INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Prime Quasi-Ideals in Ternary Seminear Rings

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OPEN ACCESS

Received: 18-08-2022

Accepted: 19-09-2022

Published: 17-10-2022

Citation: Vijayakumar R, Bharathi AD (2022) Prime Quasi-Ideals in Ternary Seminear Rings. Indian Journal of Science and Technology 15(39): 2037-2040. https://doi.org/10.17485/IJST/v15i39.1700

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives/Background: The ternary seminear ring is the generalization of seminear ring and it need not be a ternary semiring. Characterization of quotient ternary seminear rings and some structures of ternary seminear ring have been analysed and also studied ideals in ternary seminear rings. Further quasi ideals in ternary seminear rings defined and discussed about their properties. **Methods:** Properties of seminear ring and ternary semiring have been employed to carry out this research work to obtain all the characterizations of ternary seminear rings corresponding to that ternary semiring. **Findings:** We call an algebraic structure $(T,+,.)$ is a ternary seminear ring if (T,+) is a Semigroup, T is a ternary semigroup under ternary multiplication and $xy(z+u) = xyx + xyu$ for all $x,y,z,u \in T$. T is said to have an absorbing zero if there exists an element $0 \in T$ such that $x+0 = 0+x = x$ for all $x \in T$ and $xy0 = x0y = 0xy = 0$ for all $x,y \in T$. Throughout this paper T will always stand for ternary seminear ring with an absorbing zero. In this ternary structure we try to study prime quasi ideals concept and obtain their properties. **Novelty:** In this study, we define the notion of Prime quasi ideals in ternary seminear rings. We also find some of their interesting results.

AMS Subject Classification code: 16Y30,16Y99,17A40

Keywords: Ternary seminear ring; Idempotent ternary seminear ring; Ideals in ternary seminear ring; Quasi Ideals in ternary seminear ring; Prime Ideals in ternary seminear rings.

1 Introduction

Over the centuries, many mathematical theories have been introduced and one such theory is of algebraic structure. Furthermore, various great research was done and is being done by many authors in the area of seminear rings[(1–4)]. An idea on ternary seminear ring was defined by us[(5)] in 2020 and ideal of ternary seminear ring was defined as T be a ternary seminear ring (T,+,.). A non empty subset I of T is said to be a left(lateral and right) ideal of T if it holds the following conditions i) $i+j \in I$ for all $i,j \in I$ ii) t_1t_2I (respectively $t_1it_2, it_1t_2 \in I$) for all $t_1,t_2 \in T$ and $i \in I$.

If I is a left, a lateral and a right ideal of T then I is said to be an ideal of T. Later in 2021, quotient ternary seminear rings and structures of ternary seminear ring was fathered by us in [6,7]. Then we worked on ideals in ternary seminear rings especially on prime ideal in ternary seminear rings as T be a ternary seminear ring. A proper ideal I of T is said to be a prime ideal if whenever $XYZ \subseteq I$ then $X \subseteq I$ or $Y \subseteq I$ or $Z \subseteq I$, for all ideals X,Y,Z of T and semiprime ideal in ternary seminear rings as T be a ternary seminear ring. A proper ideal I of T is said to be semiprime ideal if whenever $X^3 \subseteq I$ then $X \subseteq I$, for any ideals X of T [8]. Quasi-ideals in ternary seminear rings as T be a ternary seminear ring. Let U be an additive subsemigroup of T. U is said to be a quasi-ideal of T if $UTT \cap (TUT + TTUTT) \cap TTU \subseteq U$ and also defined minimal quasi ideals as T be a ternary seminear ring. A non zero quasi ideal U of T is said to be minimal if U does not properly contain any non zero quasi ideal. we proved that an intersection of an arbitrary collection of quasi ideals of T is also a quasi ideal of T [9]. In this paper, we define a prime quasi ideals in ternary seminear rings and discuss their properties.

2 Methodology

In this research work the results of ternary seminear rings such as ideal in ternary seminear rings as prime ideal in ternary seminear rings and semiprime ideal in ternary seminear rings and structures of ternary seminear rings as idempotent in ternary seminear rings and also quasi ideals in ternary seminear ring are used to define the notion of prime quasi-ideals in ternary seminear rings and studied their characterizations.

3 Results and Discussion

This section deals with **Prime Quasi ideals in ternary seminear rings**.

DEFINITION: 3.1: Let T be a ternary seminear ring. A proper quasi ideal U of T is said to be prime quasi ideal if $XYZ \subseteq U$ which implies that $X \subseteq U$ or $Y \subseteq U$ or $Z \subseteq U$ for quasi-ideals X, Y and Z of T.

DEFINITION: 3.2: Let T be a ternary seminear ring. A proper quasi ideal U of T is said to be semiprime quasi ideal if $X^3 \subseteq U$ which implies that $X \subseteq U$ for quasi-ideal $X \in T$.

THEOREM: 3.3: Let T be a ternary seminear ring. U is a quasi ideal of T. If U is prime quasi ideal, then U is a right or lateral or left ideal of T.

Proof. Let T be a ternary seminear ring. U is a prime quasi-ideal of T. Then $(UTT)(TUT + TTUTT)(TTU) \subseteq UTT \cap (TUT + TTUTT) \cap TTU \subseteq U$. As U is prime quasi-ideal, $UTT \subseteq U$ or $TUT + TTUTT \subseteq U$ or $TTU \subseteq U$. Therefore U is a right or latral or left ideal of T.

DEFINITION: 3.4: Let T be a ternary seminear ring and $x \in T$. Then the principal quasi-ideal generated by x is defined as $\langle x\rangle_q = \{[xTT \cap (TxT+TTxTT) \cap TTx]+nx : n \,\varepsilon\, Z_0^+\}$

THEOREM: 3.5: Let T be a commutative ternary seminear ring. U is a quasi ideal of T. Then U is prime quasi ideal $\Leftrightarrow abc \in U$ which implies that $a \in U$ or $b \in U$ or $c \in U$.

Proof. If U is a prime quasi-ideal of a ternary seminear ring T. $abc \in U$. Then U is an ideal of T (from theorem 3.3). Let x ε $\langle a\rangle_u \langle b\rangle_u \langle c\rangle_u$

Then

$$x = ((aTT \cap (TaT + TTaTT) \cap TTa) + na]\,((bTT \cap (TbT + TTbTT) \cap TTb) + nb]\,((cTT \cap (TcT + TTcTT) \cap TTc) + nc]$$

As $abc \in U$ and U be an ideal of T, $x \in T$. Therefore $\langle a\rangle_u \langle b\rangle_u \langle c\rangle_u \subseteq$ U

As U is a prime quasi ideal of T. Consequently $a \in U$ or $b \in U$ or $c \in U$.

Conversely it is true.

THEOREM: 3.6: Let T be a ternary seminear ring. U is a quasi-ideal of T. Then U is prime quasi $\Leftrightarrow$ $[aTT \cap (TaT + TTaTT) \cap TTa][bTT \cap (TbT + TTbTT) \cap TTb][cTT \cap (TcT + TTcTT) \cap TTc] \subseteq U$ which implies $a \in U$ or $b \in U$ or $c \in U$.

Proof. Let T be a ternary seminear ring. U is a prime quasi ideal of T. Let $[aTT \cap (TaT + TTaTT) \cap TTa][bTT \cap (TbT + TTbTT) \cap TTb][cTT \cap (TcT + TTcTT) \cap TTc] \subseteq U$ for a, b, c *in* T. Clearly $[aTT \cap (TaT + TTaTT) \cap TTa]$, $[bTT \cap (TbT + TTbTT) \cap TTb]$ and $[cTT \cap (TcT + TTcTT) \cap TTc]$ are quasi-ideals of T. As U is prime quasi-ideal, consequently $aTT \cap (TaT + TTaTT) \cap TTa \subseteq U$ or $bTT \cap (TbT + TTbTT) \cap TTb \subseteq U$ or $cTT \cap (TcT + TTcTT) \cap TTc \subseteq U$. If $aTT \cap (TaT + TTaTT) \cap TTa \subseteq U$, then $\langle a\rangle_u \subseteq$ U, which implies that $a \in U$. $bTT \cap (TbT + TTbTT) \cap TTb \subseteq U$, then $\langle b\rangle_u \subseteq$ U, which implies that $b \in U$. $cTT \cap (TcT + TTcTT) \cap TTc \subseteq U$, then $\langle c\rangle_u \subseteq$ U, which implies that $c \in U$. Converse is true.

THEOREM: 3.7: Let T be a ternary seminear ring. Then the following criteria are equivalent

i) The quasi-ideals of T are idempotent.
ii) If X, Y, Z are quasi ideals of T such that $X \cap Y \cap Z \neq \phi$, then $X \cap Y \cap Z \subseteq XYZ$.
iii) $\langle x\rangle_u = [\langle x\rangle_u]^3 \; \forall \; x \in T$

Proof. Let T be a ternary seminear ring,

$$(i) \Rightarrow (ii)$$

Let X, Y and Z be quasi-ideal of T such that $X \cap Y \cap Z \neq \phi$. Then $X \cap Y \cap Z$ is a quasi-ideal of T. As each quasi-ideal of T is an idempotent, therefore

$X \cap Y \cap Z = (X \cap Y \cap Z)^3$
$= (X \cap Y \cap Z)\,(X \cap Y \cap Z)\,(X \cap Y \cap Z)$
$\subseteq XYZ$
$(ii) \Rightarrow (iii)$ and $(iii) \Rightarrow (i)$
$\Rightarrow$ obviously it is true.

THEOREM: 3.8: Let T be a semiprime ternary seminear ring. Every minimal quasi-ideal U of T is the intersection of a minimal right ideal A, a minimal lateral ideal B and a minimal left ideal C of T.

Proof. Let T be a semiprime ternary seminear ring. As U is a quasi-ideal of T, consequently $UTT \cap (TUT + TTUTT) \cap TTU \subseteq U$. U be minimal, so either $UTT \cap (TUT + TTUTT) \cap TTU = 0$ or $UTT \cap (TUT + TTUTT) \cap TTU = U$.

Suppose $UTT \cap (TUT + TTUTT) \cap TTU = 0$. Then either $UTT = 0$ or $UTT \neq 0$. If $UTT = 0$ then U be a non zero right ideal of T satisfying $U^3 = 0$. Thus contradiction.

If $UTT \neq 0$, then $UTTUTTU \subseteq UTT \cap (TUT + TTUTT) \cap TTU = 0$, which implies $(UTT)^3 = 0$ which makes contradiction to our assumption that 0 is a semiprime ideal of T. Hence $UTT \cap (TUT + TTUTT) \cap TTU = U$. Now, UTT is a minimal right ideal of T.

If there exist a non zero right ideal A' of T such that $A' \subseteq UTT$. Then $A'TT \cap (TUT + TTUTT) \cap TTU$ is a quasi ideal of T such that $A'TT \cap (TUT + TTUTT) \cap TTU \subseteq U$.

As U is minimal, then either $A'TT \cap (TUT + TTUTT) \cap TTU = 0$ or $A'TT \cap (TUT + TTUTT) \cap TTU = U$. If $A'TT \cap (TUT + TTUTT) \cap TTU = 0$. Then $A'UTTU \subseteq A'TT \cap (TUT + TTUTT) \cap TTU = 0$.

Now $A' \subseteq UTT \Rightarrow A'^3 \subseteq (A'UTTU)TT = 0$. By contradiction, 0 be a semiprime ideal of T. Hence $A'TT \cap (TUT + TTUTT) \cap TTU = U$, which implies that $U \subseteq A'TT \subseteq A'$. Hence $UTT \subseteq A'TT \subseteq A'$. Therefore $A' = UTT$ be a minimal right ideal of T. Similarly, we show that $TUT + TTUTT$ be a minimal lateral ideal of T and TTU be a minimal left ideal of T.

THEOREM: 3.9: Let T be a ternary seminear ring has an identity element one, then each quasi ideal of T is an intersection of a right ideal, lateral ideal and left ideal of T.

Proof. T is a ternary seminear ring with an identity element one. U is a quasi ideal of T. Then TTU is a left ideal, $TUT + TTUTT$ is a lateral ideal and UTT is a right ideal of T. Since T has an identity element I,

$\langle U\rangle_a$ = UTT, $\langle U\rangle_b$ = TUT+TTUTT and
$\langle U\rangle_c$ = TTU
So
U $\subseteq \langle U\rangle_a$ = UTT
U $\subseteq \langle U\rangle_b$ = TUT+TTUTT
and
U $\subseteq \langle U\rangle_c$ = TTU
which implies that $U \subseteq TTU \cap (TUT + TTUTT) \cap UTT \subseteq U$.
Therefore $U = TTU \cap (TUT + TTUTT) \cap UTT$.
Hence each quasi ideal of T is an intersection of right ideal, lateral ideal and left ideal.

THEOREM: 3.10: Let T be ternary seminear ring. e is an idempotent element of T. If A is a right ideal of T, then ATe is a quasi ideal of T.

Proof. Let T be a ternary seminear ring. e is an idempotent element of T. A is a right ideal of T. To prove ATe is quasi ideal of T. It is enough to prove $ATe = A \cap (TeT + TTeTT) \cap TTe$. We have $ATe \subseteq A \cap TTe$.

Let $x \in A \cap TTe$. Then $x \in A$ and $x \in TTe$. Now $x \in TTe \Rightarrow x = \sum_{i=I}^{n} S_i t_i e \quad \forall \quad S_i, t_i \in T$.

Consequently,
xee = $(\sum_{i=1}^{n} S_i t_i e)\, ee$
$= \sum_{i=1}^{n} S_i t_i\, (eee)$
$= \sum_{i=1}^{n} S_i t_i e$

$= x \Rightarrow x = xee \in Aee \subseteq ATe$
So, $A \cap TTe \subseteq ATe$. Thus $ATe = A \cap TTe$.
Again, $x = xee \in TeT$ and $o \in TTeTT \Rightarrow x + o = x \in TeT + TTeTT$.
Consequently $A \cap TTe \subseteq TeT + TTeTT$.
Therefore $ATe = A \cap (TeT + TTeTT) \cap TTe$.
Hence ATe is a quasi ideal of a ternary seminear ring.

4 Conclusion

In this research work, prime quasi ideals in ternary seminear rings are defined and their properties have been obtained. The charaterization of quasi ternary seminear ring may be extended to Bi-ideals and Bi-simple ternary seminear rings.

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