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Trend analysis and Applicability of Fuzzy Time Series Models to Night-time Luminosity Data of Uttarakhand state, India

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Abstract

Objectives: Application of trend detection and fuzzy time series models to predict the growth of Uttarakhand state using night-Time Lights (NTL) dataset (Visible Infrared Imaging Radiometer Suite (VIIRS) night-time lights). **Methods**: In this work, we propose a methodology to analyze trends and estimate different fuzzy time series models for prediction in the context of forecasting the economic growth of a low-earning income Uttarakhand state. Mann-Kendall tests are used for performing trend analysis of Uttarakhand (India) using night-time luminosity data sets. Forecasting methods based on fuzzy time series models have been presented to deal with uncertainties induced by ambiguities, vagueness, and some other non-probabilistic factors, and have widely been employed in many problems domains. The simulation is done on the Jupyter Notebook platform using night-time luminosity datasets. **Findings:** We conclude that Uttarakhand and Himachal Pradesh show positive growth in trends for their night-time lights monthly average intensity with magnitude of 0.002182 and 0.002558 respectively. Night time luminosity database can be used as a proxy indicator for the economic growth of the Uttarakhand state as the correlation between NTL time series data of Uttarakhand and its Gross Development Product (GDP) is 0.795641021. Fuzzy time series models can also be used to predict the growth of the Uttarakhand state and the High Order Fuzzy Time Series model gives the accuracy metrics of root mean square (RMSE) as 0.11. **Novelty:** No previous research work is performed on trend analysis and prediction of growth of a Uttarakhand state using night-time luminosity data sets.

Keywords: Fuzzy Systems; Fuzzy Time Series; Prediction; Night time luminosity; Satellite image

1 Introduction

People are using artificial light sources to meet the emerging needs of manufacturing sector, residential and non-residential lighting, thereby causing changes in the distribution of luminosity on the surface of the earth at night time. This surface brightness refers to night- time luminosity data sets that provide numerical measurements of the brightness of the earth at night. It is captured by remote sensing satellites referred as Night Time Luminosity (NTL) data. Note worthily, the satellite image capturing of nighttime luminosity (NTL) data sets provides promising and unbiased methods to overcome the intermittent data availability referring to socio-economic aspects. Various researchers have argued that NTL can work as a proxy indicator to evaluate the economic development of a region [1,2]. Interestingly, from the past few decades, nighttime light data sets are frequently used for forecasting, mapping, and correlating studies related to population growth, monitoring disasters, phenology of crop production, economic growth, electricity consumption, pollutant emission from automobiles and predicting human health etc [3,4]. The NTL time series data can be extracted using Google Earth Engine (GEE) [5]. Despite the enormous progress in the application of NTL images to various economic problems, the applicability of night-time luminosity data sets to rural areas of low and middle-income countries has not been explored yet. It is also difficult to establish consistent and credible economic inequality estimates at the disaggregated level in developing nations due to the lack of consistent income data at the sub-state or district level. It is well-established in research published by international and national organizations namely the World Bank [6] and Reserve Bank of India RBI [3] that Night time light and GDP are highly correlated with each other. Moreover, various usages of NTL have been studied like spatial and temporal dynamics of settlement, demographics, and socioeconomic parameters.

Uttarakhand is a low earning income state in the north of India and South of Asia continent [7]. The state benefits both the region and the country along with the economically, environmentally, and strategically backward inhabitants of the region. There are no such research studies in the past 21 years to validate and predict the growth of the economically backward Uttarakhand state. In the current predicament, the NTL data sets captured by satellite image is a useful influx of data that can be analyzed to enable better decision-making actions for any organization or policymakers for forecasting economic growth. In this work, NTL data were collected for Himachal Pradesh, Uttarakhand State, and its districts using the Google earth engine and there after Mann-Kenddle statistics were performed to find trends of NTL data of Himachal, Uttarakhand state, and Uttarakhand allied districts. We applied Fuzzy Time Series (FTS)-based algorithms to NTL time series data sets to form a forecasting model. Literature also revealed that these algorithms have also been applied for various data sets that include university enrollments, stock markets, tourism, electric load, phenological stages of apple trees etc [8]. The High-Order FTS (HFTS), Weighted FTS (WFTS), and Probabilistic Weighted FTS(PWFTS) algorithms were also applied and evaluated to a NTL data set of the Uttarakhand state [9].

2 Methodology

2.1 Study Context

The territory of Uttarakhand state is located in the south Asia continent and north of India with an area of 53,485 sq. km. The territory of Uttarakhand state shown in Figure 1 extends from 28°43' - 31°27' N latitudes and 77°34' - 81°02' E longitudes. Figure 1 show population density extracted using GEE. The orography of Uttarakhand is quite diverse with mountain systems, plains, and valleys, snow cover. Its area is covered by 86% mountainous and 65% forest. The State has 13 districts out of which ten regions (Uttarkashi, Tehri, Pithoragarh, Almora, Nainital, Rudraprayag, Chamoli, Pauri, Gopeswar, Champawat) are hilly districts, and three Regions (U. S. Nagar, Dehradun, Haridwar) plain districts [7] .

2.2 Data sets

The National Oceanic and Atmospheric Administration (NOAA) have developed an algorithm that allows it to identify stable lights by removing sunlit, glare, and moonlit. Night light data has many advantages like it is readily available, publicly, and unbiased data sets. Thus, the research on NTL images data sets is reproducible and verifiable [6]. There are presently two programs for utilizing NTL images: Defence Meteorological Satellite Program Operation Line Scan system (DMSP-OLS) [10] and Visible Infrared Imaging Radiometer Suite (VIIRS) Day-Night Band (DNB) [11]. DMSP information was readily used between (1992-2013) and thereafter discontinued. From the year 2013 onwards VIIRS-DNB system is widely used and it is operated by the National Aeronautic space agency (NASA) joint Polar satellite. At National Oceanic and Atmospheric Administration (NOAA), the VIIRS DNB average radiance composite images are available in the Version 1 Package Earth Observation Group (EOG)(11). Stay light contamination, lightning, lunar illumination, and cloud cover have all been removed from this public (DNB) data.

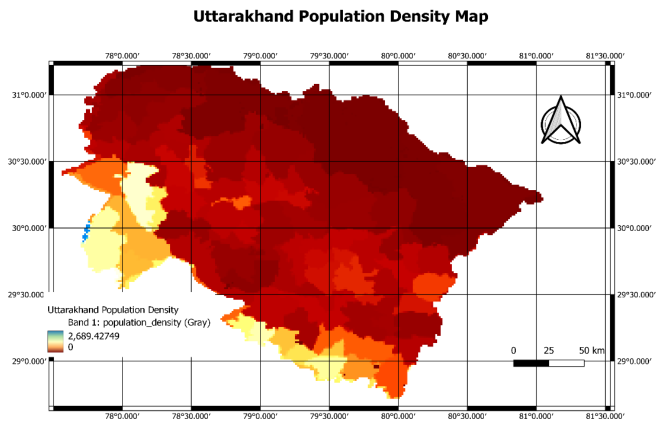

Fig 1. Study Area Uttarakhand with population density

The NTL time series of the monthly stable average radiance values (in nanoWatts/cm^2/sr) are related to Uttarakhand and its districts, Himanchal Pradesh was extracted and collected from Visible Infrared Imaging Radiometer Suite (VIIRS) NTL using a cloud computing environment readily available with VIIRS dataset called as Google Earth engine [5]. Figure 2 shows the illuminated region at night time of Uttarakhand and Himachal Pradesh using the median of the monthly average of NTL radiance for the period from (January 2014-March 2021). The monthly average of NTL radiance time series for Uttarakhand and Himachal Pradesh is shown in Figure 3. Further, district-wise NTL monthly average radiance time series data were extracted for Uttarakhand state and the monthly average of NTL radiance time series for Uttarakhand districts segregated into plain and hilly districts is shown in Figures 4 and 5.

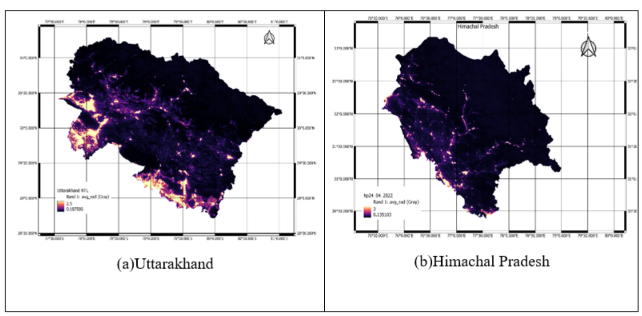

Fig 2. Median of NTL radiance extracted for the period from Jan 2014 to Mar 2021 for (a)Uttarakhand and (b)Himachal State

2.3 Methods

2.3.1 Time series analysis of NTL imagery

Anything that is seen progressively across time may be categorized as a ”time series” (TS)[9]. As instance, sales patterns, the stock market, weather predictions, and so forth. In addition to numbers, labels, &colors, these observations maybe anything else. Additionally, the times at which the observation was made might be spaced regularly or sporadically. However, it may either be discrete or continuous. It is possible to think of TS data as a collection of data points that show changes over time. The process of deriving summary and other statistical information from time-series, most notably the examination of trend and seasonality, is known as time-series analysis (TSA)[8]. Figure 6 shows the methodology followed for trend analysis and forecasting of NTL time series data extracted in section 2.2.

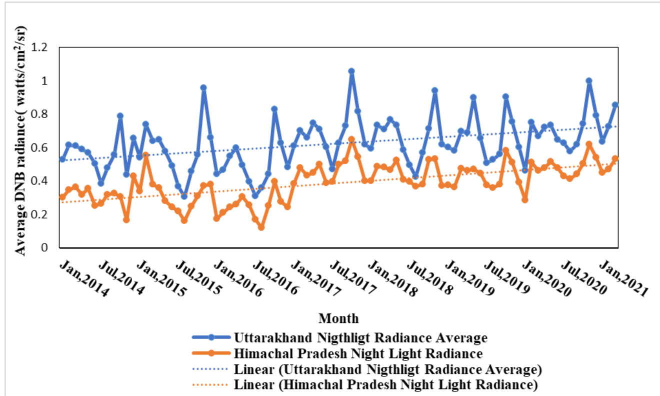

Fig 3. Monthly Average radiance of Nightlight time series for Uttarakhand and Himachal Pradesh from Jan 2014 to May 2021

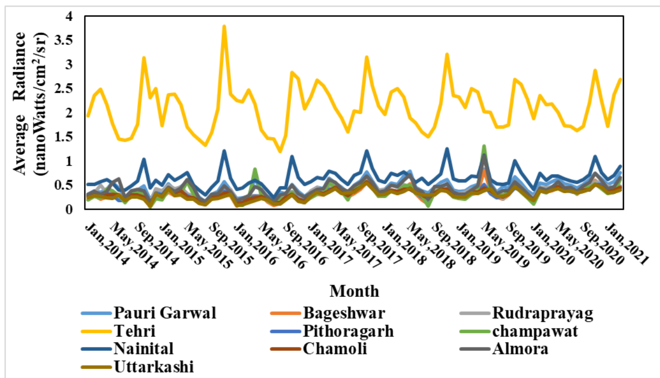

Fig 4. Time series of Average Radiance of NTL for Hilly districts of Uttarakhand

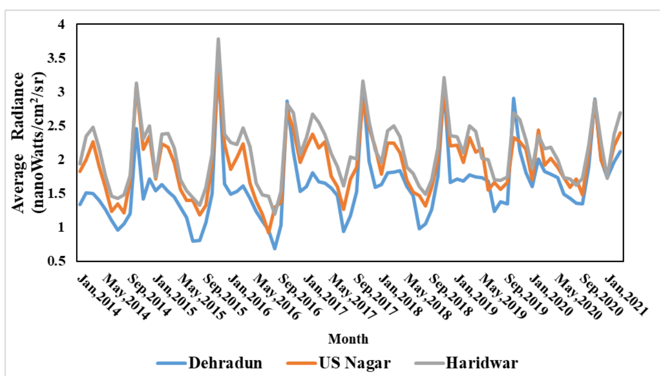

Fig 5. Time series of Average Radiance of Night time Light for Plains districts of Uttarakhand

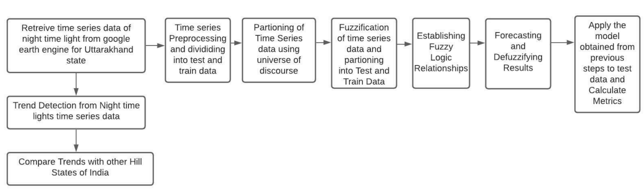

Fig 6. WorkFlow of the methodology to analyze and forecast NTL time-series data collected for Uttarakhand state

2.3.2 Mann-Kendall test

The non-parametric Mann-Kendall test is commonly used in the various datasets of meteorology and hydrology to find trends in time series data. Mann-Kendall is applied to the monthly average NTL time series to identify trends. Mann-Kendall test is a non-parametric test and does not require ordered data to be any in kind of distribution. The following steps were followed to apply the Mann-Kendall statistic:

a. Each element in time series data is compared to the following data item of the series and if there is a higher value in subsequent data, the S is increased by 1 and otherwise S is decreased by 1.

b. Calculate the value of S i.e Mann-Kendall statistic: Equations 2 and 3 below are used to calculate the value of S i.e Mann-Kendall statistic.

c. Variation of S is calculated using Equation 4.

d. Normalized statics Z is calculated using Equation 1.

e. Calculate the probability using Equation 5 associated with the Z calculated in step d.

f. The trend in time series is deduced as follows: -

i) decreasing if Z is negative and computed probability is greater than the level of significance (95% typically)

ii) increasing if Z is negative and computed probability is greater than the level of significance (95% typically)

iii) no trend If the computed probability is less than the level of significance (95% typically)

iv) A simple Sen's Method to calculate magnitude of trend is calculated using Equation 6.

$$Z = \begin{cases} \frac{S-1}{\sqrt{\mathrm{Var}(S)}} \text{ if } S > 0 \\ 0 \text{ if } S = 0 \\ \frac{S+1}{\sqrt{\mathrm{Var}(S)}} \text{ if } S < 0 \end{cases} \tag{1}$$

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} sgn(x_j - x_k) \tag{2}$$

$$\mathrm{sgn}\left(x_{\mathrm{j}} - x_k\right) = \begin{cases} +1 & \text{if } \left(x_j - x_k\right) > 0 \\ 0 & \text{if } \left(x_j - x_k\right) = 0 \\ -1 & \text{if } \left(x_j - x_k\right) < 0 \end{cases} \tag{3}$$

$$Var\,(S) = (n\,(n-1)\,(2n+5) - \sum_{i=1}^{m} t_i\,(t_i - 1)\,(2t_i + 5)]/18 \tag{4}$$

$$f\,(Z) = \frac{1}{\sqrt{2\pi}} e^{\frac{z^2}{2}} \tag{5}$$

$$\beta = median\left(\frac{x_{j-}x_k}{j-i}\right),\ j > i \tag{6}$$

where n is the number of data points; m is the number of tied groups (a set of sample data having the same value). Python library is used to perform the Mann-Kendall[(12)] test to detect trends in night light data gathered in section 2.1.

2.3.3 Fuzzy Time Series

The Fuzzy Time Series (FTS) is based on fuzzy set theory. A fuzzy time series is represented differently than a standard time series. Standard time series is made up of real-number observations while fuzzy time series is made up of fuzzy sets. The universe of discourse for the predicting issue is made up of these sets. FTS predicting methods include the following steps[(13)] :

(1) Define Linguistic Variables

a. Define Universe of Discourse and Number of Partitions: Universe of discourse is obtained from the range of values in conventional time series.

b. Data Partitioning Methods: Various methods have been researched for partitioning the universe of discourse obtained and basically falls into two fixed size and variable size partitions.

c. Create a fuzzy set for each partition and each partitioned set is given a linguistic name.

(2) Fuzzification:- This step maps the conventional time series data to the fuzzy set obtained in step 1 using membership function. For example, suppose universe of discourse (U) is partitioned as {u1,u2,u3,…un} then fuzzy sets Ai are defined on set U with corresponding membership function.

(3) Establishment of FLR & Fuzzy Logical Relationship Group (FLRG): - This step defines fuzzy temporal rules and patterns and forms a knowledge base. For example, let Ai be fuzzy set at time t and Aj be the be fuzzy set at time t+1. Then it is represented as AiAj and if there are multiple common LHS then they are grouped together. For example, Table 1 shows example of FLRG.

Table 1. Example of Fuzzy Logic Group

Group1	$A_{1\to}A_1$, $A_{1\to}A_3$
Group2	$A_{2\to}A_4$, $A_{2\to}A_5$
Group3	$A_{3\to}A_3$,$A_{3\to}$A

In this work we have used variants of FTS models namely Weighted FTS, High order FTS and PWFTS as described in [14]

(4) Defuzzification:- In this step, the forecast is done using the knowledge base obtained in step 3. The forecast provides a fuzzy set that is mapped to a corresponding numerical value.

The time series dataset is divided into training data and test data. Python library pyfts was used to apply fuzzy time series algorithms to NTL time series data set. The statistical metric Root Mean Square Error (RMSE) given in Equation 7 were applied to evaluate accuracy of the models

$$RMSE = \sqrt{\frac{1}{n}\left(\sum_{t=1}^{n}(y_t - \widehat{y_t})^2\right)} \tag{7}$$

3 Results and Discussion

The Google earth engine along with VIIRS dataset is used to extract time-series data of monthly average NTL for Uttarakhand and its Districts, and Himachal Pradesh for the period of 7 years ranging from January 2014 to March 2021. Mann-Kendall results were used to find out trends in night time lights. Table 2 shows Uttarakhand and Himachal Pradesh state has an overall increasing trend in night time lights though Himachal Pradesh's data slope is comparatively better than that of Uttarakhand. Uttarkhand state districts PauriGarwal, Dehradun, Bageshwar, Rudraprayag, Pithoragarh, Champawat, Nainital, Chamoli, Almora, and Uttarkashi also show an increasing trend while Tehri, Udhamsingh Nagar and Haridwar shows no trend for monthly average NTL. Overall Pithoragarh district shows an increasing trend with a minimum magnitude of the trend and Dehradun shows an increasing trend with a maximum magnitude of the trend.

Table 2. Mann Kendall Test performed on Night Time Light data obtained for Uttarakhand districts, Uttarakhand and Himachal Pradesh

S.NO	Uttarakhand Districts	Mann_Kendall_Test		
		Trend	Slope	Z-Value
1	PauriGarwal	increasing	0.002558	4.5535255
2	Dehradun	increasing	0.005978	3.9888462
3	Bageshwar	increasing	0.002200	4.8250035
4	Rudraprayag	increasing	0.001948	4.2566804
5	Tehri	no trend	0.001485	0.8358642
6	Pithoragarh	increasing	0.001721	4.1026552
7	Champawat	increasing	0.001721	3.6075109
8	UdhamsingNagar	no trend	0.002500	1.2611454
9	Nainital	increasing	0.002140	3.5781816
10	Chamoli	increasing	0.002000	5.0118448
11	Haridwar	no trend	0.001485	0.8358642
12	Almora	increasing	0.001914	3.4279838
13	Uttarkashi	increasing	0.002147	5.5582619
	States			

Continued on next page

Table 2 continued

1	Uttarakhand	increasing	0.002182	3.6808343
2	Himachal Pradesh	increasing	0.002558	5.6791894

Nighttime light time series data was divided into two parts: train and test data. Fuzzy Time Series methods were used to train HOFTS, WFTS and PWFTS models and then applied to test data and compared with ARIMA model. Table 3 shows RMSE metrics for FTS methods. RMSE metrics of HOFTS with order 2 have a minimum of 0.11 RMSE which can be used to forecast trends.

Table 3. RMSE results for different FTS models and along with their order

S.NO	Model	Order	RMSE
		1	0.12
1	HOFTS	2	0.11
		3	0.12
		1	0.13
2	WHOFTS	2	0.12
		3	0.13
		1	0.13
3	PWFTS	2	0.13
		3	0.15
4	ARIMA	(p=0, d=1, q=1)	0.152218

Figure 7 shows time series data obtained after using FTS trained model to generate data along with real actual test data. Following FTS models were applied

a. HOFTS: -High-order FTS (Legends HOFTS1, HOFTS2, HOFTS3 indicates results of order 1 order 2 and order 3 respectively)

b. WHOFTS: - Weighted High Order FTS (Legends WHOFTS1, WHOFTS2, WHOFTS3 indicates results of order 1 order 2 and order 3 respectively)

c. PWFTS: - Probabilistic Weighted FTS (Legends PWFTS1, PWFTS2, PWFTS3 indicate results of order 1 order 2 and order 3 respectively).

FTS models HOFTS1 time series obtained shows similarity with test data and when rmse was calculated it gives a minimum of 0.11 rmse.

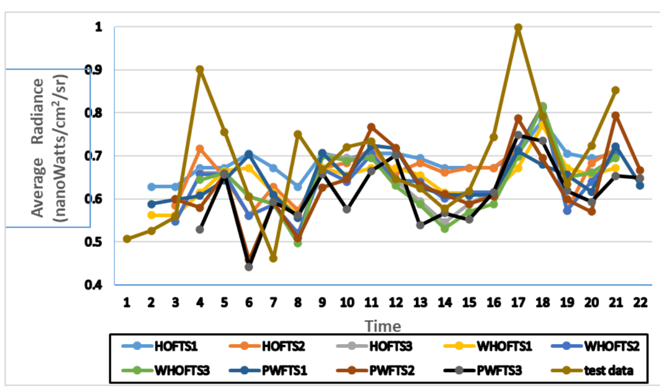

Fig 7. Test results with FTS Methods

The relationship between NTL data sets and GSDP were analyzed for all the Uttarakhand and its district for 2014-2021. The mean radiance value of Uttarakhand and its allied district has increased from 0.6 to 0.80 nW/cm^2/sr indicating economic development. Table 4 shows the yearly average NTL time-series data and Uttarakhand state Gross domestic product, these data have a correlation of 0.795641021 which indicates a significant and positive relationship exists between Night time light time series data and Uttarakhand state GDP. The percentage increase in NTL time-series data and GDP of Uttarakhand state also

Table 4. Yearly Average Night Time Light and Uttarakhand Gross State Domestic Product At Current Prices; Base year 2011-12 (In Crores) Source: NSO (https://mospi.gov.in)

S. No	Year	Average Uttarakhand Night Time Light(in nanoWatts/cm^2 /sr)	UttarakhandGSDP AT CURRENT PRICES; BASE YEAR 2011-12 (in Crores)	Percentage Increase/decreaseof Avg.Uttarakhand Night Time Light in reference to year 2014-2015	Percentage Increase/decreaseUttarakhandGSDP in reference to year 2014-2015
1	2014-15	0.573583333	161439	0	0
2	2015-16	0.539583333	177163	-5.92	9.72
3	2016-17	0.542916667	195125	-9.99	20.86
4	2017-18	0.701166667	220222	22.24	36.41
5	2018-19	0.643916667	230285	12.27	42.60
6	2019-20	0.664583333	236694	15.86	46.61
7	2020-21	0.722333333	227421	26.12	40.87

shows an increasing trend for period between 2014-2021.

Figure 8 shows percent change of NTL data and GSDP for some of the hilly states of India for the period between years 2014-15 to 2020-21 gathered using GEE. North eastern states of India shows more increase in NTL data as compared to Himachal Pradesh and Uttarakhand. The percentage change in NTL and GSDP for Uttarakhand is around 25% and 40% respectively indicating positive correlation. The percentage change in NTL data sets for Arunachal Pradesh and Manipur is higher greater than 100%. The increasing trends of NTL data sets during the period between 2014-2021, indicate that there is an increase in human activities in those regions of Uttarakhand state. Moreover, GSDP growth rate also shows positive increasing trends during this period. Hence it is conclusive that Night time luminosity data sets can act as proxy indicator of economic activity in Uttarakhand and its District and Himachal Pradesh State also. Thus this fact further conforms that the lights observed from the sky at night can be used as a proxy to monitor the process of the country's urban growth, development, and economy.

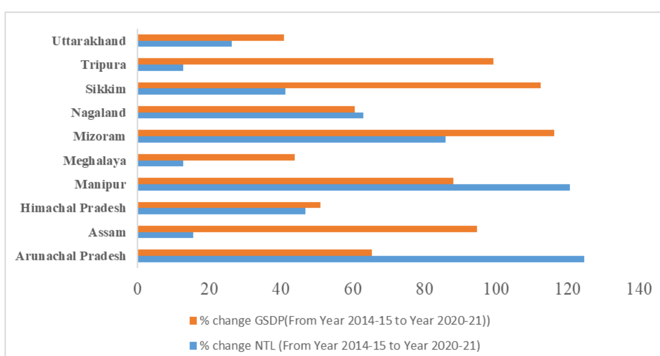

Fig 8. Percent change in NTL and GSP of hilly states from year 2014-15 to 2020-21

4 Conclusions

In this paper we used NTL time series data was used to estimate growth at the district level and as we see in trend analysis of Uttarakhand state the hilly region has lower increasing trend than that of plain regions of Uttarakhand. The same fact can be verified by comparing Figures 1 and 2a, the population density is more in plain regions of the state than in hilly regions. It is also concluded that although Pithoragarh district has increasing trend of 0.001721 nanoWatts/cm2/sr but the pace is very low and Tehri districts has no trend with slope of 0.001485 nanoWatts/cm2/sr but its average night light intensity is better than other hilly districts. This can be attributed to fact that its large area is covered by has Tehri Hydro Power Dam. Overall looking at districts of Uttarakhand, Dehradun showed increasing trend with a good slope of 0.005978 nanoWatts/cm2/sr . Moreover, NTL data sets can be attributed to the proxy indicator of economic activity in Uttarakhand regions as when the relationship between NTL and Uttarakhand GSDP were calculated it comes out as 0.795641021 which shows that NTL are positively related to Uttarakhand GSDP. NTL data for other hilly states of India were calculated and the rate of change in NTL and GSDP was calculated from the years 2014-15 to 2020-2. Uttarakhand has 40% GSDP increase and 35% average NTL increase from years 2014-15 to 2020-21. Northeast hilly states except Meghalaya have shown better growth rates in GSDP and average NTL as compared to Uttarakhand. The FTS methods were applied and HOFTS gave better RMSE value of 0.11 and this model can be used to prediction of growth of state. In the future, more time series datasets can be obtained and analyzed from Google earth engine with different datasets available, to further verify the growth of that region.

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