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# Effect of High Frequency Gain on the Performance of Optical Costas Loop in Face of Loop Delay

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## Abstract

**Objectives:** Optical costas loops (OCLs) are widely used in optical communication as homodyne receivers. Due to use of different electronic counterparts and fibre optic cable inherent loop delay always presents in the system. The steady state behaviours of optical costas loop are highly affected by the presence of loop delay. Different nonlinear behaviours may be observed due to presence of delay. There are two main objectives of this article. Firstly, how OCL can be operated as stable receiver up to some large value of loop delay by using a proportional plus integrating type loop filter (LF). Secondly, how a controlled chaotic optical signal can be generated from OCL by choosing the system parameters in correct manner. **Methods:** To investigate system behaviours of OCL both analytical and numerical methods have been used. Stability analysis of OCL has been done by Routh-Hurwitz method. From stability analysis, it is possible to predict the stable and unstable behaviour of the OCL in presence of delay and how system stability can be improved by high frequency gain value of LF. Numerical methods have also been used to solve the nonlinear equation of OCL to observe real time behaviours. **Findings:** Analytical findings show that loop stability can be improved by increasing the value of high frequency gain of LF. For large value of loop delay, chaotic oscillation of phase error may be observed in OCL. The chaotic oscillation can also be controlled by high frequency gain with certain extent value of loop delay. All the numerical findings have been properly verified with numerical results. **Novelty:** This article describes how effects of loop delay can be controlled to run OCL in steady as well as in unsteady state. From designer's point of view this study would be helpful to choose correct values of system parameters to improve the performance of OCL in optical communication. It also gives the idea for generation of chaotic optical signal, which is used in secured communications.

**Keywords:** Optical Costas loop; Optical Phase locked loop Phase detector; Loop filter; Nonlinear dynamics; and optical communications

## 1 Introduction

In electronic and fibre optic communications, to cover a large communication distance with high spectral purity homodyne receivers are widely used. Dither loop [1], balanced loop [2,3], Costas loop are the most familiar homodyne receivers, which are widely used free space and optical communication systems. In coherent optical communications, phase synchronization between the input signal and the local laser oscillator is very important. In most cases, optical costas loop (OCL) is used to fulfil the requirements [4]. Another advantage of OCL is it can recover message from supressed carrier signal. By using OCLs, it is also possible to transmit data with high speed and high energy efficiency based on digital signal processing (DSP) [5,6]. For free space satellite to ground communications phase locked loop (PLL) and optical phase locked loop (OPLL), OCL are widely used. Recently, design of PLL and OCL based coherent receivers are reported, which can efficiently recover message signals in presence of atmospheric turbulence, laser phase noise, frequency offset between transmitted and received laser signals [7]. Design of low cost and highly efficient semiconductor laser based integrated coherent optical receiver for the recovery of QPSK and BPSK signals is also reported recently [8]. How digital signal can be recover with the help of digital optical phase locked loops (DOPLLs) are also reported in some articles [9,10]. So, research on improvement and designing of PLL based homodyne receivers for electronic and fibre optic communications is increased day by day. Although, different new designs of OPLL, DOPLL, OCL have been proposed recently [7–10], but effect of time delay on the system dynamics have not been discussed with proper attention. It is impossible to design these devices with no time delay. However, value of loop delay can be made small but in some cases delay may have very large value. Actually, loop time delay arises in any electrical or electronic devices due to some factors such as, different bulk sizes of hardware components, different length of fibre optic cable used for optical signal transmission, different response times between different photonic and electronic devices used in the circuits, etc. [11]. So, to design a stable OCL as homodyne receiver, proper attention should be drawn on loop delay otherwise, different nonlinear behaviours may be observed in OCLs.

The dynamics of OCL is not linear, it is highly nonlinear and different nonlinear phenomena such as chaos, bifurcation, etc. are observed in their response [12,13]]. Dynamics of delayed optical PLL (OPLL) or OCL have been studied in the literature. Large loop delay can produce chaotic oscillations of the loop error signal in OPLL and OCL [-15]. Effects of additional loop delay on the dynamics of OPLL and OCL have been reported in literature [14,15], it has been reported that, loop performance is degraded in presence of delay. When delay exceeds some critical limit then dynamics OCL and OPLL becomes unstable. With further increase of the delay, periodic and chaotic oscillations can be observed by generation of chaotic phase modulated signals. So, for designing a chaos free stable OCL, this study is very useful and important. Again, generation of chaotic optical signal is an important research field in respect of modern scenario. These signals are used in chaotic lidar and Doppler lidar measurements [15]. Communication with chaotic signal has some advantages such as, due to noise like appearance, it can be used in cryptographic based secure communication also bandwidth can be enhanced by using chaotic optical signal with constant optical injection alone [-18]. When error signals of OCL and OPLL oscillate chaotically then they can be used as sources of chaotic optical signals. Again OCLs can be also used as receiver of chaotically modulated optical signals. These are the important facts to study nonlinear dynamics of OCL with additional loop time delay. Although, effect of loop delay on the dynamics of OCL has been discussed in [-16], how system performance can be improved by using loop filter has not described properly. In this paper, how the effects of delay can be controlled by high frequency gain of the proportional plus integrating type loop filter in place of conventional loop filter, have been studied analytically and numerically.

Condition for stable locking has been determined with the help of Routh-Hurwitz's stability analysis [16,17]. From this, permissible loop delay for stable operation of OCL can be obtained. With the help of analytical results stable and unstable zones of operation of OCLL have been detected. In unstable zone of operation, loop phase error first starts oscillation through Hopf bifurcation, which has been shown by using XPPAUT simulation software. Then by numerical integration technique by solving the equation of OCL, period-1, period-2, etc. and chaotic oscillation and how these can be controlled with high frequency gain have been discussed. The numerical results are in good agreement with analytical prediction.

The rest of the paper is organised in the following way. In section-2, block diagram of the OCL and its loop structure has been described. By using loop time delay in addition to the loop filter (LF) transfer function with high frequency gain, phase equation of the OCL has been formulated. In section-3 stable locking condition of OCL has been determined with the help of Routh-Hurwitz's analysis. With the help of analytical results, the probable behaviours of the OCL for different values of delay and high frequency gain have been discussed in this section also. In section-4, using XPPAUT software package and by solving the system equation, different nonlinear responses of the OCL have been recorded, which support analytical predictions. Finally, the paper ends with some concluding remarks in section-5.

## 2 Derivation of System equation for OCL considering loop time delay

Basic block diagram of OCL has been shown in Figure 1 a. It consists of two photo diode based phase detectors (PDs). Each phase detector is constructed by one optical coupler, two photo diodes, one amplifier and a low pass filter (LPF). Input optical signal is applied to two phase detectors, which are identified as in phase (I) and quadrature (Q) components. Optical signals with phase difference $90^0$are applied in two PD s. Depending on optical input and the signal coming from Laser oscillator (LO), two error signals are generated by two PDs. These error signals are then applied to a combiner for multiplication and after that the signal is applied to LO through a loop filter (LF) for controlling the phase of LO. Circuit diagram of the LF is shown in Figure 1b. This type of filter is known as proportional plus integrating type filter. Transfer function of this filter is written as,

$$F_l(s) = \frac{1+g\tau_1 s}{1+\tau_1 s} \tag{1}$$

Here, $g = r_2/(r_1+r_2)$, $\tau_1 = c(r_1+r_2)$ and $s = j\omega$ are high frequency loop gain, filter time constant and complex frequency respectively. Now, loop propagation delay exists in all electrical, electronics and optoelectronics systems. Although any extra circuit is not required for producing delay, but to identify loop propagation delay a delay block has been shown in Figure 1a. Time delay in any system is represented by exponential function. Considering its value as very small compared to the filter time constant, transfer function of the loop delay is considered as,

$$F_d(s) = \exp(-2\delta s) = \exp(-s\delta)/\exp(s\delta) \tag{2}$$

Here, $2\delta$ represents characteristic delay. Equivalent transfer function of the loop considering transfer function of LF and delay can be expressed as,

$$F(s) = F_l(s)F_d(s) = \frac{1+(g\tau_1-\delta)s - g\tau_1\delta s^2}{1+(\tau_1+\delta)s+\tau_1\delta s^2} \tag{3}$$

Input optical signal and reference optical signal obtained from laser LO at the in phase arm are taken as,

$$E_I = \sqrt{P_I}\exp(j\omega_i t + j\psi_n(t)) \tag{4a}$$

$$E_{LI} = \sqrt{P_{LI}}\exp(j\omega_0 t + j\zeta_n(t)) \tag{4b}$$

Where, $P_I, P_{LI}$ are the powers of input optical signal and signal coming from laser LO to the phase detector-1. $\omega_i, \omega_0$ are the frequencies of input optical signal and laser LO respectively. $\psi_n(t), \zeta_n(t)$are phase noise of laser transmitter and LO source. Phase detector-1 generates error signal depending on the phase difference between input signal and signal from LO. Phase error is written as,

$$\varphi(t) = \omega_i(t) + \psi_n(t) - \omega_0(t) - \zeta_n(t) \tag{5}$$

Now, depending on the phase error, PD-1 generates the error signal as,

$$e_1 = k_{p1}\sin\varphi(t) \tag{6}$$

Here, $k_{p1} = 2r_1 r_p\sqrt{P_I P_{LI}}$ is the gain of PD-1 and $r_1, r_p$ are trans-impedance and responsivity respectively. For quadrature arm (Q-arm), input optical signal and reference optical signal obtained from laser LO are taken as,

$$E_Q = \sqrt{P_Q}\exp(j\omega_i t + j\psi_n(t)) \tag{7a}$$

$$E_{LQ} = \sqrt{P_{LQ}}\exp(j\omega_0 t + j\zeta_n(t)) \tag{7b}$$

Where, $P_Q, P_{LQ}$ are the powers of input optical signal and signal coming from laser LO to the phase detector-2. Phase error for the PD-2 is written as,

$$\theta(t) = \omega_i t + \psi_n(t) - \omega_0 t - \zeta_n(t) - \pi/2 = \varphi(t) - \pi/2 \tag{8}$$

So, phase error signal generated from PD-2 would be as,

$$e_1 = k_{p2}\sin\theta(t) = k_{p2}\cos\varphi(t) \tag{9}$$

Here, $k_{p2} = 2r_2r_p\sqrt{P_QP_{LQ}}$ is the gain of PD-2 and $r_2, r_p$ are trans-impedance and responsivity respectively. When two error signals are multiplied in the combiner a resultant error signal would be generated which can be expressed as,

$$e = k\sin 2\varphi(t) \tag{10}$$

Where, $k = k_{p1}k_{p2}/2$ is the total gain of the OCL. Considering resultant transfer function of the LF and delay, phase error equation for lase LO is written as,

$$\frac{d\varphi}{dt} = \sigma - kF(p)\sin 2\varphi \tag{11}$$

This is operator form of equation, where frequency $s$is replaced by time derivative operator $p = d/dt$ and $\sigma = \omega_i - \omega_0$ is the frequency detuning.

Using the proper transfer functions, phase error equation in normalized form is written as,

$$\begin{gathered}\delta_n\frac{d^3\varphi}{d\tau^3} + (1+\delta_n - 2k_ng\delta_n\cos 2\varphi)\frac{d^2\varphi}{d\tau^2} + (1+2gk_n\cos 2\varphi - 2k_n\delta_n\cos 2\varphi)\frac{d\varphi}{d\tau} \\ +4k_ng\delta_n\sin 2\varphi\left(\frac{d\varphi}{d\tau}\right)^2 + k_n\sin 2\varphi = \sigma_n\end{gathered} \tag{12}$$

Where, $\tau\,(=t/\tau_1)$ is the normalized time and $k_n\,(k\tau_1), \sigma_n\,(\sigma\tau_1), \delta_n\,(\delta\tau_1)$are normalized values of loop gain, frequency detuning and loop delay respectively. Phase error dynamics can be understood by analysis of the equation (12)

## 3 Analytical Study

We want to study the effect of high frequency gain $g$ on stable and unstable behavior of OCL. To predict stable mode of operation of the loop and effects of high frequency gain on the stability of OCL, Routh- Hurwitz criterion [16,17] has been used to apply the method at first system equation is to be expressed as the combination of first order equations. The system equation (12) of OCL can be expressed in terms of first order equations of state variables $\varphi, \dot{\varphi}, y$ and $Z$as,

$$\dot{\varphi} = y \tag{12a}$$

$$\dot{y} = z \tag{12b}$$

$$\dot{z} = \frac{1}{\delta_n}\left[\sigma_n - k_n\sin 2\varphi - \{1+2k_n(g-\delta_n)\cos 2\varphi\}y - 4gk_n\delta_n\sin 2\varphi y^2 - (1+\delta_n - 2k_ng\delta_n\cos 2\varphi)z\right] \tag{12c}$$

Now, stable equilibrium point of operation would be calculated by considering as, $\dot{\varphi} = 0, \dot{y} = 0, \dot{z} = 0$. Using these conditions, stable equilibrium points would be $\varphi_1 = (1/2)\sin^{-1}(\sigma_n/k_n), \varphi_2 = \pi/2 - (1/2)\sin^{-1}(\sigma_n/k_n)$

Using equation (12) Jacobean matrix for the transformation can be written as,

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \tag{13}$$

Here, $a_3 = (2k_n/\delta_n)\cos 2\varphi, a_2 = (1/\delta_n)[1+2k_n(g-\delta_n)\cos 2\varphi]$ and $a_1 = (1/\delta_n)[1+\delta_n - 2gk_n\delta_n\cos 2\varphi]$

If, $\lambda$ be the eigenvalue of the matrix in equation (13), then characteristics equation would be as,

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{14}$$

According to Routh- Hurwitz criterion, for stable equilibrium real parts of all eigenvalues must be negative and for this the required conditions are $a_1 > 0, a_2 > 0, a_3 > 0$ and $a_1a_2 > a_3$. Using these, one can obtain the stability conditions as

$$\sigma_n < k_n \tag{15a}$$

$$\sigma_n > k_n\left[1 - \frac{1}{4k_n^2}\left(\sqrt{1+\frac{1}{g} - \frac{1}{2\delta_n} - \frac{\delta_n}{2g} + \frac{1}{g\delta_n}} - \left(1+\frac{1}{2\delta_n} - \frac{\delta_n}{2g}\right)^2\right)\right]^{1/2} \tag{15b}$$

Depending on the loop gain, loop time delay and high frequency gain there exists a range of the value for frequency detuning in which loop can be operated in stable condition. When detuning of loop exceeds beyond the range, any two roots of characteristic equation become imaginary and lead to Hopf's bifurcation of the system dynamics. At this situation, phase error starts oscillation. From the above analysis, capture range for the system is written as $\sigma_c < \sigma_n < k_n$ . Where, $\sigma_c$is the critical frequency detuning, which is expressed with the help of equation (15) as,

$$\sigma_c = k_n \left[ 1 - \frac{1}{4k_n^2} \left( \sqrt{1 + \frac{1}{g} - \frac{1}{2\delta_n} - \frac{\delta_n}{2g} + \frac{1}{g\delta_n}} - \left( 1 + \frac{1}{2\delta_n} - \frac{\delta_n}{2g} \right)^2 \right) \right]^{1/2} \tag{16}$$

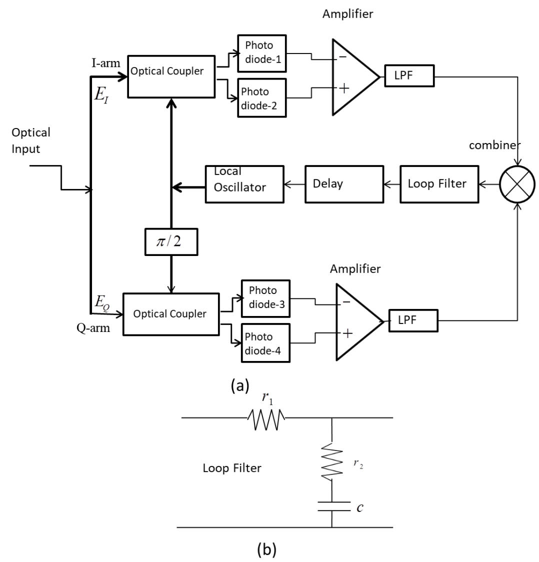


**Fig 1.** a) Block diagram of optical Costas loop showing different parts and b) circuit diagram of loop filter (bold arrow line represents optical fibre).

If stability condition is violated due to loop gain, time delay, or some other factors, then phase error starts to oscillate through Hopf's bifurcation. Effect of high frequency gain could be understood from the variations of critical detuning with normalized loop delay for different values of high frequency gain, which are shown in Figure 2. Left side, of each curve (Figure 2) represents stable zone and right side represents oscillatory zone of phase error. From the variation, it is observed that with the increase of high frequency gain the loop remains stable up to some large value of loop delay. However, effect of any amount of delay cannot be maintained by the high frequency gain of filter as shown in Figure 3. From the Figure 3, which is a parameter space drawn with the help of equation (15), it is observed that, for a specific value of high frequency gain, maximum amount of delay effect can be maintained. When loop delay is increased over some critical value for fixed value of high frequency gain and other parameters, then phase error starts to oscillate. This oscillation may be period-1, 2, 4, etc. or chaotic. However, the chaotic behavior of the loop has not been studied analytically, but it has been observed in numerical simulations.

## 4 Numerical Simulation Results

At first by using XPPAUT package software, we have solved the system equation for studying the growth of oscillation. From the bifurcation diagram as shown in Figure 4, it is observed that, when delay is increased the loop error starts oscillation through Hopf Bifurcation at some value of the loop delay as predicted analytically. A two parameter bifurcation diagram has been drawn, which shows the stable and unstable zone of operation of the loop as shown in Figure 5. This result is similar to the analytically predicted zone (shown in Figure 2). Now, to verify the analytical predictions, system equation (12) has been solved numerically according to $4^{th}$ order Runge-Kutta algorithm for different values of system parameters, high frequency gain of LF and loop time delay. In the numerical simulation, the time increment is taken as 0.0005 and to avoid transient part of the solution, sufficient numbers of initial solution points are rejected. Dynamics of the system has been studied with the help of phase plane plot and the time series plot of the phase error. Some selected simulation results are shown in Figure 6 to Figure 10. In simulation results,

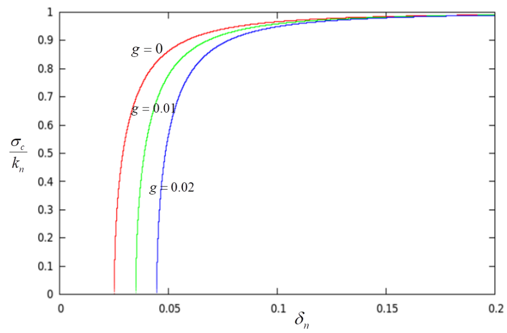


**Fig 2.** Plotting of critical detuning$(\sigma_c/k_n)$with normalised delay $(\delta_n)$for different values of high frequency gain, $[k_n = 15]$.

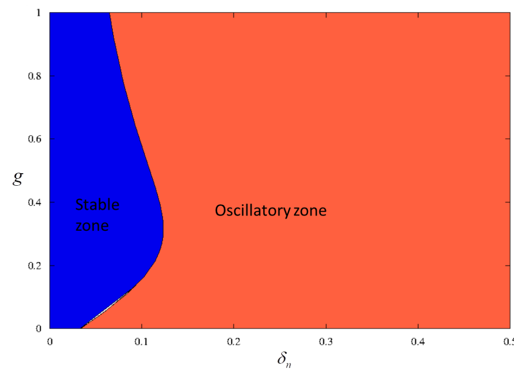


**Fig 3.** Parameter space showing stable, oscillatory zone of phase error as predicted analytically,$(\sigma_c/k_n = 0.05, k_n = 15)$.

it has been shown that how with the increase of loop delay phase error starts oscillation in period-1, period-2, period-4 and chaotic state and next, with the help of high frequency gain, it is possible to remove the oscillations to operate the device in stable mode.

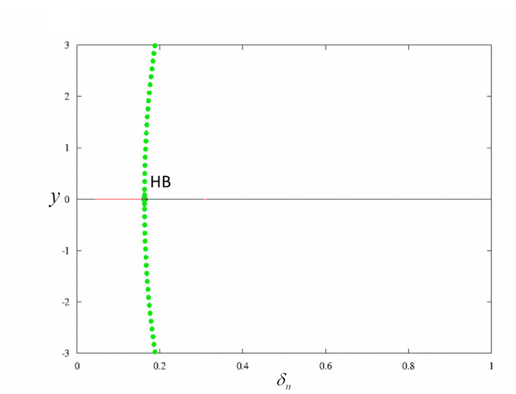


**Fig 4.** Hopf Bifurcation diagram of phase error with normalised time delay,$(\sigma_c/k_n = 0.1, k_n = 15, g = 0.2)$.

$$(\sigma_c/k_n = 0.05, k_n = 15, g = 0.2, \delta_n = 0.06)$$

For low value of loop delay $(\delta_n = 0.02)$,

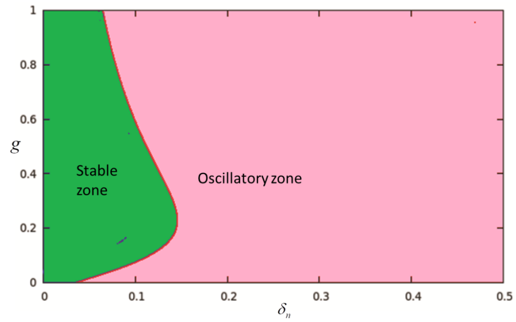


**Fig 5.** Numerically predicted parameter space showing stable, oscillatory zone of phase error, drawn by using XPPAUT, $(\sigma_c/k_n = 0.05, k_n = 15)$.

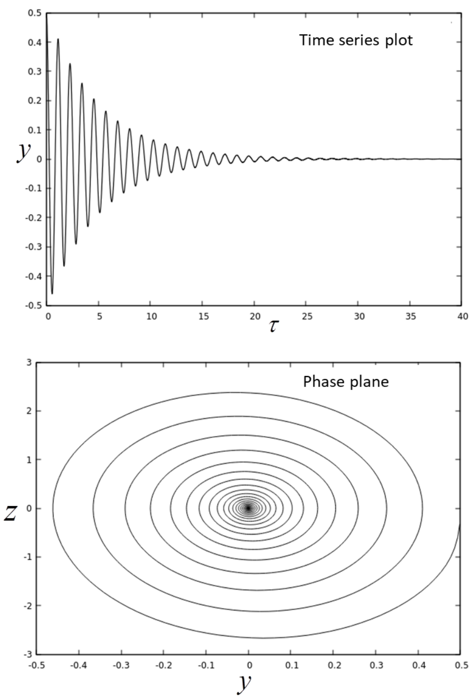


**Fig 6.** Numerically simulated Time domain and Phase plane plot,$(\sigma_c/k_n = 0.05, k_n = 15, g = 0.02, \delta_n = 0.02)$.

frequency difference of the input and reference signal remains constant and corresponding phase trajectory converges to a stable point as shown in Figure 6. In this situation, the loop would be in locked state with the input optical signal. When delay is increased by some value, $(\delta_n = 0.06)$ a closed phase trajectory is formed in phase plane and corresponding frequency difference oscillates periodically with time as shown in Figure 7. Figure 8 has been drawn for more increased values of delay $(\delta_n = 0.15)$. In the phase plane plot, bifurcation of limit cycle into two is observed, which is called limit cycle bifurcation. In this situation, phase error oscillates with two different frequencies. With further increase of delay, $(\delta_n = 0.46)$, phase trajectory is not a single closed curve but it consists of so many curves, which intersect each other and corresponding time series plot is irregular in nature as shown in Figure 9. This is known as chaotic behavior of the loop. In this condition a chaotically phase modulated laser signal is generated. For loop delay, $(\delta_n = 0.06)$and high frequency gain, g = 002 loop phase error oscillate with period one, Now when high frequency gain value of the loop filter is increased to the value 0.2, then phase error becomes steady as shown in Figure 10. It proves that by increasing high frequency gain loop stability can be improved up to some large extent of loop delay. This proves the analytical prediction. Effects of loop delay on OCL and OPLL have been discussed in author's previously published articles [14,18], where conventional LF was used, but in this article, proportional plus integrating type loop filter with high frequency gain has been used. From analytical and numerical simulation it is predicted that nonlinear dynamics

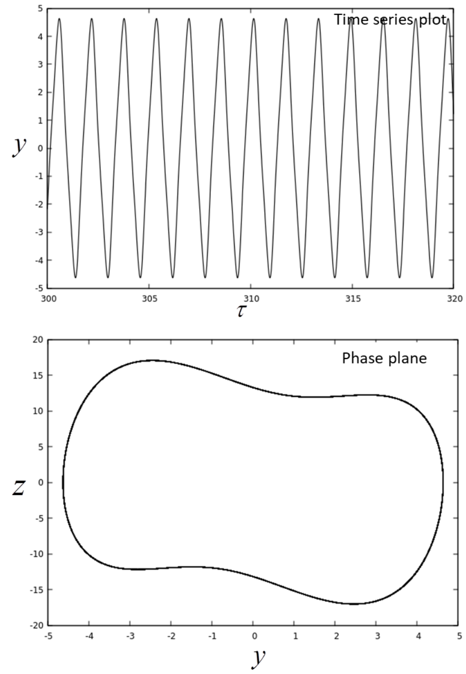


**Fig 7.** Numerically simulated Time domain and Phase plane plot,$(\sigma_c/k_n = 0.05, k_n = 15, g = 0.02, \delta_n = 0.06)$

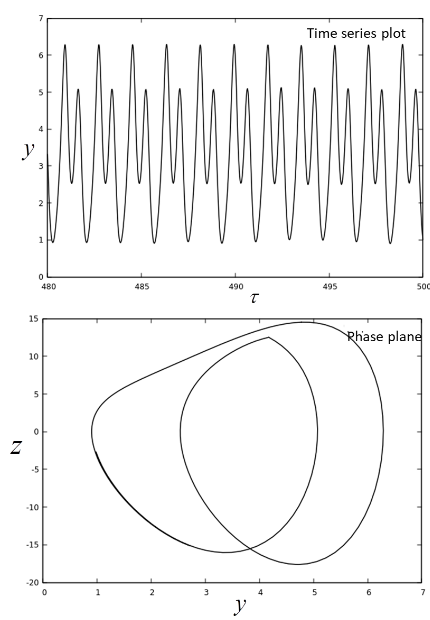


**Fig 8.** Numerically simulated Time domain and Phase plane plot,$(\sigma_c/k_n = 0.05, k_n = 15, g = 0.02, \delta_n = 0.15)$.

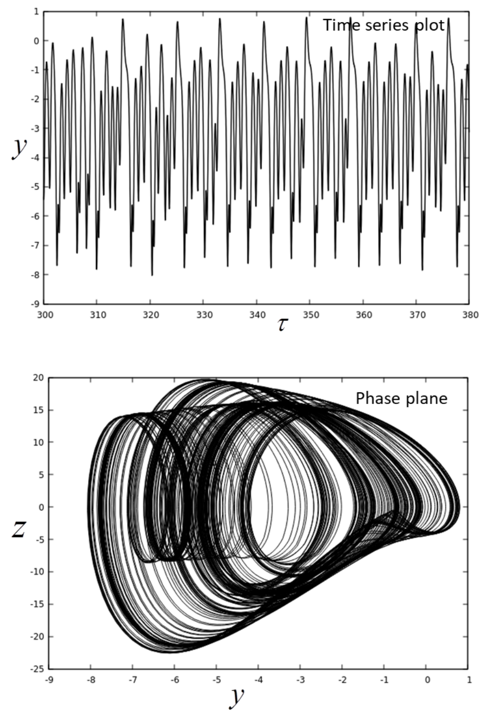


**Fig 9.** Numerically simulated Time domain and Phase plane plot,($\sigma_c/k_n = 0.05, k_n = 15, g = 0.02, \delta_n = 0.46$).

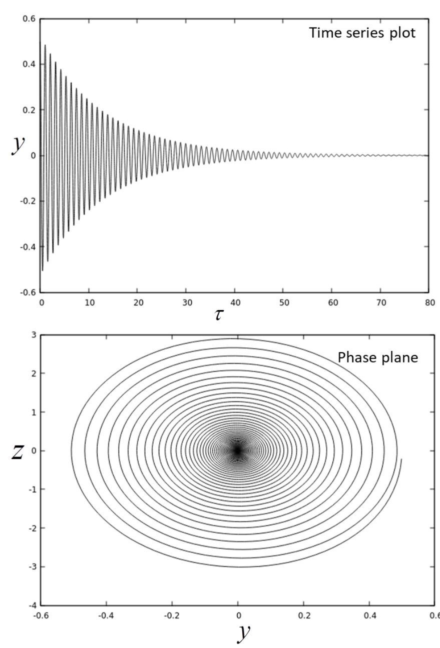


**Fig 10.** Numerically simulated Time domain and Phase plane plot

of OCL in presence of loop delay can be controlled more efficiently with the use of proportional plus integrating type loop filter.

## 5 Conclusion

In this paper, effect of high frequency gain on nonlinear dynamics of OCL having inherent loop delay has been discussed. How loop delay can be controlled by the gain of LF has been shown by analytical and numerical analysis of system equation. Different nonlinear behaviors of OCL due to presence of loop delay and controlling of those nonlinear effects have not been discussed with proper attention in different reported literatures much more. This study reports different nonlinear effects due to unwanted delay that may degrade the performance of OCL. This study also gives the solution to remove the effect of delay so that OCL may be used in stable operating condition. To design stable optical homodyne receivers such as OPLL, DOPLL or OCL effects of delay on nonlinear dynamics must be consider with proper attention otherwise, loop performance may be degraded. From analytical and numerical analysis it is observed that large loop delay can produce regular and chaotic behavior of the phase error. Chaotic phase modulated optical signal can be obtained from the loop laser oscillator for large value of loop delay. By choosing proper values of system parameters and high frequency gain value stability of OCL can be controlled up to some certain extent of delay. It is helpful to design a highly stable OCL, which can be used as stable optical receiver in optical communication system where delay present. It is also known from the study that, by proper choice of delay, high frequency gain and other factors chaotic optical signal can be generated from an OCL, which can be used in chaos based optical communication. Although use of chaotic optical signal in communication has several advantages, but synchronization between transmitter and receiver with chaotic signal is quite difficult. So, theoretical and experimental studies on synchronization for chaotic optical signal using OCL are required. Again, in different applications two or more OCL, OPLL are coupled with each other. For those coupled systems, how their behaviors are affected by loop delay is another important field of research.

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