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* **Corresponding author**.

prashant1.rnsm@gmail.com

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Evolution of Periodic Orbits in the Sun-Jupiter and Sun-Earth Systems

Prashant Kumar[1]*, Edapalapati Sai Dhanush Guptha[2], M Mourya Kishore Raju[2], V Sri Pranav[2]

1 Ajeenkya DY Patil University, Pune, India
2 Hindustan Institute of Technology, Chennai, India

Abstract

Objective: The periodic orbits around smaller and larger primaries are studied in the Sun-Jupiter and Sun-Earth systems, considering smaller primary as an oblate spheroid. **Methods:** We used the Poincare surface section method to find the periodic orbits and discussed the differences between these orbits with and without oblateness. **Findings:** Different trajectories are drawn with the change of Jacobi constant and radiation pressure. Size variations, location, and eccentricity of an orbit around smaller primary are observed due to oblateness effect of the Jupiter and Earth. Oblateness of the smaller primary and radian pressure of the bigger primary have a significant effect on eccentricity, orbit shape, size, and position in phase space. **Novelty :** Small solar system bodies, such as asteroids and comets in the Sun-Jupiter and Sun-Earth systems, can be explored using periodic orbits. Perturbation caused by oblateness of the smaller primary must be understood and addressed during human exploration missions.

Keywords: Circular Restricted Three Body Problem; Poincare Surface Sections; Jacobi Constant; Radiation Factor; Oblateness; Trajectories; Resonance

1 Introduction

In contrast to Hohmann transfer orbit, a low-energy transfer trajectory enables spacecraft to modify their current orbits while using significantly less fuel [1]**.**We need spacecraft to be in a periodic orbit around both of the primary bodies in order to create low-energy transfer trajectories. The circular restricted three body problem (CR3BP) is being a major part of rooting trajectories for spacecraft's. This CR3BP the system is a combination of motion of three bodies, where a third body of infinitesimal mass move in the influence of the two bigger primaries. Theory of orbits' is an outstanding treatise on the R3BP [2]. Periodic orbits are the most stable orbits considered for space missions. Darwin, Moulton, and Stromgren are the authors of the early significant studies on the numerical study of the periodic orbits [3–5]. Huang has explored various intriguing orbits using successive approximations, which serves as a helpful background for determining periodic orbits for the Moon probing vehicle in the actual Earth-Moon-Sun system [6]. Broucke numerically constructed a large number of periodic orbits in the RTBP framework and the orbits were divided into twelve families[7]. Oblateness effect always plays crucial role in affecting the proposed mission path. Tsirogiannis had used oblates to study Liapunov orbits in the photogravitational R3BP[8]. Mittal investigated when one of primaries is an oblate body, periodic orbits are generated by Lagrangian solutions to the R3BP[9]. Dutta calculated the PSS for the Earth-Moon system without considering any perturbations, and he also investigated the Sun-Mars system with solar radiation perturbations[10,11]. The effect of oblateness and radiation pressure has been widely considered for the space trajectories in space missions for planetary studies[12–14] and mainly from oblateness effect of Satrun-Titan R3BP by Safiya and Sharma[15]. Pathak investigated Sun and Saturn centred periodic orbits with solar radiation pressure and oblateness and discovered that solar radiation pressure has a significant effect on the position and geometry of infinitesimal particle orbits[16]. Authors have considered oblateness of the secondary primary as a parameter and results were obtained with the variation of oblateness[17]. They have considered 1 loop to 5 loop orbits of Sun-Earth and Sun-Mars system with different Jacobi constant and oblateness of the smaller primary. They studied about the variation of eccentricity and diameter of the orbits of Sun-Mars system. Recently the authors have developed a model of nonlinear multivariate regression under the structure of the CR3BP with and without perturbations for exterior and interior resonant periodic orbits of order one and found the initial position of periodic orbit without constructing PSS to save time and avoid laborious procedure[18]. We contemplated oblateness effect in different systems and able to analyse the position, time period and as well as the behaviour of the third body at the liberation points, mostly the data obtained is at the L_1 liberation point. In this paper we are discussing about CR3BP of multiple systems and the variation of orbits (size & position). The systems we considered here are: Sun-Jupiter with oblateness and radiation pressure and Sun-Earth with oblateness and radiation pressure. A widely used technique to describe the spacecraft's motion is Poincare surface of section (PSS).

2 Equations of motion

The motion of the third body (infinitesimal) under the effect of the two primaries M_1 and M_2 ($M_1 > M_2$) are shown in Figure 1. In terms of dimensionless synodic coordinate system with origin at the center of mass of the two bodies, whose locations are at ($-\mu$, 0) and ($1-\mu$, 0) with equatorial plane same as the plane of motion and the mass ratio is $\mu = \frac{m_2}{m_1+m_2}$. The equations of motion of third body are:

$$\ddot{x} - 2n\dot{y} = \frac{\partial \Omega}{\partial x}, \ddot{y} + 2n\dot{x} = \frac{\partial \Omega}{\partial y}, \tag{1}$$

where

$$\Omega = \frac{n^2}{2}\left(x^2+y^2\right) + \frac{q\left(1-\mu^2\right)}{r_1} + \frac{\mu}{r_2} + \frac{\left(1-\mu^2\right)A_2}{2{r_1}^3} \tag{2}$$

$$r_1^2 = (x+\mu)^2 + y^2 \tag{3}$$

and

$$r_2{}^2 = (x-1+\mu)^2 + y^2. \tag{4}$$

From equations (1) & (2) we get:

$$\dot{x}^2 + \dot{y}^2 = 2\Omega - C \tag{5}$$

where C is the Jacobi integral constant.

the solar radiation pressure force F_p and gravitational force F_g are inversely proportional to the distance between the bodies. The net force F acting on a body can be expressed as

$$F = F_g - F_R = F_g\left(1 - F_R/F_g\right) = qFF_g, \tag{6}$$

where q = ($1-F_p/F_g$)

when q = 1, F = F_g which means effect of solar radiation pressure force is zero. The mean perturbed motion n of a third body is given by

$$n^2 = 1 + \frac{3}{2}A_2 \tag{7}$$

$$A_2 = \left(\rho_e^2 - \rho_p^2\right)/5R^2 \tag{8}$$

Here A_2 is oblateness co-efficient of second primary. ρ_e and ρ_p are equatorial and polar radii of the second primary and R is the distance between two primaries.

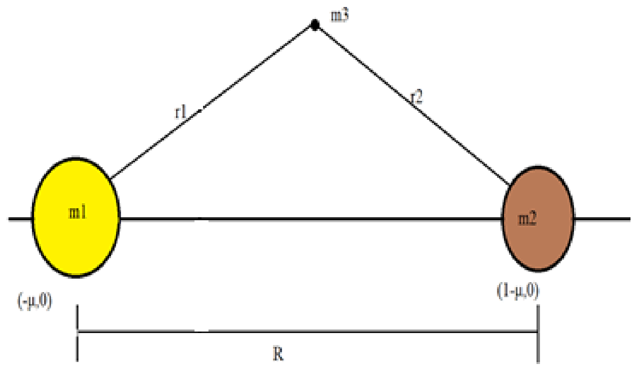

Fig 1. Representation of CR3BP

3 Results and Discussion

3.1 Sun-Jupiter System with oblateness and radiation effect

The effect of radiation pressure of bigger primary on periodic obits in the photo gravitational Sun-Jupiter system was studied[5]. In order to see the effect of Jupiter's oblateness and the Sun's radiation influence, the Sun-Jupiter system is taken into consideration. The mass of Sun m_1 = 1.9891×10^{30} kg and mass of Jupiter m_2= 1.898×10^{27} kg. The distance between Sun and Jupiter is R = 745.85×10^6 kms. The equatorial radius is 71492 kms, The polar radius is 66854 kms, So oblateness of the Jupiter is 0.000231 and the q varies from 1 to 0.9. The mass ratio of the Sun-Jupiter system μ = 0.000953. The Jacobi integral constant varies from 2.8 to 3.0. The equations of motion are solved using Runge Kutta Grill method and graphs are plotted using MATLAB software. Using PSS, we investigated the effect of q on the location and eccentricity of the orbits of the Sun-Jupiter system for various values of the Jacobi constant C. The PSS of the system are generated using different values of C and q. The PSS for C = 3 and q = 1 is shown in Figure 2. The PSS graph is drawn between x and $\dot{x}$. Further we have plotted PSS for C = 2.9 and 2.95 with q = 0.9, 0.95 and 1. Our objective is to use the PSS technique to compare the effect of C and radiation pressure on various parameters of the orbits of the Sun-Jupiter system. We found orbits at x = 0.44613, 0.524705, 0.692455 for C = 2.9 at q = 1, considering this as a first case the variation of orbits are compared. The eccentricity of the orbits is found with the following equations,

$$\mathrm{e}=\sqrt{1-\frac{h^2}{a(1-\mu)}} \tag{3.1}$$

$$\mathrm{h}=(\mathrm{x}+\mu)(\mathrm{y}+\dot{\mathrm{x}}+\mu) \tag{3.2}$$

$$\mathrm{a}=1\div\left[\frac{2}{\mathrm{r}_1}-\frac{\mathrm{v}^2}{1-\mu}\right] \tag{3.3}$$

$$V^2=\left(\dot{x}^2+(y+\dot{x}+\mu)^2\right) \tag{3.4}$$

It is observed that as C increases from 2.9 to 3.0 the eccentricity of the orbit decreases. Variations in eccentricity for C = 2.9 to 3.0 are shown in Table 1. The table shows that a change in C in the range (2.9, 3) affects the location of the periodic orbits. The periodic orbit's location is also affected by C. Sun centred period orbits move towards the Jupiter with increase the radiation pressure and eccentricity increases with radiation pressure of the Sun[5]. We observed a different kind of variation for C = 3.0 the eccentricity of the orbit increased for some extend and then decreased. This variation is shown through graphical representation in Figure 3. We also observed that there are some different kind of orbits which started as Jupiter centered and then slowly changed as Sun centered orbits as shown in Figure 4.

Table 1. Variation in eccentricity of the orbits at q = 1

C	Orbit	x	e
	3 loop	0.44613	0.99863
2.9	4 loop	0.524705	0.9994
	elliptic	0.692455	0.99881
	3 loop	0.49915	0.9984
2.95	4 loop	0.588955	0.99993
	Elliptic	0.7856	0.99943
	3 loop	0.568383	0.99911
3.0	5 loop	0.758019	0.99962
	Elliptic	0.944268	0.99927

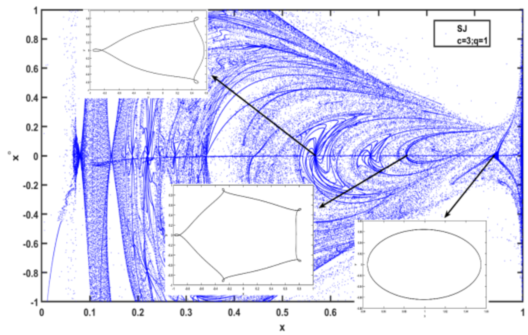

Fig 2. PSS and trajectories of Sun-Jupiter System

Table 2. Variation in eccentricity of the orbits at q = 0.9

C	Orbit	x	e
	3 loop	0.43460	0.99969
2.9	5 loop	0.20901	0.99979
	elliptic	0.98715	0.99948
	5 loop	0.29920	0.99971
2.95	elliptic	0.52391	0.99961
	elliptic	0.99103	0.99970
3.0	elliptic	0.99313	0.99952
	3 loop	0.56769	0.99949

Table 3. Variation in eccentricity of the orbits at q =1

C	Orbit	x	e
	4 loop	0.6739	0.99969
2.9	3 loop	0.560157	0.99873
	Elliptic	0.93585	0.99956
	5 loop	0.239866	0.99912
2.95	8 loop	0.204828	1.00000
	Elliptic	0.9801	0.99951
3.0	Elliptic	0.98868	0.99672

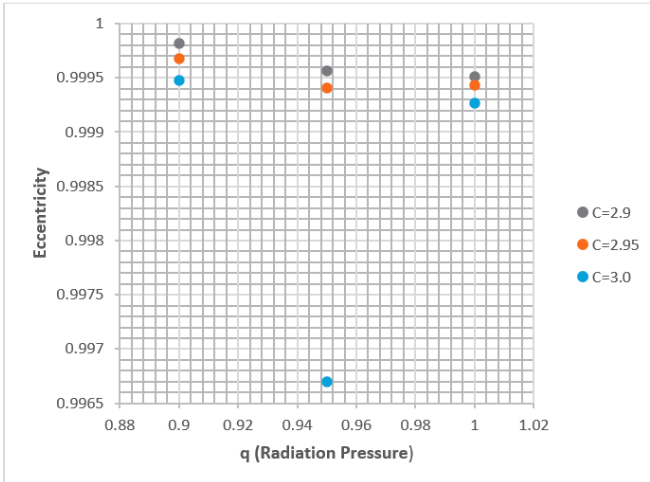

Fig 3. Elliptic orbit at different q and C levels

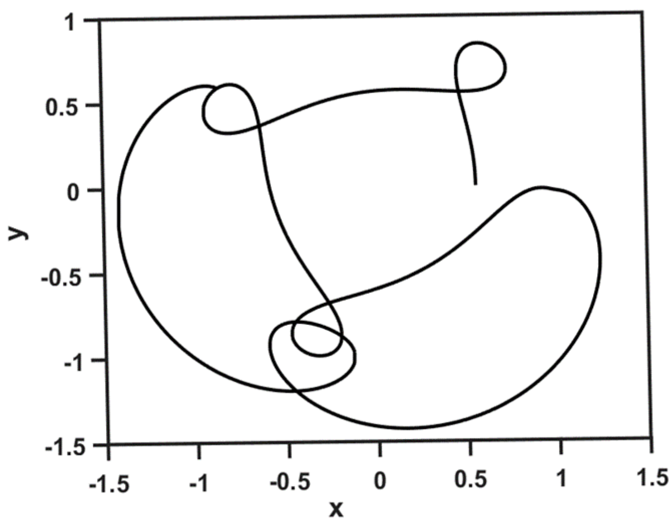

Fig 4. Orbit transfer from Jupiter centered to Sun centered

3.2 Sun-Jupiter System with variation of oblateness

We investigated how the position of periodic orbits and eccentricity around the Sun-Jupiter system varies due to variations in oblateness and Jacobi constants. The C or Jacobi constant often known as energy is used to obtain outcomes. The three cases for C = 2.9, 2.95, and 3.0 with oblateness are considered. The system's radiation pressure is ignored (q = 1). PSS graphs were generated for each of the three cases and PSS are shown in Figures 5, 6 and 7, by considering constant value of oblateness and changing the values of C from 2.9 to 3.0. As per the Kolmogorov-Arnold-Moser, If there are smooth, well defined islands, then the trajectory is likely to be regular and the islands correspond to oscillation around a periodic orbit and we found several periodic orbits: elliptic, two-loop, three-loop, and four-loop orbits. The elliptic orbit is close to Jupiter and Jupiter-centered periodic orbit, whereas the three and four loop orbits are Sun-centered periodic orbits. There are some four and higher loop orbits formed at higher energy, i.e., at C = 3.0, and lower oblateness, i.e., at A_2 = 0.0001 and A_2 = 0.00001. As the oblateness increases from 0.0001 to 0.01, the PSS graph becomes distracted and the orbit vanished. To study the oblateness effect of the Jupiter, we have explored Jupiter centered elliptic orbits for C = 2.9, C = 2.95 and C = 3.0. From the Figure 8, it is observed that the size of the periodic orbit undergo significant changes due to C and eccentricity decreases. Elliptic, three, and four loop orbits are considered for the analysis. These orbits are periodic in nature and the time period, location, and eccentricity are provided in Table 4. It is obvious that as the C increase, the orbits' position recedes away from the Sun[17].

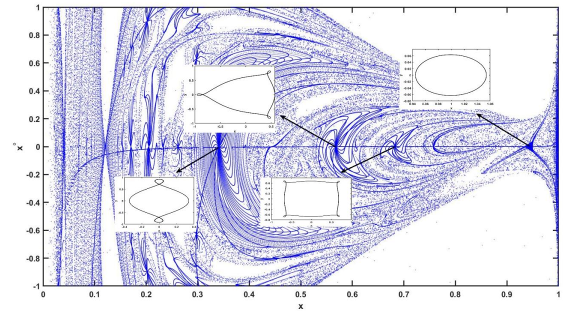

Fig 5. PSS of Sun-Jupiter system for oblateness A_2 = 0.00001 and C=3.0

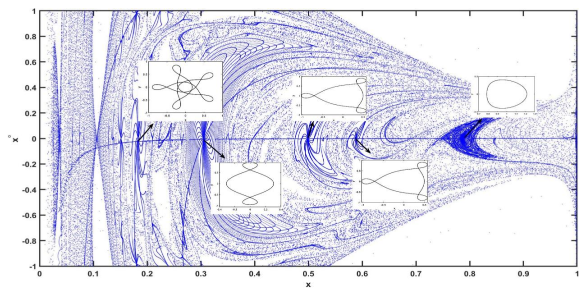

Fig 6. PSS of Sun-Jupiter system for oblateness A_2 = 0.00001 and C=2.95

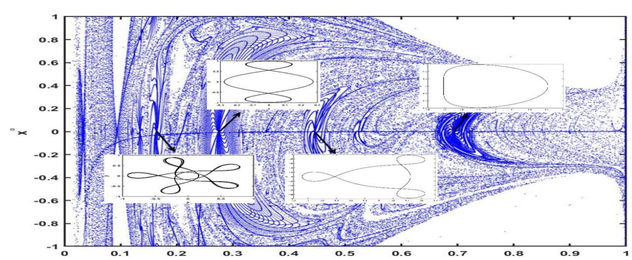

Fig 7. PSS of Sun-Jupiter system for oblateness A_2 =0.00001 and C=2.9

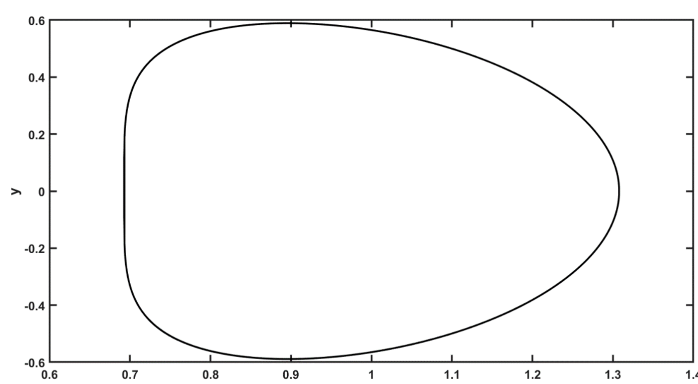

Fig 8. Change in Jupiter centered orbits with Jacobi constant C and oblateness

Table 4. Eccentricity of various orbits for different Cat q =1

C	Orbit	x	e
2.9	Elliptic	0.692698	0.99945
	3 loop	0.45964	0.99956
	2 loop	0.274403	0.99981
	5 loop	0.163642	0.99984
2.95	Elliptic	0.786077	0.99939
	4 loop	0.5885014	0.99958
	3 loop	0.499022	0.9986
	2 loop	0.305416	0.99856
	5 loop	0.1833	0.99987
3.0	Elliptic	0.944725	0.99946
	4 loop	0.683	0.99946
	3 loop	0.568304	0.99927
	2 loop	0.341199	0.99112

Table 5. ccentricity of various orbits for different oblateness at q = 1.

C	Orbit	x	e
2.7	3 loop	0.300295	0.99967
	Elliptic	0.4791	0.45969
	elliptic	0.682446	0.89168
2.8	2 loop	0.180598	0.97883
	2 loop	0.2234	0.98723
	3 loop	0.364276	0.99965
	Elliptic	0.5641	0.99413

Table 6. Eccentricity of various orbits for different oblateness at q = 0.97

C	Orbit	x	e
2.7	2 loop	0.201199	0.99993
	3 loop	0.330702	0.99486
	elliptic	0.5155	0.99999
2.8	2 loop	0.2495	0.95567
	3 loop	0.4044	0.97999
	elliptic	0.622847	0.99984

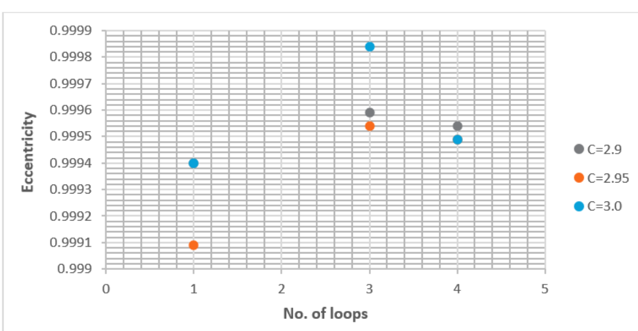

Fig 9. Variation of orbits at different C levels

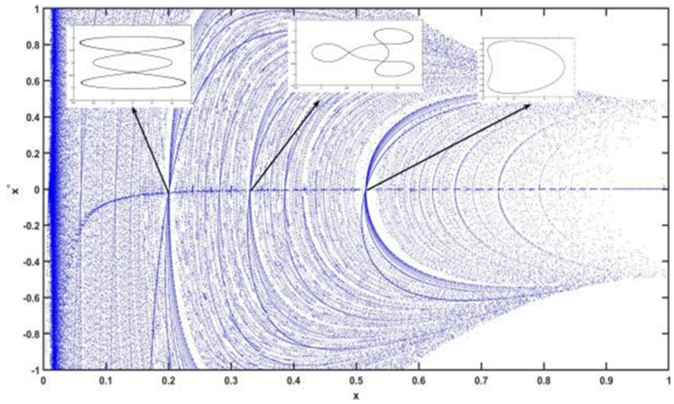

Fig 10. PSS of Sun-Earth System

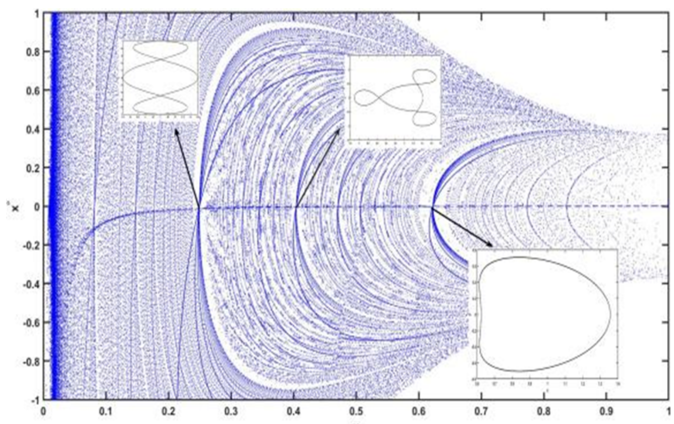

Fig 11. PSS of Sun-Earthsystem

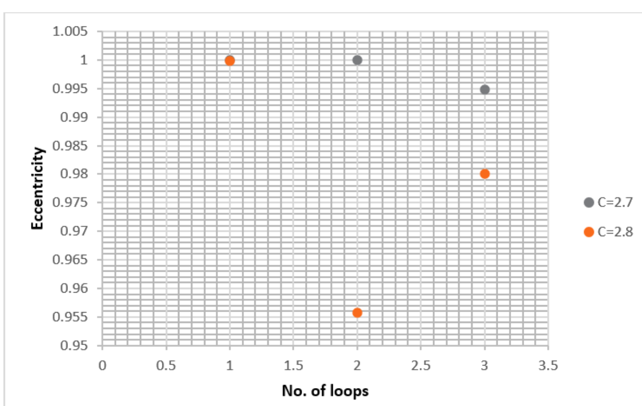

Fig 12. Variation of eccentricity of orbit at q = 0.97

3.3 Sun-Earth System

The effect of oblateness on the position, shape and size of periodic orbits was studied by Pathak and Thomas[7] for Sun-Earth system. In our study, we have considered the effects of solar radiation pressure of bigger primary and actual oblateness of smaller primary on periodic orbits in the Sun-Earth system. The mass of the Sun m_1 = 1.9891×10^{30} kg and the mass of the Earth m_2=5.9722×10^{24} kg. The distance between Earth and Sun R = 147.92×10^{6} kms. The equatorial radii of the Earth ρ_e= 6378.137 kms and the polar radii ρ_p= 6356.75 kms. The oblateness of the Earth is 2.489×10^{-12}. The mass ratio of the Sun-Earth system μ=3.002×10$^{-6.}$ Periodic orbits in Sun-Earth system have great amount of significance for the vast amount of astronomical information that can be gathered for research purpose. These orbits are also sources of asteroids that are of huge scientific and economic value. Hence periodic orbits of Sun-Earth system are found using Poincare maps and further research is done on the sensitivities of position of these periodic orbits due to the effect of radiation pressure from the bigger primary i.e., Sun and the oblateness of the smaller primary i.e., Earth using CR3BP. Initially the Jacobi constant which corresponds to the energy of system is taken as C = 2.7 and the radiation pressure is taken as zero q = 1 and the PSS is plotted and shown in Figure 10. The periodic orbits are found at x = 0.180598, 0.300295, 0.4791 and 0.682446. Now for the energy level i.e., C = 2.7 considering the radiation pressure caused by the Sun q = 0.97 we plot the PSS and shown in Figure 11. We have located the periodic orbits at x = 0.201199, 0.330702 and 0.5155. Similarly we have generated PSS for Jacobi constant C = 2.8 with (q = 0.97) and without the radiation pressure. Tables 5 and 6 provide a comparison of the eccentricity and location of the periodic orbits. The periodic orbits found at these different conditions are unique in nature i.e., these orbits can be elliptical, 2 loop or 3 loop and can also be Sun-centered or Earth-centered. The sensitivities rose due to the effect of radiation pressure and oblateness can be observed by comparing the factors like change in position before and after applying radiation pressure, time period of the orbit and eccentricity of the orbit. We observed that as the energy of the system increases the orbit moves towards the smaller primary and the eccentricity of the orbit decreases. Similarly for the same energy as we increase the radiation pressure the position of orbit is moving further towards the smaller primary and the eccentricity of the orbit is also increasing. Under the same assumptions, our results also agree with those of Pathak & Thomas[7] when the smaller primary is an oblate spheroid. We can say that the characteristics of the periodic orbits are changing based on the application perturbed forces.

3.4 Investigate the impact of the new mean motion

To study the effect of the new mean motion $n = \sqrt{1+3A}$, A is the oblateness coefficient, we have obtained PSS for the Sun-Earth system whose mass ratio μ = 3.002×10^{-6}. In this case, we plotted the two, three, and elliptical periodic orbits and found that their shape remained the same as in the previous case. Table 7 provides a comparison of the location of the periodic orbits for C = 2.7 and 2.8. It should be noted that the mean motion has no effect on the location of the periodic orbits.

Table 7. Initial location of the period orbits

C	Orbit	$n = \sqrt{1+\frac{3}{2}A_2}$	$n = \sqrt{1+3A}$
2.7	2 loop	0.180	0.180
	3 loop	0.300	0.301
	Elliptic	0.479	0.474
2.8	2 loop	0.223	0.223
	3 loop	0.364	0.364
	Elliptic	0.564	0.564

4 Conclusion

We have analysed the motion of a particle in Sun-Jupiter and Sun-Earth system incorporating the effect of oblateness of Jupiter and Earth within the frame work of RTBP, using Poincare surfaces of section method**.** For Sun-Jupiter system we have shown that as radiation pressure increases eccentricity decreases and the orbits' shapes change and as the C increase, the orbits' position recedes away from the Sun. Similarly, as C increases the eccentricity also increases. For the Sun-Earth system we have found elliptic, three-loop and four-loop periodic orbits. The eccentricity of these periodic orbits increases as Jacobian constant increases from 2.9 to 3.0. Size variations, location, and eccentricity of an orbit around Jupiter and Earth are noticed due to oblateness effect of the smaller primary. These periodic orbits could be useful for future mission design in both the systems.

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