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RESEARCH ARTICLE

An Improved Class of Mixed Estimators of Population Mean under Double Sampling

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Abstract

Objectives: To estimate the finite populations mean using two auxiliary variables in double sampling as well as the efficiency of the proposed class of estimators. **Methods:** The mixing of estimators became more popular in developing more efficient estimators for estimating finite population parameters but while mixing two or more estimators, we should consider the basic purpose and the conditions under which the individual estimators are developed and are efficient. This paper deals with a class of mixed estimators of population mean by mixing ratio estimator and dual to product estimator in two phase sampling scheme using SRSWOR scheme to select the sample units at both the cases, i.e. Case-I: When $\bar{X}$ is unknown but $\bar{Z}$ is known and Case-II: When both $\bar{X}$ and $\bar{Z}$ are unknown. The purpose of mixing these two estimators is that both these estimators are designed to be used effectively for population mean when the population correlation coefficient between the study variable and the auxiliary variable is highly positive. **Results:** We observe that the proposed class of estimators is more efficient than the existing estimators which are available in literature and the empirical study indicates that the proposed class of estimators t_{01} and t_{02} performs better than the other existing estimators of $t_1^{'}$, $t_2^{'}$, $t_3^{'}$, $t_4^{'}$, $t_5^{'}$, $t_6^{'}$, $t_7^{'}$, $t_8^{'}$ and $t_9^{'}$. **Novelty:** The percent relative efficiency (PRE) of the proposed class of estimators in Case-II i.e., t_{02} is superior than the estimator proposed in Case-I t_{01} for all the populations except Population 1 and 10, which needs further rigorous attention to compare the performances of t_{01} and t_{02}.

Keywords: Double sampling; Class of estimators; Bias; Mean square error (MSE); Percent relative efficiency (PRE)

1 Introduction

In sample survey literature, estimation of finite population parameters with a greater accuracy has become a matter of concern among the researchers for a long decade. The auxiliary information is being utilized in designing more efficient estimators specifically with a prior knowledge on the population total of the auxiliary character. When the information on the population mean of the auxiliary variable is not available in advance, a cost-efficient technique is the use of double sampling scheme or two-phase sampling. It is used to obtain the information about the auxiliary variable cheaply from a bigger sample at the first stage and relatively a smaller sample at the second stage can be found in [1]. The traditional ratio estimator was modified in several ways earlier by chain ratio-type, regression in ratio-type estimator using prior information of two auxiliary variables x and z. [2] and [3] proposed "dual to ratio estimator and dual to product estimator using the transformation of an auxiliary variable in double sampling". [4] suggested "a class of estimators for estimating finite population mean using two auxiliary variables" and [5] "proposed dual to ratio-cum-dual to product estimator for Y using two-auxiliary variables". [6] suggested "some classes of estimators through predictive approach in presence of two auxiliary variables". [7] proposed "an improved chain-ratio type estimator in double sampling using two-auxiliary variables. [8] introduced an improved chain ratio-product type estimator that has been developed for estimating the population mean of the study variable using two auxiliary variables under double sampling scheme". [9] suggested "an improved estimator of population mean in stratified double sampling using two-auxiliary variables". [10] proposed "a predictive estimator of finite population mean in two phase sampling using two auxiliary variables". [11] suggested "a new family of exponential estimators in the two-phase sampling using the information of an auxiliary attribute". [12] suggested "an estimator for estimating population means using a double sampling scheme using two auxiliary variables". Similarly [13,14], and [15] proposed different estimators for estimating population mean in two-phase sampling. Recently, [16] proposed combination of exponential and ln functions under two-phase sampling using two auxiliary variables for estimating population mean. [17] suggested two-exponential shrinkage estimator using two stage two phase sampling for estimating population mean of study variable and [18] suggested generalized class of exponential type estimators for estimating the finite population means using two auxiliary attributes under simple random sampling and stratified random sampling. [19] proposed a class of dual to ratio estimator for estimating the finite population mean in presence of missing values in two-phase sampling design and two different sampling designs in two-phase sampling are compared under imputed data. [20] suggested a class of estimators employing known non-traditional parameters of auxiliary variable along with some traditional as well and compared the suggested class of estimators with the competing class of estimators using auxiliary variable. They obtained the conditions under which proposed class is more efficient then the competing estimators and these conditions of efficiency are verified using primary data collected from Siddhaur Block of Barabanki District of Uttar Pradesh State in INDIA. [21] proposed exponential ratio estimators in the stratified two-phase sampling by utilizing an auxiliary attribute. It is a common approach to adopt the statistical measures of the auxiliary variable such as correlation, coefficient of variation etc. for estimating the population mean. So, [22] proposed novel estimators by adding an exponential parameter on the auxiliary variable. [23] proposed new class of exponential-type estimators in simple random sampling for the estimation of the population mean of the study variable using information of the population proportion possessing certain attributes.

Consider a finite population having N distinct and identifiable units and y_i is the value of the study variable for the $i-th$ unit of the population $(i = 1, 2, .., N)$ with unknown population mean $\bar{Y}$. In order to estimate $\bar{Y}$, we select a random sample of size n from this population using the SRSWOR scheme. Suppose the class of estimators t is based on the known values of $\bar{X}$ and $\bar{Z}$, i.e., the population means of auxiliary variables x and z. But, when the values are not known to us in advance, we use the method of double sampling. In this procedure, we draw a large preliminary sample known as first phase sample s' of size $n'(< N)$ for the population of size N to observe both the auxiliary variable x and z. Then we draw another sample of size $n(< n')$ from the selected first phase sample usually known as the second phase sample s to observe the study variable y only. At this stage, we are avoiding the case when both the samples are selected independently and directly from the population as the cases are very much similar to this case in the sense that the performance of our proposed class of estimators is like for both the cases. We consider that both these first phase and second phase samples are selected by the SRSWOR scheme.

Case-I: When $\bar{X}$ is unknown but $\bar{Z}$ is known.

Case-II: When both $\bar{X}$ and $\bar{Z}$ are unknown.

We now extend our proposed class of estimators under these two cases as follows:

2 The Proposed Class of Estimators

2.1 Case-I: When \overset-$\bar{X}$ is unknown but \overset-$\bar{Z}$ is known

In this case, we study only the auxiliary variable x for the first phase sample in order to estimate $\bar{X}$. Let $\bar{x}'$ be the mean of the x variable for the n' first phase sample units. So, our proposed class of estimators t_{01} to estimate $\bar{Y}$ under double sampling is given by

$$t_{01} = \bar{y}\left[\alpha_1 \frac{\bar{x}'}{\bar{x}} + (1-\alpha_1)\frac{\bar{x}'}{\bar{x}^*_d}\right]\left[\beta_1 \frac{\bar{Z}}{\bar{z}} + (1-\beta_1)\frac{\bar{Z}}{\bar{z}^*_d}\right] \tag{1}$$

where $\alpha_1,\ \beta_1$ are two real constants and $\bar{x}^*_d = \frac{\bar{x}'-f'\bar{x}}{1-f} = \left(1+g'\right)\bar{x}' - g'\bar{x}$ as an unbiased estimator for $\bar{X}$, $g' = \frac{f^*}{1-f^*}$, $f^* = \frac{n}{n'}$, $f' = \frac{n'}{N}$, $\bar{z}^*_d = \left(1+g'\right)\bar{z}' - g'\bar{z}$. In order to study the large sample behaviour of this estimator, we consider $\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{x}' = \bar{X}\left(1+e_1'\right)$, $\bar{z}' = \bar{Z}\left(1+e_2'\right)$ with $E\left(e_1'\right) = E\left(e_2'\right) = 0$, so e_i's are the sampling errors associated with respective statistics, Thus we have

$$\begin{gathered} E\left(e_0^2\right) = \theta C_{yy}, E\left(e_1^2\right) = \theta C_{xx}, E\left(e_1'^2\right) = \theta' C_{xx}, \quad E\left(e_2^2\right) = \theta C_{zz} \\ E\left(e_2'^2\right) = \theta' C_{zz}, \quad E\left(e_0^2\right) = \theta C_{yy}, E\left(e_1^2\right) = \theta C_{xx}, E\left(e_1'^2\right) = \theta' C_{xx} \\ E\left(e_2^2\right) = \theta C_{zz}, \quad E\left(e_2'^2\right) = \theta' C_{zz}, E\left(e_0 e_1\right) = \theta C_{yx}, E\left(e_0 e_1'\right) = \theta' C_{yx}, \\ E\left(e_0 e_2\right) = \theta C_{yz}, \quad E\left(e_0 e_2'\right) = \theta' C_{yz}, \quad E\left(e_1 e_1'\right) = \theta C_{xx}, \quad E\left(e_1 e_2\right) = \theta C_{xz} \\ E\left(e_1 e_2'\right) = \theta' C_{xz} \end{gathered} \tag{2}$$

and assuming the sampling errors are very small, the bias and MSE of t_{01} terms up to $o(n^{-1})$ are given by

$$\begin{aligned} B(t_{01}) = \bar{Y}[& g' C_{yx}\left(\theta-\theta'\right) + g'^2 C_{xx}\left(\theta-\theta'\right) + \theta g^2 C_{zz} + g\theta C_{yz} + gg' C_{xz}\left(\theta-\theta'\right) \\ & + \alpha_1\left\{\left(1+g'\right)\left(\theta'-\theta\right) + g'^2 C_{xx}\left(\theta'-\theta\right) + g\left(1+g'\right)\left(\theta-\theta'\right) C_{xz}\right\} \\ & + \beta_1\left\{\left(1-g^2\right)\theta C_{zz} - (1+g)\theta C_{yz} - g'(1+g)\left(\theta-\theta'\right) C_{xz}\right\} \\ & + \alpha_1\beta_1(1+g)\left(1+g'\right)\left(\theta-\theta'\right) C_{xz}] \end{aligned} \tag{3}$$

and

$$\begin{aligned} \mathrm{MSE}(t_{01}) = \bar{Y}^2 & \left[\theta C_{yy} + \left(\theta-\theta'\right)\left\{g' - \alpha_1\left(1+g'\right)\right\}^2 C_{xx} + \beta_1\left\{g - \alpha_1(1+g)\right\}^2 \theta C_{zz}\right. \\ & + 2\left\{g' - \alpha_1\left(1+g'\right)\right\}\left(\theta-\theta'\right) C_{yx} + 2\beta_1\left\{g - \alpha_1(1+g)\right\}\theta C_{yz} \\ & \left. + 2\left\{g' - \alpha_1\left(1+g'\right)\right\}\left\{g - \alpha_1(1+g)\right\}\left(\theta-\theta'\right) C_{xz}\right] \end{aligned} \tag{4}$$

respectively. The class of estimators t_{01} attains a minimum mean square error when

$$\alpha_1 = \frac{g'}{1+g'} + \frac{\theta(\rho_{yx} - \rho_{yz}\rho_{xz})}{(1+g')(\theta - \left(\theta-\theta'\right)\rho_{xz}^2)}\sqrt{\frac{C_{yy}}{C_{xx}}} = \alpha_1{}^{(o)} \tag{5}$$

and

$$\beta_1 = -\frac{g}{1+g} + \frac{\left(\left(\theta-\theta'\right)\rho_{yx}\rho_{xz} - \theta\rho_{yz}\right\}}{(1+g)(\theta - \left(\theta-\theta'\right)\rho_{xz}^2}\sqrt{\frac{C_{yy}}{C_{zz}}} = \beta_1{}^{(o)} \tag{6}$$

The minimum MSE of t_{01} is given by

$$MSE\left(t_{01}{}^{(o)}\right) = \bar{Y}^2\theta C_{yy}\left(1 - \frac{\left(\theta-\theta'\right)\rho_{yx}^2 + \theta\rho_{yz}^2 - 2\left(\theta-\theta'\right)\rho_{yx}\rho_{yz}\rho_{xz}}{\left(\theta - \left(\theta-\theta'\right)\rho_{xz}^2\right.}\right]. \tag{7}$$

Case-II: When both \overset-$\bar{X}$ and \overset-$\bar{Z}$

In this case, we study both the auxiliary variables x and z for the selected first phase sample of n' units. Suppose $\bar{x}'$ and $\bar{z}'$ be the mean of x and z-variables respectively for the n' first phase sample units. So, our proposed class of estimators t_{02} to estimate $\bar{Y}$

is now modified to

$$t_{02}=\bar{y}\left[\alpha_2\frac{\bar{x}'}{\bar{x}}+(1-\alpha_2)\frac{\bar{x}'}{\bar{x}_d^*}\right]\left[\beta_2\frac{\bar{z}'}{\bar{z}}+(1-\beta_2)\frac{\bar{z}'}{\bar{z}_d^*}\right] \tag{8}$$

where α_2 and β_2 are two real constants. Following the same procedure as in the case of t_{01}, the bias of t_{02} up to order $o(n^{-1})$ as

$$\begin{aligned}B(t_{02})=\bar{Y}[&\theta g'^2(C_{xx}+C_{zz})-\theta' g'^2(C_{xx}+C_{zz})+(\theta-\theta')g'C_{yx}+(\theta-\theta')g'C_{yz}\\&+(\theta-g'\theta')g'C_{xz}\\&+\alpha_2\{(1+g')(\theta'-\theta)C_{yx}-(\theta-\theta')g'^2C_{xx}+g'(1+g')(\theta'-\theta)C_{xz}\}\\&+\beta_2\{(1+g')(\theta'-\theta)C_{yz}-(\theta-\theta')g'^2C_{zz}+g'(1+g')(\theta'-\theta)C_{xz}\}\\&+\alpha_2\beta_2(1+g')^2(\theta-\theta')C_{xz}]\end{aligned} \tag{9}$$

with MSE as

$$\begin{aligned}\mathrm{MSE}(t_{02})=\bar{Y}^2\Big[&\theta C_{yy}+\{g'-\alpha_2(1+g')\}^2(\theta-\theta')C_{xx}\\&+\{g'-\beta_2(1+g')\}^2(\theta-\theta')C_{zz}+2\{g'-\alpha_2(1+g')\}(\theta-\theta')C_{yx}\\&+2\{g'-\beta_2(1+g')\}(\theta-\theta')C_{yz}\\&+2\{g'-\alpha_2(1+g')\}\{g'-\beta_2(1+g')\}(\theta-\theta')C_{xz}\Big]\end{aligned} \tag{10}$$

The values of α_2 and β_2 for which we get the MVB of this class are given by

$$\alpha_2=\frac{g'}{1+g'}+\frac{(\rho_{yx}-\rho_{yz}\rho_{xz})}{1-\rho_{xz}^2}\sqrt{\frac{C_{yy}}{C_{xx}}}={\alpha_2}^* \tag{11}$$

and

$$\beta_2=\frac{g'}{1+g'}+\frac{(\rho_{yz}-\rho_{yx}\rho_{xz})}{1-\rho_{xz}^2}\sqrt{\frac{C_{yy}}{C_{zz}}}={\beta_2}^* \tag{12}$$

and the minimum MSE of t_{02} becomes

$$MSE\left({t_{02}}^{(o)}\right)=\bar{Y}^2C_{yy}\left(\theta-\left(\theta-\theta'\right)\rho_{y.xz}^2\right]. \tag{13}$$

3 Comparison with Different Existing Estimators

We compare the MSE of the class of estimators t_{01} and t_{02} of equations (7) and (13), with the estimators are mentioned in the review of the literature

3.1 With Simple Mean Estimator

The variance of simple mean estimator is given by

$$V(t_0)=\bar{Y}^2\theta C_{yy}. \tag{14}$$

From equation (14), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to the simple mean estimator t_0.

3.2 With Sukhatme (1962) Double Sampling Ratio Estimator

Using one auxiliary variable, "the ratio estimator in double sampling" proposed by Sukhatme is given by

$$t_1'=\bar{y}\frac{\bar{x}'}{\bar{x}} \tag{15}$$

whose bias and MSE are given by

$$B(t_1^{'}) = \bar{Y}(\theta - \theta^{'})\,[C_{xx} - C_{yx}] \tag{16}$$

and

$$MSE(t_1^{'}) = \bar{Y}^2\left[\theta C_{yy} + (\theta - \theta^{'})\,(C_{xx} - 2C_{yx})\right]. \tag{17}$$

respectively. From (17), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to ratio estimator $t_1^{'}$ in double sampling if

$$\frac{\theta C_{yy}(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz})}{\theta - (\theta - \theta^{'})\rho_{xz}^2} + C_{xx} - 2C_{yx} > 0 \tag{18}$$

and

$$C_{yy}\rho_{y.xz}^2 + C_{xx} - 2C_{yx} > 0 \tag{19}$$

respectively. where $\theta^{**} = \frac{\theta}{\theta - \theta^{'}}$.

3.3 With Chand (1975) Chain Ratio-type Estimator

"Chain ratio-type estimator using prior information of two auxiliary variables x and z" proposed by Chand

$$t_2^{'} = \bar{y}\frac{\bar{x}^{'}}{\bar{x}}\frac{\bar{Z}}{\bar{z}^{'}} \tag{20}$$

The bias and MSE of this estimator are

$$B\left(t_2^{'}\right) = \bar{Y}\left[\theta^{'}C_{yx} - \theta C_{yx} - \theta^{'}C_{yz} - (\theta - \theta^{'})C_{xx}\right], \tag{21}$$

and

$$MSE\left(t_2^{'}\right) = \bar{Y}^2\left[\theta C_{yy} + \left(\theta - \theta^{'}\right)(C_{xx} - 2C_{yx}) + \theta^{'}(C_{zz} - 2C_{yz})\right]. \tag{22}$$

respectively. From (22), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to chain ratio-type estimator $t_2^{'}$ if

$$\left(\theta^{'} - \theta\right)(C_{xx} - 2C_{yx}) - \theta^{'}(C_{zz} - 2C_{yz}) - \frac{\theta\left(\theta - \theta^{'}\right)C_{yy}\left(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta - \left(\theta - \theta^{'}\right)\rho_{xz}^2} > 0 \tag{23}$$

and

$$\left(\theta^{'} - \theta\right)\left(C_{yy}\rho_{yx}^2 + C_{xx} - 2C_{yx}\right) - \theta^{'}(C_{zz} - 2C_{yz}) > 0 \tag{24}$$

respectively.

3.4 With Kiregyera (1980) Regression in Ratio Estimator

Kiregyera proposed an estimator of $\bar{Y}$ by considering a regression estimation of first auxiliary variable x on second auxiliary variable z for the first phase sample as

$$t_3^{'} = \bar{y}\frac{\left[\bar{x}^{'} + b_{yx}\left(\bar{Z} - \bar{z}^{'}\right)\right]}{\bar{x}}, \tag{25}$$

where b_{yx} is the sample regression coefficient of y on x. The bias and MSE, to the first order of approximations are

$$B\left(t_3^{'}\right) = \bar{Y}\left[\theta^{'}\rho_{xz}C_x\left(C_x - C_{yz}\right) - \left(\theta - \theta^{'}\right)C_{yx} - \theta^{'}C_{xx}\right] \tag{26}$$

and

$$MSE\left(t_3^{'}\right) = \bar{Y}^2\left[\theta C_{yy} + \left(\theta - \theta^{'}\right)\left(C_{xx} - 2C_{yx}\right) + \theta^{'}\rho_{xz}C_x\left(\rho_{xz}C_x - C_{yz}C_y\right).\right] \tag{27}$$

respectively. From (27), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to double sampling regression in ratio-type estimator $t_3^{'}$ if

$$\left(\theta^{'} - \theta\right)\left(C_{xx} - 2C_{yx}\right) - \theta^{'}\rho_{xz}C_x\left(\rho_{xz}C_x - C_{yz}C_y\right) - \frac{\theta\left(\theta - \theta^{'}\right)C_{yy}\left(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta - \left(\theta - \theta^{'}\right)\rho_{xz}^2} > 0 \tag{28}$$

and

$$\left(\theta^{'} - \theta\right)\left(C_{yy}\rho_{y.xz}^2 + C_{xx} - 2C_{yx}\right) - \theta^{'}\rho_{xz}C_x\left(\rho_{xz}C_x - C_{yz}C_y\right) > 0 \tag{30}$$

respectively.

3.5 With Kumar and Bahl (2006 Dual to Ratio Estimator

[2] introduced "dual to ratio estimator" of $\bar{Y}$ as

$$t_4^{'} = \bar{y}\frac{\bar{x}_d^*}{\bar{x}^{'}} \tag{31}$$

whose bias and MSE of the estimator $t_4^{'}$ becomes

$$B(t_4^{'}) = -\bar{Y}\left(\theta - \theta^{'}\right)g^{'}C_{yx} \tag{32}$$

and

$$MSE(t_4^{'}) = \bar{Y}^2\left[\theta C_{yy} + \left(\theta - \theta^{'}\right)g^{'}\left(g^{'}C_{xx} - 2C_{yx}\right)\right] \tag{33}$$

respectively. From (33), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to dual to ratio estimator $t_4^{'}$ if

$$\frac{\theta C_{yy}\left(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta - \left(\theta - \theta^{'}\right)\rho_{xz}^2} + g^{'2}C_{xx} - 2g^{'}C_{yx} > 0 \tag{34}$$

and

$$C_{yy}\rho_{y.xz}^2 + g^{'2}C_{xx} - 2g^{'}C_{yx} > 0 \tag{35}$$

respectively.

3.6 With Singh and Choudhury (2012) Dual to Product Estimator

[3] proposed, in the light of[2], dual to product estimator of $\bar{Y}$ as,

$$t_5^{'} = \bar{y}\frac{\bar{x}^{'}}{\bar{x}_d^*} \tag{36}$$

whose bias and MSE becomes

$$B(t_5^{'}) = \bar{Y}\left(\theta - \theta^{'}\right) g^{'}(g^{'}C_{xx} + C_{yx}) \tag{37}$$

and

$$MSE(t_5^{'}) = \bar{Y}^2\left[\theta C_{yy} + \left(\theta - \theta^{'}\right) g^{'}\left(g^{'}C_{xx} + 2C_{yx}\right)\right]. \tag{38}$$

From (38), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to dual to product estimator $t_5^{'}$ if

$$\frac{\theta C_{yy}\left(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta - \left(\theta - \theta'\right)\rho_{xz}^2} + g^{'2}C_{xx} + 2g^{'}C_{yx} > 0 \tag{39}$$

and

$$C_{yy}\rho_{y.xz}^2 + g^{'2}C_{xx} + 2g^{'}C_{yx} > 0 \tag{40}$$

respectively.

3.7 With Choudhury and Singh (2012) Class of Estimators

(12) proposed a class of estimator as

$$t_6^{'} = \bar{y}\left[\alpha\frac{\bar{x}^{'}}{\bar{x}}\frac{\bar{Z}}{\bar{z}^{'}} + (1-\alpha)\frac{\bar{x}^{*}}{\bar{x}^{'}}\frac{\bar{z}^{'}}{\bar{Z}}\right] \tag{41}$$

where α is any constant so as to minimize the MSE of $t_6^{'}$.

Whose bias and MSE becomes

$$B(t_6^{'}) = \bar{Y}\left[\alpha\left(\theta C_{xx} - \theta^{'}C_{xx} + \theta^{'}C_{zz}\right) - \left(\frac{n}{n^{'}-n} + \alpha\left(\frac{n^{'}-2n}{n^{'}-n}\right)\right)\left(\left(\theta - \theta^{'}\right)C_{yx} + \theta^{'}C_{yz}\right)\right] \tag{42}$$

and

$$MSE(t_6^{'}) = \bar{Y}^2\left[C_{yy} - 2\left(\frac{n}{n^{'}-n} + \alpha\left(\frac{n^{'}-2n}{n^{'}-n}\right)\right)\left(\left(\theta - \theta^{'}\right)C_{yx} + \theta^{'}C_{yz}\right) + \left(\frac{n}{n^{'}-n} + \alpha\left(\frac{n^{'}-2n}{n^{'}-n}\right)\right)^2\left(\left(\theta - \theta^{'}\right)C_{xx} + \theta^{'}C_{zz}\right)\right] \tag{43}$$

The optimum value of α becomes,

$\alpha^{(o)} = \frac{(\theta-\theta')C_{yx}+\theta'C_{yz}}{\left(\frac{n'-2n}{n'-n}\right)((\theta-\theta')C_{xx}+\theta'C_{zz})} - \frac{n}{n'-2n'}$

resulting in an MVB estimator of this class.

The MVB of this class is

$$MSE(t_6^{'(o)}) = \bar{Y}^2\left[\theta C_{yy} - (\left(\theta - \theta^{'}\right)C_{yx} + \theta^{'}C_{yz})\left(\theta - \theta^{'}\right)C_{xx}\right]. \tag{44}$$

From (44), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to double sampling ratio-type estimator $t_6^{'}$ if

$$C_{xx}\left(\left(\theta - \theta^{'}\right)C_{yx} + \theta^{'}C_{yz}\right) - \frac{\theta C_{yy}\left(\rho_{yx}^2 + \theta^{**}\rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta - \left(\theta - \theta'\right)\rho_{xz}^2} > 0 \tag{45}$$

and

$$C_{yy}\rho_{y.xz}^2 - C_{xx}\left(\theta - \theta^{'}\right)C_{yx} + \theta^{'}C_{yz} > 0 \tag{46}$$

respectively.

3.8 With Chanu and Singh (2014) Class of Estimators

[4] proposed "a class of estimators for estimating finite population mean using two auxiliary variables" as

$$t_7' = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\frac{\bar{z}}{\bar{z}'}\right)^{\alpha} \tag{47}$$

where α is any constant. The bias and MSE of t_7' is given by

$$B(t_7') = \bar{Y}\frac{\left(\theta-\theta'\right)}{2}\left[\alpha^2\left(C_{xx}+C_{zz}-2C_{xz}\right)+\alpha\left(C_{xx}-C_{zz}\right)-2\alpha\left(C_{yx}+C_{yz}\right)\right] \tag{48}$$

And

$$MSE(t_7') = \bar{Y}^2\theta C_{yy}+\bar{Y}^2\left(\theta-\theta'\right)\left[\alpha^2\left(C_{xx}+C_{zz}-2C_{xz}\right)-2\alpha C_{yx}+2\alpha C_{yz}\right] \tag{49}$$

respectively. The optimum value of α becomes
$\alpha^{(o)} = \frac{C_{yx}-C_{yz}}{C_{xx}+C_{zz}-2C_{xz}}$.
The MVB of this proposed class is

$$MSE\left(t_7'^{(o)}\right) = \bar{Y}^2\theta C_{yy}-\frac{\bar{Y}^2\theta'\left(C_{yx}-C_{yz}\right)^2}{C_{xx}+C_{zz}-2C_{xz}}. \tag{50}$$

From (50), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to estimator t_7' if

$$\frac{\theta'\left(C_{yx}-C_{yz}\right)^2}{C_{xx}+C_{zz}-2C_{xz}}-\frac{\theta\left(\theta-\theta'\right)C_{yy}\left(\rho_{yx}^2+\theta^{**}\rho_{yz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta-\left(\theta-\theta'\right)\rho_{xz}^2}>0 \tag{51}$$

and

$$\left(\theta-\theta'\right)C_{yy}\rho_{y.xz}^2+\frac{\theta'\left(C_{yx}-C_{yz}\right)^2}{C_{xx}+C_{zz}-2C_{xz}}>0 \tag{52}$$

respectively.

3.9 With Choudhury and Singh (2015) Estimator

[5] proposed an estimator

$$t_8' = \bar{y}\left(\frac{\bar{x}_d^*}{\bar{x}'}\right)\left(\frac{\bar{z}'}{\bar{z}_d^*}\right) \tag{53}$$

whose bias and MSE becomes

$$B(t_8') = \bar{Y}\left(\theta-\theta'\right)g'\left[-C_{yx}+C_{yz}+g'C_{zz}\right] \tag{54}$$

and

$$MSE(t_8') = \bar{Y}^2\left[\theta C_{yy}+\left(\theta-\theta'\right)g'\left(2(C_{yz}-C_{yx})+g'\left(C_{xx}+C_{zz}-2C_{xz}\right)\right\}\right] \tag{55}$$

From (55), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to estimator t_8' if

$$\frac{\theta\left(\theta-\theta'\right)C_{yy}\left(\rho_{yx}^2+\theta^{**}\rho_{yz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta-\left(\theta-\theta'\right)\rho_{xz}^2}+2g'\left(C_{yz}-C_{yx}\right)+g'\left(C_{xx}+C_{zz}-2C_{xz}\right)>0 \tag{56}$$

and

$$C_{yy}\rho_{y.xz}^{2}+2g^{'}(C_{yz}-C_{yx})+\ g^{'}(C_{xx}+C_{zz}-2C_{xz})>0 \tag{57}$$

respectively.

3.10 With Guha and Chandra (2019) Class of Estimators

(7) proposed “improved chain-ratio type estimator in double sampling” as

$$t_9^{'}=\bar{y}\left[\frac{\bar{Z}}{\bar{z}}+(1-\alpha)\frac{\bar{x}^{'}}{\bar{x}}\frac{\bar{Z}}{\bar{z}^{'}}\right] \tag{58}$$

where $\alpha(0<\alpha<1)$ is any constant so as to determine by minimizing the MSE of $t_9^{'}$. Its bias and MSE are given as

$$B(t_9^{'})=\bar{Y}\left[(\alpha-1)\left(\theta^{'}C_{xx}+(\theta-\theta^{'})C_{yx}\right)-\alpha\left(\theta-\theta^{'}\right)C_{yz}-\theta^{'}C_{yz}\right] \tag{59}$$

and

$$MSE\left(t_9^{'}\right)=\bar{Y}^{2}\left[\alpha^2\theta\left(C_{yy}+C_{zz}-2C_{yz}\right)+(1-\alpha)^2\left(\theta C_{yy}+\left(\theta-\theta^{'}\right)\left(C_{xx}-2C_{yx}\right)+\theta^{'}\left(C_{zz}-2C_{yz}\right)\right\}\right] \tag{60}$$

respectively. The optimum value of α leading to an MVB estimator of this class as
$\alpha^{(o)}=\frac{\theta C_{yy}+\left(\theta-\theta^{'}\right)\left(C_{xx}-2C_{yx}\right)+\theta^{'}\left(C_{zz}-2C_{yz}\right)}{\theta\left(C_{yy}+C_{zz}-2C_{yz}\right)+\theta C_{yy}+\left(\theta-\theta^{'}\right)\left(C_{xx}-2C_{yx}\right)+\theta^{'}\left(C_{zz}-2C_{yz}\right)}$
The MVB of this class becomes

$$MSE\left(t_9^{'(o)}\right)=\bar{Y}^{2}\frac{D_1D_2}{D_1+D_2}. \tag{61}$$

From (61), (7), and (13), the class of estimators t_{01} and t_{02} are always preferred to estimator $t_9^{'}$ if

$$\theta C_{yy}\left[1-\frac{\left(\theta-\theta^{'}\right)\left(\rho_{yx}^2+\theta^{**}\rho_{yz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)}{\theta-\left(\theta-\theta^{'}\right)\rho_{xz}^2}\right]-\frac{D_1D_2}{D_1+D_2}>0 \tag{62}$$

and

$$C_{yy}\theta\left[-\left(\theta-\theta^{'}\right)\rho_{y.xz}^2\right]-\frac{D_1D_2}{D_1+D_2}>0 \tag{63}$$

respectively, where
$D_1=\theta\left(C_{yy}+C_{zz}-2C_{yz}\right)$ and $D_2=\theta C_{yy}+\left(\theta-\theta^{'}\right)\left(C_{xx}-2C_{yx}\right)+\theta^{'}\left(C_{zz}-2C_{yz}\right).$

4 Empirical Study

To study the performance of the proposed classes of estimators t_{01} and t_{02} with other estimators as well as classes of estimators under double sampling, we proceed as in the case of single-phase sampling. Table 1. represents the source and description of the populations, which are available in different textbooks. Table 2. represents some selected characteristics of different populations. We can see that the proposed classes of estimators perform better than other competing classes on the basis of bias (Table 3), MSE (Table 4), optimum values of the proposed classes of estimators (Table 5) as well as PRE (Table 6). Again, the percent relative efficiency (PRE) of the proposed class of estimators in Case-II i.e., t_{02} is superior than the estimator proposed in Case-I t_{01} for all the populations except Population 1 and 10, which needs further rigorous attention to compare the performances of t_{01} and t_{02}.

Table 1. Sources and Descriptions

P. No.	*y*	*x*	*z*	Sources
1	Actual Inflation Rate	Unemployment rate	Unexpected inflation rate	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 203
2	"Real Gross Product (Millions of NT$)"	"Labour days (Millions of days)"	"Real capital Input (Millions of NT$)"	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 216
3	Real Gross Product (millions of NT$)	Labour input(per a thousand persons)	Real Capital input (Millions of NT$)	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 224
4	Defense budget outlay for years *t*$/billions	GNP in different years, $/billions	U.S. military sales/assistance	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 227
5	Defense budget outlay for years *t*$/billions	GNP in different years, $/billions	Average industry sales	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 227
6	"Per capita consumption of Chicken, Ibs"	"Real disposable income per capita,$"	"Real retail price of chicken per Ib"	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 228
7	"Per capita consumption of Chicken, Ibs"	"Real disposable income per capita,$"	"Real retail price of pork per Ib"	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 216
8	"Per capita consumption of Chicken, Ibs"	"Real disposable income per capita, $"	"Real retail price of beef per Ib"	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 216
9	"Per capita consumption of Chicken, Ibs"	"Real disposable income per capita ,$"	"Composite real price of chicken substitutes per Ib"	Gujarati, N.D. (1945). Basic Econometrics. Tata McGraw-Hill: New Delhi. p. 216
10	"Mean yield of rice per plant"	"Number of tillers"	"Percentage of sterility"	Swain, A.K.P.C. (2003). Finite Population Sampling Theory and Methods. South Asian Publishers: New Delhi. p. 286
11	"Area under wheat in 1964 (in acres)"	"Area under wheat in 1963 (in acres)"	"Cultivated area in 1961 (in acres)"	Murthy M.N. (1967). Sampling Theory and Methods. Statistical Publishing Society: Calcutta, India. p. 399
12	Output of the Factory	The number of workers	Fixed capital	Murthy M.N. (1967). Sampling Theory and Methods. Statistical Publishing Society: Calcutta, India. p. 399
13	"Season average price per pound during1996"	"Season average price per pound during1995"	"Season average price per pound during1994"	Singh S. (2003). Advanced sampling theory with applications. Kluwer Academic Publishers: The Netherlands. p. 1115
14	"Number of 'placebo' Children"	"Number of paralytic polio cases in the 'placebo' group"	"Number of paralytic polio cases in the 'not inoculated' group"	Cochran W.G. (1977). Sampling Techniques. III Edition, Wiley Eastern Limited: New Delhi, p.182

Table 2. Some selected characteristics of the populations)

P. No.	N	n'	n	g'	$\bar{Y}$	$\bar{X}$	$\bar{Z}$	C_{yy}	C_{xx}	C_{zz}
1	13	8	3	-1.60	7.757	6.654	6.686	0.167	0.051	0.152
2	15	10	4	-2.00	24735.333	287.347	25506.633	0.042	0.003	0.089
3	15	9	3	-1.50	24292.527	578.613	159919.33	0.349	0.191	0.062
4	20	16	7	-4.0	83.860	1358.155	6.287	0.126	0.299	1.051
5	20	15	4	-3.0	83.860	1358.155	29.145	0.126	0.299	0.243
6	23	18	5	-3.6	39.670	1035.065	47.996	0.036	0.373	0.056
7	23	17	3	-2.83	39.670	1035.065	90.400	0.036	0.373	0.159
8	23	19	6	-4.75	39.670	1035.065	124.448	0.036	0.373	0.179
9	23	19	6	-4.75	39.670	1035.065	107.857	0.036	0.373	0.142
10	50	35	15	-2.33	12.830	9.040	18.762	0.164	0.072	0.010
11	34	18	6	-1.12	199.441	747.588	208.882	0.584	0.911	0.571
12	80	55	18	-2.20	5182.637	285.125	1126.463	0.127	0.911	0.571
13	36	28	11	-3.50	0.203	0.186	0.171	0.161	0.169	0.142
14	34	26	12	-3.25	4.924	2.488	2.912	1.079	1.567	1.358

Table 3. Bias of Different Estimators

P. No.	t_1'	t_2'	t_3'	t_4'	t_5'	t_6'
1	0.065	-0.147	-0.03	-0.005	0.013	-0.091
2	-22.797	-88.274	-35.711	-12.128	13.524	-56.091
3	-309.401	-2529.816	-1414.671	-335.173	399.628	3654.900
4	0.679	-3.560	-1.425	-0.669	1.253	-7.444
5	1.754	-7.665	-2.982	-0.709	0.996	3.678
6	1.505	-2.782	-0.652	-0.175	0.340	-0.114
7	2.860	-5.295	-1.231	-0.179	0.271	0.800
8	1.188	-2.209	-0.515	-0.175	0.385	-0.404
9	1.188	-2.206	-0.514	-0.175	0.385	-0.392
10	-0.003	-0.072	-0.045	-0.016	0.023	0.060
11	-1.183	-20.143	-10.972	-1.979	2.349	19.121
12	116.157	-244.203	-67.642	-17.504	32.372	-39.234
13	0.000	-0.004	-0.002	-0.001	0.001	0.002
14	0.136	-0.591	-0.262	-0.115	0.218	-1.445

Table 4. Continued

P. No.	$t_7{}'$	$t_8{}'$	$t_9{}'$	t_{01}	t_{02}
1	0.501	0.078	-0.282	-1.027	-1.335
2	296.861	103.948	201.694	241.762	-1119.863
3	-3544.604	-117.006	-1748.835	-22732.870	-6805.933
4	1.888	2.442	-1.547	-12.112	-122.697
5	-2.026	0.183	-3.588	-37.406	-77.934
6	0.121	-0.090	-1.056	-15.337	-22.637
7	-0.349	-0.028	-1.649	-16.062	-49.604
8	-0.263	0.046	-0.741	-16.062	-46.203
9	-0.126	0.012	-0.734	-17.625	-41.588
10	0.004	-0.017	-0.099	-0.100	-0.078
11	33.641	1.168	-13.889	-113.170	-117.266
12	-72.724	6.065	-85.066	-1134.357	-2165.942
13	-0.001	0.000	-0.002	-0.021	-0.038
14	-0.064	0.068	-0.365	-2.760	-5.727

Table 5. MSE of Different Estimators

P. No.	t_0	$t_1{}'$	$t_2{}'$	$t_3{}'$	$t_4{}'$	$t_5{}'$
1	2.570	2.942	2.657	2.733	0.765	2.709
2	4666665.0	3277796.2	2890251.6	3056112.3	246353.4	5301173.0
3	54923189	14838296	8792268	6505011	15344563	72773359
4	82.077	42.334	83.555	38.965	131.635	243.245
5	176.781	86.107	74.428	75.308	325.084	319.245
6	8.895	43.626	43.255	46.321	84.414	29.302
7	16.473	82.472	82.973	85.825	159.297	34.331
8	7.001	34.420	34.832	36.554	66.624	29.244
9	7.001	34.420	34.832	36.554	66.624	29.244
10	1.256	0.739	0.781	0.741	0.220	1.754
11	3190.947	1113.374	529.874	571.633	1268.124	4053.989
12	146918.30	436453.87	446183.60	475668.52	949137.68	405428.19
13	0.000	0.000	0.000	0.000	0.000	0.001
14	1.410	1.042	0.998	0.952	1.425	3.048

Table 6. Continued

P. No.	$t_6{}'$	$t_7{}'$	$t_8{}'$	$t_9{}'$	t_{01}	t_{02}
1	2.375	0.743	3.534	1.601	0.648	0.740
2	3602755.8	2097358.2	8529039.4	28847920.7	16329412.8	1193805.4
3	5407979	23378240	48580038	9907215	4002731681	9610981
4	37.289	63.405	203.679	494.824	12099.978	16.690
5	16.149	169.813	170.875	100.082	59317.573	20.105
6	1.198	1.485	20589	83.990	3155.345	1.418
7	1.825	5.549	11.827	136.663	10181.089	2.471
8	0.799	1.908	3.559	57.841	17295050	1.186
9	0.796	1.759	2.992	53.241	15310.429	1.179
10	0.784	0.664	0.880	5.130	0.652	0.664
11	613.715	2401.719	3459.216	637.513	58497.467	707.097
12	24519.92	87317.84	117980.20	264669.54	40172659.68	32601.52
13	0.000	0.000	0.000	0.000	0.037	0.000
14	0.884	1.374	1.504	2.361	12.530	0.736

Table 7. Optimum values of the Proposed Class of Estimators

P. No.	α_1^*	β_1^*	α_2^*	β_2^*
1	1.056	-1.390	1.194	-1.267
2	-1.543	-1.726	-1.719	-0.416
3	-0.018	-22.614	-0.058	-2.265
4	-0.498	-4.266	-0.258	-2.265
5	-0.091	-13.794	-0.155	-0.543
6	-0.258	-15.212	-0.388	0.258
7	-0.174	-12.051	-0.274	-0.033
8	-0.077	-22.638	-0.278	-0.025
9	-0.076	-23.904	-0.236	-0.097
10	-1.083	0.897	-1.083	1.088
11	-0.050	-5.465	-0.096	-0.953
12	0.038	-8.286	0.250	-0.757
13	-0.372	-10.559	-0.531	-0.405
14	-0.399	-2.915	-0.457	-0.237

Table 8. PRE of Different Estimators

P. No.	t_0	t_1'	t_2'	t_3'	t_4'	t_5'
1	100	87.378	96.742	94.053	335.972	94.892
2	100	142.372	161.462	152.699	1894.297	88.031
3	100	370.145	624.676	844.321	357.933	75.472
4	100	193.881	98.231	210.643	62.352	33.742
5	100	205.303	237.520	234.743	54.380	55.289
6	100	20.390	20.565	19.204	10.538	30.358
7	100	19.973	19.853	19.194	10.341	47.983
8	100	20.340	20.099	19.152	10.508	23.940
9	100	20.340	20.272	19.168	10.508	23.940
10	100	170.000	160.867	169.592	571.277	71.630
11	100	286.602	602.209	588.216	251.627	78.711
12	100	33.662	32.928	30.887	15.479	36.238
13	100	289.348	393.803	394.712	186.767	53.497
14	100	135.362	141.246	148.155	98.985	46.267

Table 9. Continued

P. No.	t_6'	t_7'	t_8'	t_9'	t_{01}	t_{02}
1	108.245	345.928	72.726	160.527	396.568	347.499
2	129.530	222.502	54.715	16.177	28.578	390.907
3	1015.595	234.933	113.057	554.376	1.372	57.463
4	220.107	129.449	40.297	16.587	0.678	491.770
5	1094.679	104.103	103.456	176.363	0.298	879.265
6	742.339	599.064	343.533	10.591	0.282	627.123
7	902.528	296.882	139.281	12.054	0.162	666.757
8	901.624	366.925	196.717	12.104	0.040	590.153
9	879.517	398.031	233.970	13.149	0.046	593.975
10	160.228	189.193	142.685	24.486	192.655	189.194
11	519.940	132.861	92.245	500.531	5.455	451.274
12	599.170	168.257	124.528	55.510	0.366	450.649
13	391.593	103.347	103.296	148.992	1.143	335.583
14	167.007	102.607	93.796	59.732	11.255	191.650

5 Conclusion and Further Study

In this article, we proposed a class of estimators for estimating the finite population mean by mixing of ratio and dual to product estimators in double sampling scheme. These two estimators which are mixed were designed to be used in case of positive correlation coefficient between the study variable y and the auxiliary variables x and z. We have used double sampling procedure in designing two different classes of estimators t_{01} and t_{02}. In order to judge the performance of the suggested class of estimators, we analyze the large sample properties in terms of bias and mean square error up to first order approximation. The optimality conditions were derived for the proposed class of estimators and the minimum mean square errors were calculated for the purpose of comparison with several competing estimators namely, with simple mean estimator t_0, double sampling ratio estimator $t_1^{'}$, Chand's chain-ratio type estimator $t_2^{'}$, Kiregyera's estimator $t_3^{'}$, Kumar and Bahl (2006) estimator $t_4^{'}$, Singh and Choudhury (2012) estimator $t_5^{'}$, Choudhury and Singh (2012) estimator $t_6^{'}$, Chanu and Singh (2014) estimator $t_7^{'}$, Choudhury and Singh (2015) estimator $t_8^{'}$ and Guha and Chandra (2019) estimator $t_9^{'}$. Again for the numerical comparion of these estimators, we have considered 14 different natural populations available in literature and are specified in Table 1. Both theoretically as well as numerically, we have evidenced that the performance of the proposed classes of estimators is always better than other competing estimators /classes of estimators on the basis of its bias, mean square error and percent relative efficiency (PRE). The estimation techniques developed in the present article may be further extended to estimating the other population parameters such as population total, ratio, variance, quantiles, correlation and regression coefficients etc.

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