{R+1}{2}\right)^2\right)
\end{aligned} \tag{28}$$

Equations in (27) constitute a system of four linear equations in four arbitrary constants C_1 ,E_1, E_2 *a*nd D_2. If the determinant of coefficients is zero, this system has a non-trivial solution. Thus we obtain

$$tan\lambda h = \frac{LN' A\gamma - 2N' B}{(N')^2 A\lambda - 2LB} \tag{29}$$

Case (1) If $\varepsilon \to 0$ *and* $\xi \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and the secular equation (29) becomes

$$\tan\left(\sqrt{\frac{\omega^2}{V_2^2} - k^2}\right) h = \frac{\mu_2 N' \left[8\left(1-\frac{P_2}{\mu_2}\right) + \frac{C^2}{C_l^2}\right] k + LN' \sqrt{\frac{2N\omega^2}{LV_1^2} - \frac{(N-P_1}{L} k^2} \sqrt{\left(1-\frac{P_2}{\mu_2}\right)}}{(N')^2 \sqrt{\left(1-\frac{P_2}{\mu_2}\right)} \sqrt{\frac{\omega^2}{V_2^2} - k^2 - \mu_2 L \left[8\left(1-\frac{P_2}{\mu_2}\right) + \frac{C^2}{C_l^2}\right] k}}$$

where

$$\begin{aligned}
\gamma^2 &= k^2 \frac{(N-P)}{L} - \frac{2N\omega^2}{LV_1^2}\\
\lambda^2 &= \left(\frac{\omega^2}{V_2^2} - k^2\right)\\
\beta^2 &= 1 - \frac{P_2}{\mu_2(1+\varepsilon z)} - \frac{2c^2}{cf}\frac{\xi}{\varepsilon}
\end{aligned} \tag{30}$$

Case (1.1) If $\varepsilon \to 0$,$\xi \to 0$ and $P_2 \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and the secular equation (29) becomes

$$tan\left(\sqrt{\frac{\omega^2}{V_2^2} - k^2}\right) h = \frac{\mu_2 N' \left[8 + \frac{C^2}{C_l^2}\right] k + LN' \sqrt{\frac{2N^2}{LV_1^2} - \frac{(N-P)}{L} k^2}}{(N')^2 \sqrt{\frac{\omega^2}{V_2^2} - k^2 - \mu_2 L \left[8 + \frac{C^2}{C_l^2}\right] k}} \tag{31}$$

Case (1.2) If $\varepsilon \to 0$,$\xi \to 0$ and $P \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and the secular equation (29) becomes

$$tan\left(\sqrt{\frac{\omega^2}{V_2^2} - k^2}\right) h = \frac{\mu_2 N' \left[8\left(1-\frac{P_2}{\mu_2}\right) + \frac{C^2}{C_l^2}\right] k + N' \sqrt{NL} \sqrt{\frac{2^2}{V_1^2} - k^2} \sqrt{\left(1-\frac{P_2}{\mu_2}\right)}}{(N')^2 \sqrt{\left(1-\frac{P_2}{\mu_2}\right)} \sqrt{\frac{\omega^2}{V_2^2} - k^2 - \mu_2 L \left[8\left(1-\frac{P_2}{\mu_2}\right) + \frac{C^2}{C_l^2}\right] k}} \tag{32}$$

Case (1.3) If $\varepsilon \to 0$,$\xi \to 0$, $P_2 \to 0$ and $P \to 0$ then the non-homogeneous half-space reduces to homogeneous half-space and the secular equation (29) becomes

$$tan\left(\sqrt{\frac{\omega^2}{V_2^2}-k^2}\right)h = \frac{\mu_2 N' \quad \left[8+\frac{C^2}{C_l^2}\right]k + N' \sqrt{NL}\sqrt{\frac{2\omega^2}{V_1^2}-k^2}}{(N')^2 \sqrt{\frac{\omega^2}{V_2^2}-k^2-\mu_2 L} + \left[8+\frac{C^2}{C_l^2}\right]k} \tag{33}$$

Case (2) Upper half space is isotropic poroelastic: If $L = N$ then the upper transversely isotropic poroelastic half space will become poroelastic half space and equation (29) reduces to

$$tan\lambda h = \frac{NN'A\sqrt{\frac{2\omega^2}{V_1^2}-\left(1-\frac{P}{N}\right)k^2} - 2N'B}{(N')^2 A\lambda - 2NB} \tag{34}$$

Case (2.1) Upper half space is isotropic elastic: In addition to $L = N$ if $\rho_{11} = \rho, \rho_{12} \to 0$, $N = \mu$ then the upper poroelastic half space will become elastic half space and equation (34) reduces to

$$tan\lambda h = \frac{\mu N' A\sqrt{\frac{2\omega^2\rho}{\mu}-\left(1-\frac{P}{\mu}\right)k^2} - 2N' B}{(N')^2 A\lambda - 2\mu B} \tag{35}$$

where μ, ρ are Lame constants of upper elastic half space.

Case (3) Layer is isotropic elastic: If $\rho'_{11} = \rho', \rho'_{12} \to 0$, $N' = \mu'$ then the poroelastic layer will become elastic layer and equation (34) reduces to

$$tan\left(\sqrt{\frac{\omega^2\rho'}{\mu'}-k^2}\right)h = \frac{L\mu' A\gamma - 2\mu' B}{(\mu')^2 A\sqrt{\frac{\omega^2\rho'}{\mu'}-k^2} - 2LB} \tag{36}$$

where μ', ρ' are Lame constants of elastic layer.

Case (3.1) Upper half space is isotropic elastic and Layer is isotropic elastic: If $L = N$, $\rho_{11} = \rho$, $\rho_{12} \to 0$, $N = \mu$ and $\rho'_{11} = \rho', \rho'_{12} \to 0$, $N' = \mu'$ then the upper transversely isotropic poroelastic half space will become elastic half space and the poroelastic layer will become elastic layer. The secular equation (29) reduces to

$$tan\left(\sqrt{\frac{\omega^2\rho'}{\mu'}-k^2}\right)h = \frac{\mu\mu' A\sqrt{\frac{2\omega^2\rho}{\mu}-\left(1-\frac{P}{\mu}\right)k^2} - 2\mu' B}{(\mu')^2 A\sqrt{\frac{\omega^2\rho'}{\mu'}-k^2} - 2LB} \tag{37}$$

where μ, ρ are Lame constants of upper elastic half space and μ', ρ' are Lame constants of elastic layer.

3 Results and Discussion

The impact of initial stress of the both lower and upper half-spaces and inhomogeneous parameter of lower half-space has been discussed. Few authors[12,13] have studied the impact of initial stresses and inhomogeneous parameters on elastic layers sandwiched between different elastic half spaces i.e. they have considered the solids as elastic in nature. In present study, we have considered poroelastic and transversely isotropic poroelastic solids. In particular upper half space is transversely isotropic poroelastic and layer is isotropic poroelastic. Thus, the impact of initial stress parameters and inhomogeneous parameters is discussed when upper half space is transversely isotropic and layer is poroelastic. So, in the present paper the impact of transverse poroelasticity of upper half space and poroelasticity of layer on phase velocity is studied.

Neglecting poroelastic constants in the poroelastic layer reduces it into elastic layer and the same is discussed as a particular case. Similarly, frequency equations are obtained by reducing upper transversely isotropic poroelastic half-space into poroelastic and elastic half-spaces as other particular cases. Non-dimensional phase velocity against non-dimensional wave number for various cases is calculated, plotted and discussed.

Figures 2 and 3 depict phase velocity for various values of non-dimensional in-homogeneous parameters (ep $= \varepsilon/k$, $si = \xi/k$) taking constant non-dimensional initial stresses $P_1 = 0.5$, $P_3 = 1$ respectively for upper and lower half-spaces. In particular, Figure 2 is plotted for fixed $\xi/k = 0.2$ and different values of $\varepsilon/k = 0.05, 0.1, 0.2, 0.3, 0.4$ i.e. for constant density parameter and different rigidity parameters. With an increase in wave number, phase velocity is shown to steadily decrease. For higher values of the parameterε/k, phase velocity is more. It shows there is inverse effect of wave number on phase velocity and for a particular wave number there is direct effect of rigidity on phase velocity.

Fixed value $\varepsilon/k = 0.3$ and different values for $\xi/k = 0,\ 0.1,\ 0.2, 0.3, 0.4$ i.e. constant rigidity parameter and different density parameters are considered in Figure 3. Although phase velocity falls as wave number rises, it is less for higher values of $\xi/k = 0.1$. Also, it is noted that the differences in phase velocity for different values of ξ/k is little, particularly for higher values of wave number. It shows there is inverse effect of wave number on phase velocity. For a particular wave number there is inverse effect of density on phase velocity but the effect is less when compared with the effect of rigidity.

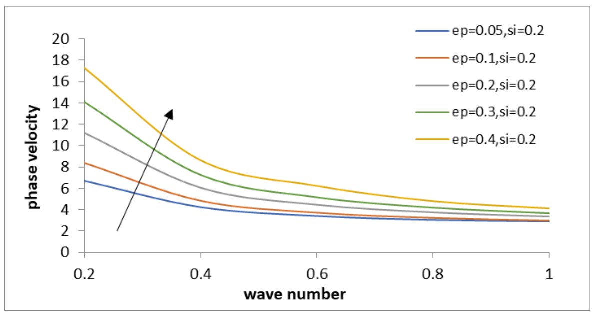

Fig 2. Phase velocity in the intermediate layer for various values of in-homogeneous parameter $ep = \varepsilon/k$. and constant values of $si = \xi/k$

Phase velocity against wave number in the case of poroelastic layer for fixed initial stress parameter $P_1 = 3$ of upper half-space and for various values of initial stress parameter $P_3 = 0,\ 0.5,\ 1,\ 1.5,\ 2$ of lower half-space is presented in Figure 4.

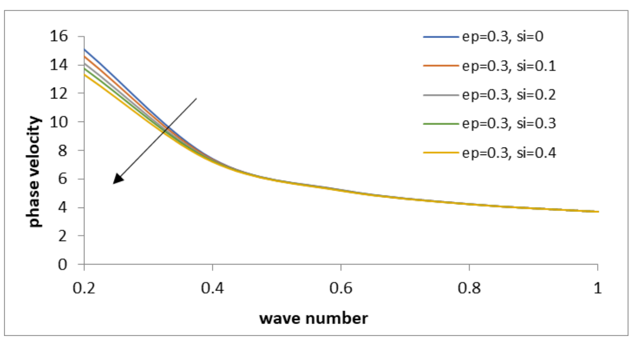

Fig 3. Phase velocity in the intermediate layer for various values of in-homogeneous parameter $ep = \varepsilon/k$. and different values of $si = \xi/k$

The non-dimensional in-homogeneous parameters are taken as $\varepsilon/k = 0.5$, $\xi/k = 0.1$. The phase velocity is low for higher values of initial stress P_3 of the lower half-space. But phase velocity is constant for fixed value of initial stress P_3 of the lower half-space and different values of initial stress P_1 of the upper half-space are considered. Thus, there direct impact of initial stress of lower half-space on phase velocity and phase velocity is independent of the variation of initial stress of the upper half-space.

Figure 5 presents phase velocity against wave number for homogeneous lower half-space. Phase velocity of the poroelastic layer is calculated for fixed initial stress parameter $P_1 = 3$ of upper half-space and different values of initial stress parameter $P_3 = 0,\ 0.2, 0.5, 0.7,\ 2$ of lower half-space. It is noted that the more the initial stress P_3 is taken, the less phase velocity is obtained. Also, the phase velocity is steady when the wave number varies from 0.2 to 0.6 and a sudden decrease is found between 0.6 to 0.8. The phase velocity is nearly same when wave number varies from 0.8 to 1.

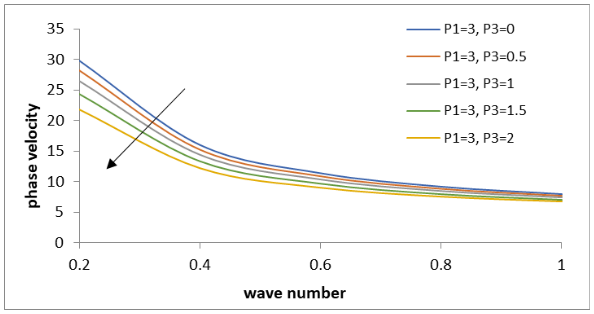

Fig 4. Phase velocity in the intermediate layer for constant value of initial stress P_1 of upper half-space and different values of initial stress P_3 of lower half-space

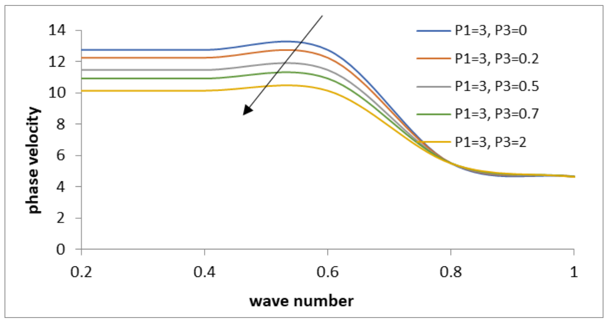

Fig 5. Phase velocity in the intermediate layer for a fixed value of initial stress P_1 of upper half-space and various values of initial stress P_3 of lower half-space when lower half-space is homogeneous

Phase velocity is calculated when upper half-space is poroelastic and results are depicted in figure 6. Phase velocity displays a similar pattern of behaviour to that of transversely isotropic poroelastic upper half-space shown in the Figure 2. However, compared to transversely isotropic poroelastic upper half-space, phase velocity is higher in the case of poroelastic upper half-space. Hence, transverse poroelasticity of the upper half-space reduces the phase velocity.

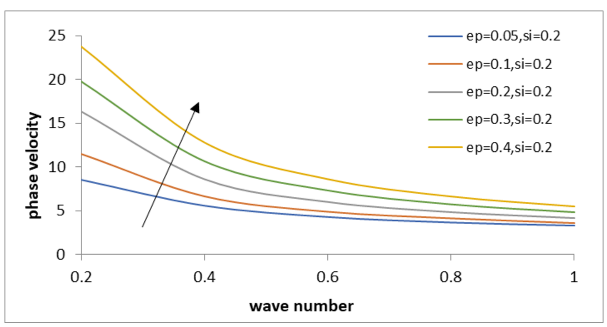

Fig 6. Phase velocity in the intermediate layer for different values of in-homogeneous parameter $ep = \varepsilon/k$. and constant values of $si = \xi/k$when upper half-space is poroelastic

4 Conclusion

The propagation of love waves in poroelastic layer bound to inhomogeneous isotropic half-space and transversely isotropic poroelastic half-space is discussed. Frequency equations of various other particular cases are obtained. The effect of initial stress and in-homogeneity is observed. The results obtained in particular cases are coincided with the results of earlier study. This work can be extended further by discussing effect of gravity, imperfect bonding between the solids.

1. As the wavenumber increases, phase velocity decreases.
2. As the inhomogeneous rigidity parameter increases, so does the phase velocity.
3. Phase velocity drops as density inhomogeneous parameter ξ/k increases. As the ratio of inhomogeneous parameters rises, it is clear that phase velocity rises.
4. Lower inhomogeneous elastic half-space with higher initial stress values generate lower phase velocities.
5. Inhomogeneous lower half-space has a lower phase velocity than homogeneous lower half-space.
6. The phase velocity is decreased as the initial stress of the bottom half-space increases.
7. The phase velocity is not significantly affected by the upper half space's initial stress.
8. When the upper half-space is poroelastic, phase velocity is higher than it is when the upper half-space is transversely isotropic.

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