

SYSTEMATIC REVIEW

 OPEN ACCESS

Received: 05.07.2021

Accepted: 19.11.2021

Published: 28.12.2021

Citation: Mishra N, Singh SP (2021) Dynamic Modelling and Control of Flexible Link Manipulators: Methods and Scope - Part 2. Indian Journal of Science and Technology 14(48): 3494-3508. https://doi.org/10.17485/IJST/v14i48.1148_ii

* **Corresponding author.**

natrajmishra@gmail.com

Funding: APC is partially deferred by Indian Society for Education and Environment

Competing Interests: None

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ISSN

Print: 0974-6846

Electronic: 0974-5645

Dynamic Modelling and Control of Flexible Link Manipulators: Methods and Scope - Part 2

Natraj Mishra^{1*}, S P Singh²¹ Assistant Professor, School of Engineering, UPES Dehradun, India² Professor, Department of Mechanical Engineering, IIT Delhi, India

Abstract

Objectives: This paper addresses two key issues in the area of flexible robotics. The issues are vibration control of flexible links and trajectory control of flexible robots. A brief, yet, significant review is provided that addresses these two issues. **Methods:** For vibration control of flexible links, possibilities of the use of passive and active damping methods are explored in the literature. After that, the effect of proper trajectory planning to ensure positional accuracy at the end-effector is studied. **Findings:** After a review of 181 research papers from the year 1970 to 2021, it has been found that the vibration suppression of flexible links can be achieved through the application of viscoelastic materials, piezoelectric materials, and optimum trajectory planning. Recent trends in research in the area of flexible manipulators show that an optimal trajectory can significantly help in reduction of link vibrations and achievement of positional accuracy simultaneously. **Novelty:** The novelty of the present work lies in exploring the possible application of passive and active damping control methods for vibration suppression of flexible link manipulators. Besides that, the survey also highlights how well planned trajectory may help achieve accurate tip positioning of flexible robots.

Keywords: Flexible manipulator; viscoelastic damping; active vibration control; trajectory planning and control

1 Introduction

In Part 1⁽¹⁾ of the review on flexible manipulators, the issues related to dynamic modelling and control were discussed. In the present work (Part 2 of the review work), vibration suppression techniques using passive and active vibration control means are discussed. Besides that, work done by various researchers in the area of trajectory planning and control is also reviewed. The aim of the present work is to provide an insight to the readers for achieving tip position control of flexible link manipulators through vibration suppression of links and optimum planning of trajectory. The approaches for trajectory planning discussed within this review are same as adopted for rigid robots. These techniques will have to be modified suitably to incorporate the effects of flexibility. Flexibility within a system may arise either due to its structural design or

due to its motion. Structural flexibility can be dealt with by incorporating passive damping using viscoelastic materials and through active damping using piezoelectric materials. On the other hand, flexibility due to motion may be dealt with by proper trajectory planning. Part 1 of this review broadly classified the areas of study of flexible manipulators. The same is provided here also for the convenience of the readers (refer to Fig. 1 below).

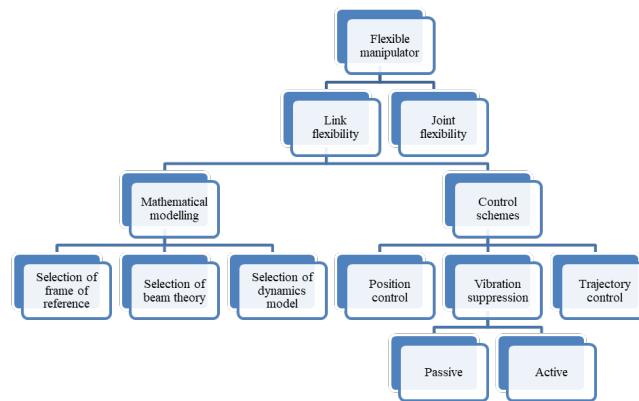


Fig 1. Study of flexible manipulator

There can be two types of flexibilities present within a flexible manipulator: joint flexibility and link flexibility. Only link flexibility has been considered in the present review on flexible manipulators. A review on mathematical modelling and position control schemes has been discussed in Part 1. In Part 2 of the study, the survey progressively provides information about research work done by authors in the area of flexible robotics and vibration control using passive and active damping methods. Besides that, significance of trajectory planning and control in achieving the desired position is also highlighted. The present survey is thus divided into the following sub-headings:

- i. Passive and active control of vibrations of flexible links
 - a. Review on viscoelastic damping (passive vibration control)
 - b. Review on active damping (active vibration control)
- ii. Trajectory planning and control of robots

2 Passive and active control of vibrations of flexible links

The positional accuracy at the tip of flexible manipulators can be improved by using passive and active vibration control methods. Hence, a survey on work done in the area of passive and active control of vibrations of structures was conducted. Firstly, a review on viscoelastic damping is provided and then a brief study of work done by various authors on active vibration control using piezoelectric materials is provided.

2.1 Review on viscoelastic damping

Zhou et al. [2016]⁽²⁾ presented a review on methods and models for imparting viscoelastic damping to the structures (Fig. 2). Grootenhuis [1970]⁽³⁾ discussed the efficient ways of damping vibrations of structures through the application of viscoelastic materials either in an unconstrained fashion or by making a multilayer sandwich (refer to Fig. 2 provided below).

Jones et al. [1972]⁽⁴⁾ used a resonance method to develop a scheme for measuring the complex-moduli-based properties of viscoelastic materials attached to thin sheets of metals. Kapur et al. [1977]⁽⁵⁾ experimentally verified the ‘four-element model’ to analyze beams with viscoelastic damping subjected to shock excitations. Trompette et al. 1978]⁽⁶⁾ advocated that the dynamic behaviour of a three-layer beam with a viscoelastic core depends upon the boundary conditions and longitudinal displacements in the beam. Ioannides and Grootenhuis [1979]⁽⁷⁾ used triangular finite elements to find out the dynamic stiffness of a constrained viscoelastic layer. Again, Ioannides and Grootenhuis [1982]⁽⁸⁾ proved that the analysis of viscoelastically damped structures using integral equations is faster than by using differential equations. Tzou [1988]⁽⁹⁾ presented a study on nonlinear contact dynamics and controls for eccentrically supported masses and simply supported beams using analytical and finite element methods. The shear moduli of viscoelastic materials are vulnerable to any change in impressed frequency, hence, Xisheng et al. [1995]⁽¹⁰⁾ developed a ‘finite element perturbation method’ that made use of a frequency-dependent stiffness matrix. At the same time, they also pointed out the importance of the optimal placement of viscoelastic layers and

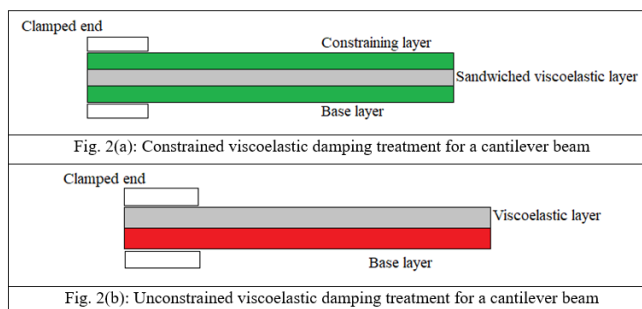


Fig 2. Set-up for imparting viscoelastic damping to structures

optimal selection of viscoelastic materials. Thomas et al. [1998]⁽¹¹⁾ developed a new finite element for constrained layer beams with moderately thick cores. Barkanov [1999]⁽¹²⁾ used fast Fourier transform to study the transient response of viscoelastically damped structures. Lei et al. [2006]⁽¹³⁾ considered the effects of time and spatial hysteresis while modelling beams and plates. The equations of motion were represented using the integro-partial-differential equation. Lepoittevin and Kress [2010]⁽¹⁴⁾ improved the damping capabilities of segmented constrained viscoelastic layers using Nelder-Mead simplex. Dutt and Roy [2010]⁽¹⁵⁾ presented a generic method of representing the viscoelasticity using differential time operator and finite element method. Navin and Singh [2010]⁽¹⁶⁾ devised a method for optimal placement of constrained viscoelastic layers using the approach of modal strain energy for damping of vibrations of a curved panel. Palmeri and Adhikari [2011]⁽¹⁷⁾ studied the transverse vibrations of constrained viscoelastic structures using state-space representation. This helped them to handle inhomogeneity, different types of boundary conditions, and rate-dependent constitutive law for viscoelastic structures. Lei et al. [2013]⁽¹⁸⁾ developed a transfer function method to obtain the closed-form solution of beams. They made use of standard three-parameter viscoelastic models and nonlocal EB beam theory. Hujare and Sahasrabudhe [2014]⁽¹⁹⁾ followed the ASTM standards and obtained the damping factors for constrained viscoelastic structures. LI et al. [2015]⁽²⁰⁾ used both the mode superposition method and Fourier transform method for calculating the dynamic response of viscoelastically damped system. Freundlich [2016]⁽²¹⁾ performed the dynamic modelling of a viscoelastic beam using fractional derivatives. Adhikari [2013]⁽²²⁾ has done a significant job in the area of analysis and identification of damping. Damping plays an important role in the vibration attenuation of structures. An important characteristic of viscoelastic materials is their loss factor. The ‘loss factor’ is directly proportional to the energy dissipated per cycle to the maximum energy stored per cycle. Ghiringhelli and Terraneo [2015]⁽²³⁾ calculated the loss factor of sandwiched viscoelastic specimens using the modal strain energy technique. Bonfiglio et al. [2016]⁽²⁴⁾ determined the complex moduli of viscoelastic materials using the transfer matrix approach. Once the complex modulus is obtained, damping offered by the viscoelastic material can be easily determined. Vergassola et al. [2018]⁽²⁵⁾ provided an experimental method for the determination of loss factor for the viscoelastic materials for naval applications. Hamdaoui et al. [2019]⁽²⁶⁾ identified the viscoelastic parameters based upon Adjoint method. They described their viscoelastic model as a non-linear eigenvalue problem.

For continuous systems like beams, researchers have used various types of damping models. These are the viscous air damping model, Kelvin-Voigt damping model, time hysteresis damping model, spatial hysteresis damping model, and Friswell damping model. Besides the above damping models, literature also describes Anelastic Displacement Field (ADF) models of viscoelastic damping for constrained layer sandwich beams. A survey about these damping methods reveals that the energy dissipation from materials is only a weak function of frequency and almost directly proportional to q^n where q represents the degree of freedom of the vibratory system and index n lies between 2 and 3. For mild steel, $n = 2.3$. Real systems have complex eigenvalues. Hence, analysis of damped systems can be done using the state-space method and the configuration-space method. Table 1 provides the list of various authors who have used either of these two methods.

The phenomenon of viscoelasticity can be modelled using the combination of linear spring and linear dashpot elements. Few of these viscoelastic models are provided in Figure 3.

In Figure 3 provided below, σ = stress within the element; ϵ = strain within the element; F = force applied on the element; u = extension within the element; η = dynamic viscosity of dashpot element; E = spring constant of the element and, D represents $\frac{d}{dt}$. It can also be found from the table that, the stress-strain equation of a linear viscoelastic material can be expressed in the following form:

$$P\sigma = Q\epsilon \tag{1}$$

Table 1. Methods for analysis of viscoelastically damped structures

Method	Research papers	Remarks
State-space method	[Li and Hu, 2016] ⁽²⁷⁾ ; [Ding et al., 2016] ⁽²⁸⁾ ; [Sheoran et al., 2016] ⁽²⁹⁾ ; [Ezzat and El-Bary, 2017] ⁽³⁰⁾ ; [Norouzi and Alibeigloo, 2017] ⁽³¹⁾ ; [Oskouie et al., 2017] ⁽³²⁾ ; [Hien and Lam, 2018] ⁽³³⁾ ; [Aldawody et al., 2019] ⁽³⁴⁾ ; [Feri et al., 2021] ⁽³⁵⁾	By using this method an nth order differential equation can be reduced to a set of n first-order differential equations which makes it an efficient tool to apply for both the linear and non-linear systems. It can be applied to both time-invariant and time-variant systems. But, the method is computationally intensive.
Configuration space method	[Hammami et al, 2016] ⁽³⁶⁾ ; [Yamamoto, 2017] ⁽³⁷⁾ ; [Jiang et al, 2017] ⁽³⁸⁾ ; [Zeng et al, 2017] ⁽³⁹⁾ ; [Mehnert et al, 2018] ⁽⁴⁰⁾ ; [Drainville, 2019] ⁽⁴¹⁾ ; [Goryacheva and Miftakhova, 2019] ⁽⁴²⁾ ; [Jamshidi et al, 2020] ⁽⁴³⁾ ; [Kromer and Roubicek, 2020] ⁽⁴⁴⁾ ; [Brighenti et al, 2021] ⁽⁴⁵⁾ ;	The configuration space of a physical system under consideration is specified by its degrees of freedom. The governing equation to be solved is treated like an eigenvalue problem. An nth order differential equation yields a 2n order polynomial equation.

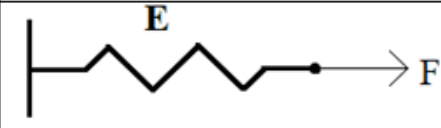
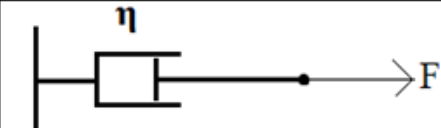
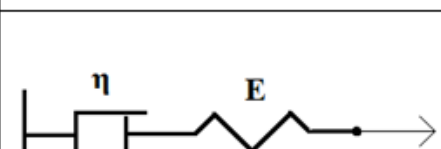
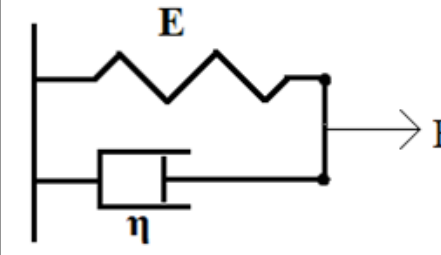
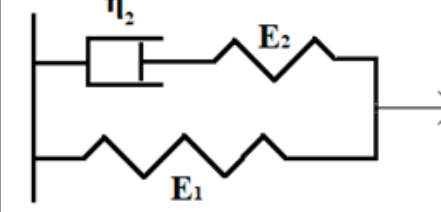
S. No.	Type of element	Physical representation	Mathematical model
1	Spring		$\sigma = E\epsilon$ or $F = Eu$
2	Dashpot		$\sigma = \eta D\epsilon$ or $F = \eta Du$
3	Maxwell		$D\epsilon = \frac{1}{E} D\sigma + \frac{1}{\eta} \sigma$; or $Du = \frac{1}{E} DF + \frac{1}{\eta} F$
4	Kelvin-Voigt		$\sigma = E\epsilon + \eta D\epsilon$; or $F = Eu + \eta Du$
5	Three-element (elastic)		$\left(D + \frac{E_2}{\eta_2}\right)F = \left[(E_1 + E_2)D + \frac{E_1 E_2}{\eta_2}\right]u$

Fig 3. Model representation of viscoelasticity

where, P and Q are polynomials with constant coefficients in the operator D. From the governing equation described in equation-1, information about complex modulus, Y, and complex compliance, J can be obtained as follows.

$$Y = \frac{Q}{P} \text{ and } J = \frac{P}{Q} \quad (2)$$

The models described above help in understanding the viscoelastic damping present in the structures. The recent study in the area of viscoelasticity is dedicated to finding the relationship between the coefficient of restitution and the damping ratio of viscoelastic materials⁽⁴⁶⁾.

2.2 Review on active damping of vibrations

Benjeddou [2000]⁽⁴⁷⁾ presented a comprehensive survey highlighting the advances and trends in finite element formulations and applications of adaptive structural elements. Chopra [2002]⁽⁴⁸⁾ provided a state-of-art review on smart structures. Cannon and Schmitz [1984]⁽⁴⁹⁾ controlled the tip vibrations of a single-link flexible manipulator with non-collocated sensor-actuator pairs. Sakawa et al. [1985]⁽⁵⁰⁾ achieved the position control as well as vibration control of a flexible link by controlling the motor torque. Goh and Caughey [1985]⁽⁵¹⁾ stressed using position feedback rather than velocity feedback for active vibration control due to better stability and great ability to handle unmodelled modes. Baz and Poh [1988]⁽⁵²⁾ presented a modified independent modal space control (MIMSC) method for selection of optimal location, control gains, and excitation voltage of piezoelectric actuators for active vibration control of a flexible beam. Tzou [1989]⁽⁵³⁾ described a general theory based on finite element modelling for active vibration control of smart structures using sensors and actuators in an integrated manner. Again in the year 1991, Tzou [1991]⁽⁵⁴⁾ achieved the position control of active structures using variable feedback control gains. Murozono and Sumi [1994]⁽⁵⁵⁾ were able to control the first mode vibrations of a cantilever using foil strain gauges that generated thermal bending moment. Lesieutre and Lee [1996]⁽⁵⁶⁾ used finite element modelling of beams with segmented active constrained layer (ACL). The segmented ACL showed enhanced damping performance than the continuous layer. The researchers also included viscoelasticity using the anelastic displacement fields (ADF) method. Chen et al. [1997]⁽⁵⁷⁾ investigated the dynamic stability of smart structures using state-space formulation and velocity feedback. Aldraihem and Wetherhold [1997]⁽⁵⁸⁾ performed the analyses of smart structures undergoing both bending and twisting vibrations based upon modal cost and controllability. They carried out their study on active control using two different types of materials viz. lead zirconate titanate (PZT) and PZT/epoxy piezoelectric composite (PZT/Ep). The authors concluded that PZT/Ep provided the best bending-twisting actuation for vibration damping. Benjeddou et al. [1997]⁽⁵⁹⁾ compared extension and shear actuation mechanisms for smart structures and concluded that the shear-actuated beam showed less deformation. Peng et al. [1998]⁽⁶⁰⁾ and Xu and Koko [2004]⁽⁶¹⁾ showed that the number of sensor and actuator pairs and their relative placement play a key role in vibration suppression of smart structures. Singh et al [2003]⁽⁶²⁾ described few efficient algorithms for active vibration control of a cantilever beam. Sun et al. [2005]⁽⁶³⁾ proposed a method for placement of piezoelectric actuators based upon mode shape functions of an actively controlled structure. Gardonio and Elliott [2005]⁽⁶⁴⁾ studied the effect of high values of control gains on vibration level and Eigen values of the smart structure. Sharma et al. [2005]⁽⁶⁵⁾ provided a novel approach based on independent modal space control and fuzzy logic control for active vibration control of a cantilever. Gatti et al. [2007]⁽⁶⁶⁾ highlighted the effects of control system hardware and dynamics of transducers on the stability of actively controlled structures. Vasques and Rodrigues [2008]⁽⁶⁷⁾ assessed and discussed the performances of feedback, feed-forward, and hybrid controllers for an actively controlled beam with active constrained layer damping (ACLD) treatment. Belouettar et al. [2008]⁽⁶⁸⁾ investigated the influence of feedback parameters during active vibration control of sandwich beams. Qiu et al. [2009]⁽⁶⁹⁾ suppressed the first two vibration modes using proportional feedback control and sliding mode control. They successfully reduced the problems of phase hysteresis and time delay in actively controlled structures. Mirzaee et al. [2010]⁽⁷⁰⁾ dealt with maneuver control and vibration suppression of a two-link flexible arm with embedded piezoelectric sensors and actuators. They designed their control system using variable structure control along with Lyapunov control for vibration suppression of the links. Gupta et al. [2011]⁽⁷¹⁾ showed that the effectiveness of active vibration control decreases with increase in temperature. Therefore, it is necessary to consider the temperature-dependence of piezoelectric stress coefficient and permittivity during the mathematical modelling of piezoelectricity. Yavuz et al. [2016]⁽⁷²⁾ focused their research on control of residual vibrations. According to them, residual vibrations of a flexible manipulator can be controlled through proper selection of parameters like acceleration time, constant time and deceleration time. Malgaca et al. [2016]⁽⁷³⁾ followed the same approach for a curved flexible link. The effect of nonlinearities present in piezoelectric actuation under the presence of strong electric field was studied by Yasin and Kapuria⁽⁷⁴⁾. He et al. [2017]⁽⁷⁵⁾ used radial basis function neural network for vibration suppression of a single-link flexible manipulator with input deadzone. Deadzone is a nonlinearity common with vibration control systems which makes them insensitive to small signals. Fuzzy active vibration control was achieved by Alavi et al [2017]⁽⁷⁶⁾. Lu et al. [2018]⁽⁷⁷⁾ proposed the criterion

of optimal placement of piezoelectric actuators on a single-link flexible manipulator for vibration suppression based on modal H_2 norm, change rate of natural frequencies, and improved particle swarm optimization algorithm. Utilization of piezoelectric materials can be done for vibration damping of links of the Flexible manipulator. For this, the sensors and actuators can be applied in two ways on the links, viz., collocated and non-collocated (refer to Figure 4 given below).

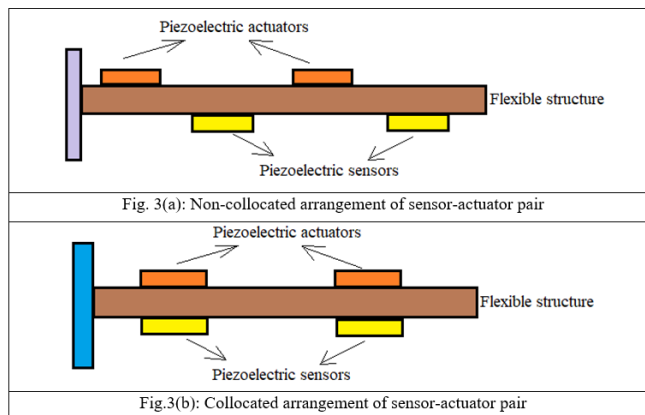


Fig 4. Relative arrangement sensor-actuator pair on a flexible structure for active damping

The sensed degrees of freedom and the controlled degrees of freedom will be the same in collocated arrangement while they will be different in non-collocated arrangement. According to Santos et al [2019] the amount of damping offered by piezoelectric materials of different thicknesses can be optimally found out using Neural Networks and Genetic Algorithm⁽⁷⁸⁾. Hamed et al [2020]⁽⁷⁹⁾ utilized the technique of perturbation analysis for active vibration control of helicopter blades undergoing nonlinear vibrations. Similar approach can be applied for controlling the vibrations of a flexible link manipulator⁽⁸⁰⁾. Table 2 provides the names of few researchers who applied different types of piezoelectric actuation mechanisms for active vibration control of smart beams.

Table 2. Piezoelectric actuation mechanisms

Piezoelectric actuation mechanism	Researchers	Remarks
Extension actuation mechanism	[Varma et al., 2017] ⁽⁸¹⁾ ; [Medeiros et al., 2017] ⁽⁸²⁾ ; [Chuaqui et al., 2018] ⁽⁸³⁾ ; [Carrera et al., 2018] ⁽⁸⁴⁾ ; [Zoric et al., 2019] ⁽⁸⁵⁾ ; [Goncalves et al., 2020] ⁽⁸⁶⁾ ; [Shakir and Saber, 2020] ⁽⁸⁷⁾ ; [Reddy et al., 2021] ⁽⁸⁸⁾ ; [Singh et al., 2021] ⁽⁸⁹⁾	The piezoelectric actuator is poled in the direction of the applied electric field. It results in the development of axial strains. There exists electromechanical coupling between axial strain and transverse electric field vector. The actuator based on the 'extension mechanism' is mounted on the surface of the smart structure. It results in the generation of concentrated forces and moments at the boundaries of the surface of the structure.
Shear actuation mechanism	[Sakib et al, 2017] ⁽⁹⁰⁾ ; [Carrera et al, 2018] ⁽⁸⁴⁾ ; [Dubey and Panda, 2019] ⁽⁹¹⁾ ; [Altammar et al, 2019] ⁽⁹²⁾ ; [Carrison et al, 2020] ⁽⁹³⁾ ; [Reddy et al, 2021] ⁽⁸⁸⁾ ; [Gupta et al, 2021] ⁽⁹⁴⁾	The piezoelectric actuator is poled in the direction perpendicular to the applied electric field. This induces shear stresses within the piezoelectric material. Here, electromechanical coupling exists between shear strain and transverse electric field. The actuator based on the 'shear mechanism' is sandwiched within the smart structure. Thus, it induces distributed moments within the structure.

Researchers have used various types of feedback control mechanisms during the active control of structures. The names of few researchers are provided below in Table 3.

Table 3. Types of feedback control used for active vibration control (AVC) of smart structures

Type of feedback control used for AVC	Researchers
Position feedback	[Marinangeli et al, 2017] ⁽⁹⁵⁾ ; [Yuan et al, 2017] ⁽⁹⁶⁾ ; [Zhang and He, 2018] ⁽⁹⁷⁾ ; [Zhang et al, 2019] ⁽⁹⁸⁾ ; [Perez et al, 2019] ⁽⁹⁹⁾ ;
Velocity feedback	[Wang et al, 2017] ⁽¹⁰⁰⁾ ; [Rahman et al, 2018] ⁽¹⁰¹⁾ ; [Chuaqui et al, 2018] ⁽⁸³⁾ ; [Ma et al, 2019] ⁽¹⁰²⁾ ; [Selim et al, 2019] ⁽¹⁰³⁾ ; [Kleinwort et al, 2021] ⁽¹⁰⁴⁾
Acceleration feedback	[Yang et al, 2017] ⁽¹⁰⁵⁾ ; [Ahmed and Saeed, 2017] ⁽¹⁰⁶⁾ ; [Mohanty and Dwivedy, 2018] ⁽¹⁰⁷⁾ ; [Peukert et al, 2019] ⁽¹⁰⁸⁾ ; [Zhao and Zhou, 2020] ⁽¹⁰⁹⁾ ; [Shin et al, 2021] ⁽¹¹⁰⁾

3 Trajectory control of Flexible manipulators

By proper planning of trajectory, link vibrations can be controlled up to a certain extent. Good trajectory planning requires optimization of joint torque requirements, jerk, tip position accuracy, and operation speed. Hence, a brief literature survey on trajectory control of both rigid and flexible robots was conducted. Luca [1998]⁽¹¹¹⁾ provided some techniques based on feedback control law for trajectory tracking of flexible manipulators. Due to non-linear dynamics, linear control is difficult to apply to such systems. To apply linear control theory for joint motion control of robots, a high gear ratio is used so that a linear PD or PID controller may be used. A computed-torque controller is better than a linear PD or PID controller. It makes use of ‘feedback linearization’ of non-linear systems and appears in various types of control techniques like robust control, adaptive control, etc. [Lewis et al., 2004]⁽¹¹²⁾. The motion of the joints of a robot can be controlled either by the approach of ‘independent joint control’ or by the approach of ‘multivariable control’ [Spong et al., 2005]⁽¹¹³⁾. In independent joint control, each joint of a robot is controlled by considering it as a Single input/single-output (SISO) system. The coupling effects due to the motion of other links are treated as disturbances. On the other hand, the multivariable control facilitates the design of robust [Spong and Vidyasagar, 1987]⁽¹¹⁴⁾ and adaptive [Ortega and Spong, 1989]⁽¹¹⁵⁾ nonlinear controllers that guarantee more stability and better tracking of arbitrary trajectories. Harris and Wang [1988] proposed mathematical models for the stabilization of closed-loop constrained robots. The motion planning of robots is very important from the point of view of performing desired tasks. Slotine and Yang [1989]⁽¹¹⁶⁾ presented a computationally efficient time-optimal path-following algorithm for robots under actuator constraints. Few authors made use of the concept of decoupled and invariant dynamics of manipulator’s arm during the design stage for achieving high-speed trajectory control of direct-drive manipulators [Youcef and Kuo, 1993]⁽¹¹⁷⁾ and the concept of dynamic isotropy for decoupling of inertia matrix for obtaining robust control [Ma and Angeles, 1993]⁽¹¹⁸⁾. Other authors-Piazzzi and Visioli [2000]⁽¹¹⁹⁾, Constantinescu and Croft [2000]⁽¹²⁰⁾, Gasparetto and Zanotto [2007]⁽¹²¹⁾, and Marcello et al. [2015]⁽¹²²⁾ tried to formulate algorithms based on minimization of an objective function that helped in finding out the time-optimal trajectories for the robots. For this, they considered kinematic, dynamic, and payload constraints for the formulation of the objective function. As per Green and Sasiadek [2004]⁽¹²³⁾, it is necessary to know the type of boundary condition applicable to the given structure of the flexible manipulator. This helps in achieving good trajectory control of the manipulator. Xavier et al. [2010]⁽¹²⁴⁾ described how to build autonomous robots that could perform service tasks safely within the vicinity of humans. Such robots are composed of trajectory planners and controllers. Hu and Zhang [2015]⁽¹²⁵⁾ tried to control the end-point vibrations by using two controllers- one for trajectory control and the other for vibration suppression. For this, they used the variable-speed control moment gyros. Sato et al. [2016]⁽¹²⁶⁾ provided a method for trajectory control based upon a minimum energy approach which led to energy saving. Sun [2016]⁽¹²⁷⁾ showed that the tracking accuracy during trajectory planning of a two-link flexible manipulator could be increased by properly lubricating the revolute joint. Lismonde et al. [2019]⁽¹²⁸⁾ tried to optimize the trajectory of flexible manipulators using the geometric optimization method and feedforward control action. They also tried to modify the real-time trajectory of such robots based on their research. Giorgio and Vescovo [2019]⁽¹²⁹⁾ proposed an energy-based method to track the trajectory of a flexible manipulator taking care of end-effector vibrations. Currently, researchers are using particle swarm optimization algorithm for tuning of controllers for efficient trajectory tracking and vibration suppression simultaneously (⁽¹³⁰⁾, ⁽¹³¹⁾, ⁽¹³²⁾, ⁽¹³³⁾).

3.1 Effect of trajectory on tip response

Trajectory planning for the manipulators can be done using polynomial trajectories in joint-space. It is found that a low-order trajectory introduces an initial jerk at the tip of the manipulator. This jerk is absent when a high-order trajectory is used. Besides that, high amplitude of residual vibration can be observed at the tip when low-order trajectories are used. On the other hand, the amplitude of residual vibration is less when high-order trajectories are used. The tip response of manipulators depend also on

the value of trajectory time used. The decision about the trajectory time has to be taken keeping in view the speed and accuracy of operation required. Optimization of trajectories (¹³⁴), (¹³⁵) can significantly reduce the tip vibrations of manipulators and thereby increase tip position accuracy.

4 Conclusions from literature survey

In this section, conclusions from the literature survey will be provided. There are three sub-sections. In the first sub-section, conclusions on dynamic modelling and control of flexible manipulators are provided while in the second sub-section; conclusions on passive and active control of vibrations are provided. The third sub-section provides a conclusion on trajectory control of manipulators.

4.1 Conclusions on passive and active vibration control

From the literature survey, it is found that the phenomenon of viscoelasticity is due to shear strain and depends upon the frequency of the input source. The viscoelastic properties are also found out to be temperature-dependent (¹³⁶). The viscoelastic materials can be applied to vibrating structures either in a constrained or unconstrained fashion. For developing an effective model of viscoelasticity it is necessary to establish a relationship between the coefficient of restitution, loss factor, and the damping ratio. During active vibration control of structures, piezoelectric sensors and actuators are applied in a collocated and non-collocated fashion. Besides that, a suitable control scheme like position or velocity feedback is employed. There are four main steps involved in the vibration control of active structures. These are modelling of the smart structure, accurate positioning of sensors and actuators, determination of optimal feedback gain, and performance evaluation of controller design [Karagulle et al., 2004] (¹³⁷). The performance of a smart structure greatly depends upon the stiffness of the bounding piezoelectric layer (¹³⁸). From the literature, it is found that Boley's method is more suitable than Euler-Bernoulli beam theory for deriving an accurate mathematical of piezoelectric phenomenon (¹³⁹). Furthermore, most of the research on active vibration control was performed using the extension actuation mechanism. Piezoelectric actuation is of two types: extension actuation mechanism and shear actuation mechanism. The 'extension actuation mechanism' is caused by mounting the piezo-actuators on the surface of flexible structures while the 'shear actuation mechanism' is caused by sandwiched or embedded piezoelectric actuators. The surface-mounted piezo-actuators act through boundary point axial forces and bending moments while the embedded piezo-actuators act through distributed interface shear forces and bending moments [Benjeddou et al., 1997] (⁵⁹). The surface-mounted extension piezoelectric actuators are more effective for flexible structures while embedded shear actuators are more effective for comparatively stiffer structures [Trindade, 2007] (¹⁴⁰). In the extension actuation mechanism, the piezoelectric constant d_{31} is used while in the shear actuation mechanism, piezoelectric constant d_{15} is used. As per the literature, the constant d_{15} may be used to produce large angular displacement and torque [Benjeddou et al., 1999] (¹⁴¹). For a cantilever beam, the extension actuator yields maximum tip deflection when it is placed at the clamped end whereas the shear actuator yields maximum tip deflection when it is placed at a distance of one-tenth of the beam length from the clamped end [Kapur and Hagedorn, 2007] (¹⁴²). It is also found that while using active vibration control of flexible manipulators using piezoelectric actuators, the 'negative derivative feedback' controller is more effective than the 'positive position feedback' controller (¹⁴³).

4.2 Conclusions from literature survey on trajectory control

From the above survey on trajectory control of robot manipulators it is found that for performing desired tasks, it is necessary to plan trajectories optimally. This can be done in three ways; viz., minimum-time trajectory planning, minimum-energy trajectory planning, and minimum-jerk trajectory planning. This creates the requirement for efficient trajectory planning algorithms for robots (¹⁴⁴). The choice of an appropriate trajectory planning method depends upon the geometric path followed by the end-effector of the robot. There are three ways to define the geometric path. These are expression trees, splines, and arrays of points. The problem of trajectory planning (¹⁴⁵) for a robot may be defined as: Given a curve in the joint space of a robot, dynamic properties of the robot, and the actuator characteristics of the robot, what set of signals to the actuators will drive the robot from its current state to a defined final state with minimum cost? Table 4 summarizes the three schemes of trajectory planning.

The trajectory planning schemes described in the table above, require the implementation of a suitable trajectory planning algorithm. The algorithms may be based on three different methods. These are phase plane method, dynamic programming, and perturbation trajectory improvement algorithm (¹⁴⁵). It is also found that dynamic decoupling of system dynamics makes it easy to design a good control system. Furthermore, the sensitivity of unmodelled dynamics and bounds on actuator are the key issues in controller design [Kobilarov, 2014] (¹⁷⁹) for good trajectory control. Current research is focused on the application of computer-vision (¹⁸⁰) and, reinforcement learning (¹⁸¹) for effective trajectory control of flexible manipulators.

Table 4. Trajectory planning schemes

Scheme	Researchers	Remarks
Minimum-time trajectory planning	[Valente et al., 2017] ⁽¹⁴⁶⁾ ; [Yao et al., 2017] ⁽¹⁴⁷⁾ ; [Shen et al., 2017] ⁽¹⁴⁸⁾ ; [Dakka et al., 2017] ⁽¹⁴⁹⁾ ; [Reiter et al., 2018] ⁽¹⁵⁰⁾ ; [Shen et al., 2018] ⁽¹⁵¹⁾ ; [Zhao et al., 2019] ⁽¹⁵²⁾ ; [Kim and Croft, 2019] ⁽¹⁵³⁾ ; [Liu et al., 2020] ⁽¹⁵⁴⁾ ; [Pei et al., 2020] ⁽¹⁵⁵⁾ ; [Zhang et al., 2021] ⁽¹⁵⁶⁾ ; [Faroni et al., 2021] ⁽¹⁵⁷⁾	The scheme makes use of the following steps ⁽¹⁵⁸⁾ . a) Obtain the equation of path in parametric form. b) Determine the torque constraints imposed by the actuators. c) Plan the various possible trajectories within the actuator torque limits. d) Search for the trajectory with minimum time using a suitable algorithm.
Minimum-energy trajectory planning	[Huang et al., 2011] ⁽¹⁵⁹⁾ ; [Gregory et al., 2012] ⁽¹⁶⁰⁾ ; [Korayem et al., 2012] ⁽¹⁶¹⁾ ; [Wigstrom et al., 2013] ⁽¹⁶²⁾ ; [Du et al., 2015] ⁽¹⁶³⁾ ; [Wang et al., 2018] ⁽¹⁶⁴⁾ ; [Carabin et al., 2019] ⁽¹⁶⁵⁾ ; [Wu et al., 2020] ⁽¹⁶⁶⁾	The steps are as follows ⁽¹⁶³⁾ , ⁽¹⁶⁷⁾ . a) Identify the work process of the robot. b) Determine the configurations of the robot for the given work process through inverse kinematics. c) Define the equation of trajectory to be followed by each joint of the robot. d) Using the expressions of joint torques from the dynamics model of the given robot, find out the torque requirement at each joint. e) Calculate the total energy consumption at each joint. f) Minimize the energy function by minimizing the joint torque requirement.
Minimum-jerk trajectory planning	[Ramabalan et al., 2017] ⁽¹⁶⁸⁾ ; [Lu et al., 2017] ⁽¹⁶⁹⁾ ; [Zribi et al., 2018] ⁽¹⁷⁰⁾ ; [Huang et al., 2018] ⁽¹⁷¹⁾ ; [Rout et al., 2018] ⁽¹⁷²⁾ ; [Rojas et al., 2019] ⁽¹⁷³⁾ ; [Fang et al., 2019] ⁽¹⁷⁴⁾ ; [Zhou et al., 2020] ⁽¹⁷⁵⁾ ; [Oliveira et al., 2020] ⁽¹⁷⁶⁾ ; [Devi et al., 2021] ⁽¹⁷⁷⁾	Minimum-jerk trajectories produce motions similar to that of human joints ⁽¹¹⁹⁾ . They also minimize vibrations. Hence, they are highly desirable. The steps to be followed for minimum-jerk trajectory planning are as follows ⁽¹⁷⁸⁾ . a) Identify the work process. b) Determine the configurations of the robot using inverse kinematics. c) Identify the velocity and acceleration constraints. d) Formulate the trajectory. e) Obtain the maximum jerk. f) Define a cost function that will help in minimizing the maximum jerk. Jerk can also be minimized by selecting high-order polynomial trajectories.

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