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Dynamic modelling and control of flexible link manipulators: methods and scope- Part-1

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Abstract

Objectives: This paper addresses two key issues in the area of flexible robotics. The issues are dynamic modelling and control of flexible link robots. A brief, yet, significant review is provided that addresses these issues. **Methods:** The various approaches used by researchers for dynamic modelling and control of flexible robots are presented. Besides that, methods used for achieving optimal control are also discussed. **Findings:** After a review of 153 research papers from the year 1975 to 2021, it has been found that a good dynamic model of flexible manipulator helps in reducing the control and computational efforts. Recent trends in research in the area of flexible manipulators are towards the use of sliding mode control and vision-based control techniques. **Novelty:** Inclusion of the effect of torsional vibrations besides lateral vibrations on the positional accuracy of flexible manipulators makes the current research work novel.

Keywords: Flexible manipulator; modelling; dynamics; control

1 Introduction

Nowadays, robots are used in various fields like marine applications, medicine, the agricultural sector, space explorations, industry, and many other diverse fields. Many of these applications of robots require them to be lightweight, fast, and at the same time accurate and precise. The potential advantages offered by elastic link manipulators are higher operation speed and greater payload-to-manipulator weight ratio than the rigid robots. But, the flexibility of arms leads to deformation and vibrations at the tips of the links during the motion. As a consequence, dynamics modelling becomes extremely complicated. Also, the appearance of oscillations makes the control of such manipulators an arduous task. Thus, consideration of structural flexibility in robotic arms is a real challenge. The study in the area of flexible manipulators can be broadly classified as shown in Figure 1 below.
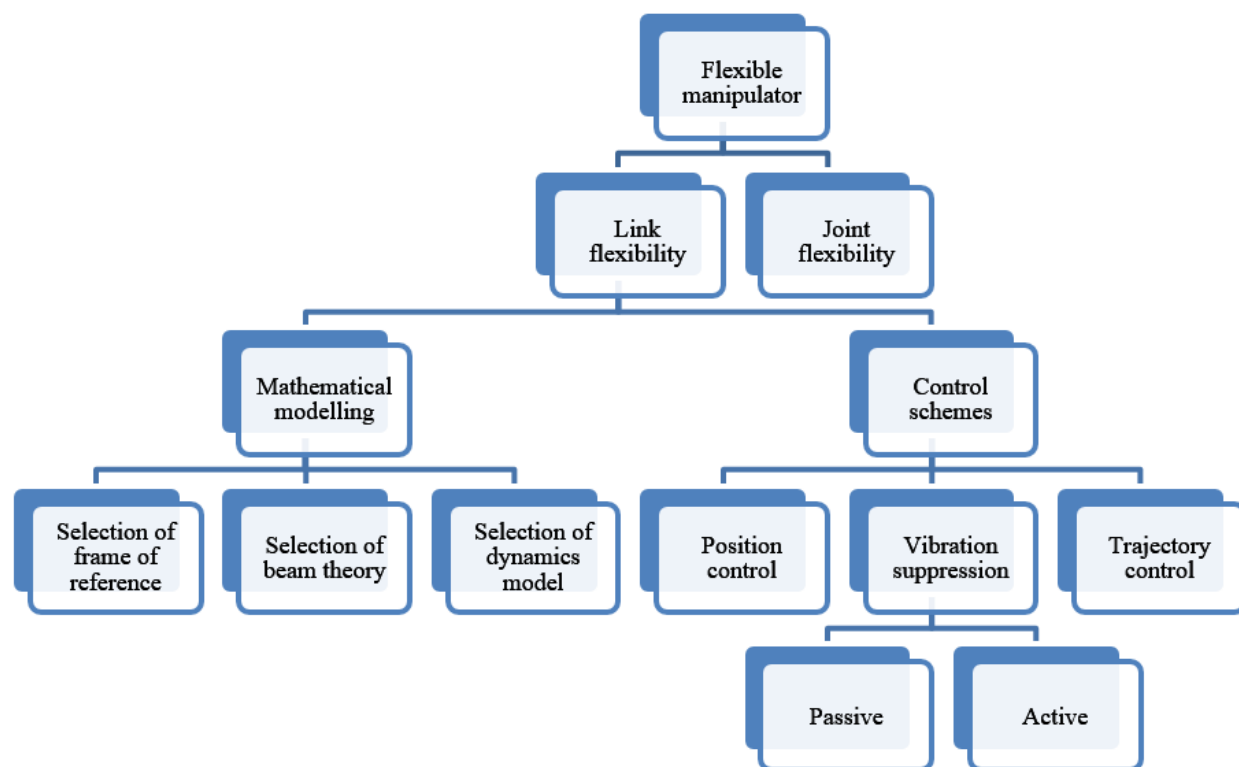
Fig 1. Study of flexible manipulator

There can be two types of flexibilities present within a flexible manipulator: joint flexibility and link flexibility. Only link flexibility has been considered in this paper. The present literature survey gives a brief description of work done by various authors regarding various approaches adopted by them during the mathematical modelling of flexible manipulators. It also provides information about the application of control schemes used by researchers for accurate tip position control of flexible manipulators. The techniques of vibration suppression and trajectory control are reviewed in Part-2 of the study. For the establishment of an accurate mathematical model, firstly the frame of reference is chosen. Three different types of frames of reference are available in the literature. These are: floating frame of reference, corotational frame of reference, and inertial frame of reference. The flexible links can be modelled using Euler-Bernoulli beam theory, Timoshenko beam theory, and non-linear beam theory available in the literature. If the length to thickness ratio of the beam is more than 10 then Euler-Bernoulli beam theory is suitable otherwise Timoshenko beam theory gives accurate results. It is advisable to incorporate the effect of geometrical stiffening during mathematical modelling for accurate results. Dwivedy and Eberhard [2006][1], Benosman and Vey [2004][2], Lochan et al. [2016][3], and Jing et al. [2019][4] provided a comprehensive review on various techniques used by different researchers for modelling and control of flexible multi-body systems. A deep and significant insight is provided by Tokhi and Azad [2017][5] with regards to the modelling, simulation and control of flexible-link manipulators that serve as a lighthouse for the researchers working in the area of flexible robotics. Many industrial applications require the use of moving platforms comprised of flexible mechanisms. Precise control of these platforms is necessary as it directly impacts the industrial production and hence the economy of any country[6].

The present survey is divided into the following sub-headings:

1. Dynamic modelling using assumed modes method (AMM)
2. Dynamic modelling using finite element method (FEM)
3. Other approaches for dynamic modelling
4. Control strategies for accurate tip positionin
5. Optimization techniques

2 Dynamic modelling using assumed modes method

The initial work on flexible robots started with the study of a single flexible link. Euler-Bernoulli (EB) beam theory was used for modelling the lateral vibrations. The solution to the elastic deformation of the flexible link was obtained using the assumed modes method (AMM). From the literature, it was found that the most preferred approach for developing the mathematical model of flexible manipulators is: Lagrangian dynamics. Book et al. [1975] [7] pioneered the research in this area. They developed the frequency domain model and the time domain model and used the linear feedback schemes for vibration control. The correct input torques for the flexible manipulator were found out by Luh et al. [1980] [8] using the resolved-motion-rate-controls (feedback) method. Book and Majette [1983] [9] determined the feedback gains of the controller using frequency-domain and state-space representations. Judd and Falkenburg [1985] [10] studied the control problem from the point of view of generating the desired trajectory and avoiding singularities simultaneously. Kanoh et al. [1986] [11] developed the technique of quasilinearization for implementing a non-linear PID controller. Bakr and Shabana [1986] [12] developed a method for the dynamic analysis of geometrically nonlinear and inertia-variant flexible systems using Lagrange's multipliers. The issues involved in utilizing end-point sensing for Two-Link Flexible manipulators were raised by Oakley and Cannon [1989] [13]. With the onset of the nineties, the researchers forayed into the research of multi-link flexible manipulators. Luca and Siciliano [1991] [14] derived a closed-form finite-dimensional dynamic model for planar multilink lightweight robots. Li and Sankar [1993] [15] developed methods for 'forward dynamics' computation of flexible manipulators that were systematic and efficient. DU et al. [1996] [16] focused upon geometric non-linearity caused by large elastic deflections of a flexible Euler-Bernoulli beam. Theodore and Ghosal [1997] [17] performed dynamic modelling of a flexible Euler Bernoulli link with a prismatic joint. Yuksel and Aksoy [2009] [18] scrutinized the bending vibrations of a flexible linear EB beam subjected to different base excitations. Ata et al. [2012] [19] showed the importance of inverse dynamic analysis for flexible manipulators. They also showed the effect of boundary conditions used during the mathematical modelling of flexible manipulators on actuator torques. Furthermore, various nonlinear phenomena like friction and gravity greatly affect the modal vibrations of a flexible manipulator [20]. Thus, it is desirable to have an accurate dynamic model of the flexible manipulator from the point of view of control effectiveness. A mathematical model of a Two-Link Flexible manipulator having two revolute joints undergoing both small bending and small torsional deformations is described by Mishra and Singh [2019] [21]. An investigation into the effect of payload on the modal response of the flexible manipulator is carried out by Kumar and Pratiher [2020] [22].
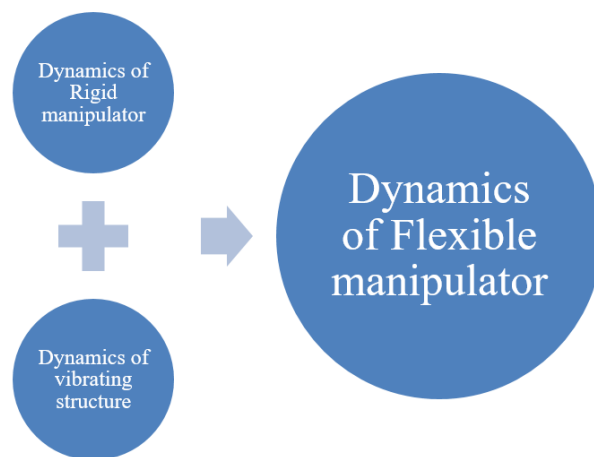

Fig 2. Procedure of obtaining the dynamic model of aflexible manipulator

From Figure 2, it can be seen that the dynamics of a flexible manipulator can be obtained by superimposing the dynamics of the rigid manipulator and the dynamics of a vibrating structure. The vibrating structure is usually taken as a beam which is a distributed parameter system. Distributed parameter systems exhibit infinite modes and hence it is necessary to use AMM. AMM helps in incorporating a sufficient number of modes that can help describe the vibration behaviour of the flexible system satisfactorily. But as the structure of the flexible manipulator becomes complex, it becomes difficult to use the AMM. The solution obtained using AMM is prone to the number of modes used and the type of boundary conditions chosen. From the literature, it is found that the mathematical model of a flexible manipulator depends upon the type of modelling technique used [3]. In the following section, a survey on dynamic modelling using FEM is provided.

3 Dynamic modelling using finite element method

While many researchers were using AMM for modelling flexible manipulators, Sunada and Dubowsky [1981][23] developed a method of dynamic analysis based upon finite elements. To reduce the size of dynamic equations obtained by FEM, component mode synthesis (CMS) was used by them. After 1986, most of the authors started to use the finite element method along with Lagrangian dynamics for dynamic modelling. Dado and Soni [1986][24] carried out the forward and the inverse dynamics analyses of flexible manipulators. Naganathan and Soni [1986][25] used the Newton-Euler approach for dynamic modelling of flexible links modelled as Timoshenko beam. The response of a Two-Link Flexible manipulator under gravity was observed by Usoro et al. [1986][26]. Bayo [1987][27] found the joint torque of a single flexible link for producing the desired tip motion. He also performed the frequency domain analysis of the flexible manipulator system. Simo and Vu-Quoc [1987][28] highlighted the role of nonlinear beam theory in transient dynamic analysis of flexible structures. Tzou and Wan [1990][29] implemented Rayleigh and viscoelastic damping for preparing the mathematical model of flexible manipulators. Chedmail et al. [1991][30] used modified Denavit-Hartenberg representation for kinematic analysis of a Two-Link flexible manipulator. Gaultier and Cleghorn [1992][31] developed and applied a spatially translating and rotating-beam finite element to model flexible manipulators. Alberts et al. [1992][32] evaluated the effectiveness of viscoelastic damping methods using finite element analysis. Hu and Ulsoy [1994][33] used Galerkin's approach for preparing the mathematical model of a rigid-flexible spherical-coordinate robot from the point of view of the design of a robust controller. Stylianou and Tabarrok [1994][34] developed governing equations of motion for an axially moving flexible beam and investigated its dynamic stability characteristics under the effects of physical damping, tip mass, tip support, and wall flexibility. They performed discretization using 'variable domain finite beam elements'. Theodore and Ghosal [2003][35] modelled the flexible links using finite element approach and discussed important issues like robustness and stability for the controller used for trajectory control of a flexible manipulator. Fotouhi [2007][36] studied the vibration analysis of a rotating flexible robotic arm undergoing large deformations. Before him all the authors considered only the small elastic deformation case. Grazioso et al. [2016][37] made use of FEM based upon geometrically exact beam theory for obtaining the most accurate mathematical model for the flexible manipulators. Bamdad and Feyzollahzadeh [2020][38] used discrete time transfer matrix method to analyze the large deformations of a flexible manipulator. They tried to reduce the computational complexity associated with such manipulators. The links of the flexible manipulator possess distributed parameters. Hence, to describe the dynamics of these links, discretization can be done using the finite element method. This involves the division of flexible links into some finite number of elements and finding the inertia and stiffness matrices that govern the dynamics of the system under consideration. Discretization of flexible links[21] can be done using Space-Frame Elements [Chandrupatla and Belegundu, 2012][39]. A 'space-frame element' has two nodes with each node having six degrees of freedom: three translational and three rotational. Figure 3 shows the procedure of obtaining a finite element model for the flexible links.
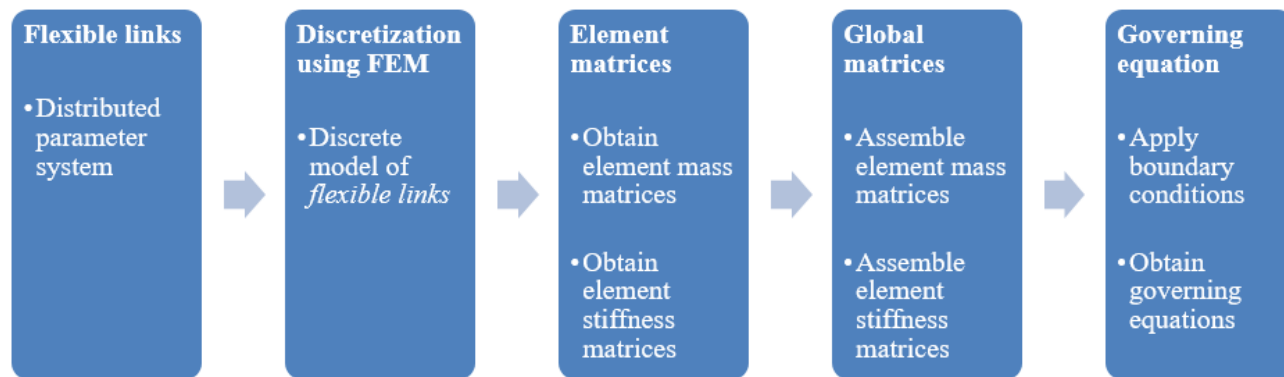
Flexible links	**Discretization using FEM**	**Element matrices**	**Global matrices**	**Governing equation**
• Distributed parameter system	• Discrete model of *flexible links*	• Obtain element mass matrices • Obtain element stiffness matrices	• Assemble element mass matrices • Assemble element stiffness matrices	• Apply boundary conditions • Obtain governing equations

Fig 3. Procedure of obtaining a finite element model for flexible links

The derivation of the finite element model depends upon the type of finite element chosen for discretization, the number of finite elements used for discretization, and the type of boundary conditions chosen. The use of FEM facilitates the easy handling of boundary conditions. The boundary conditions exhibited by the flexible manipulator are time-dependent. This is due to time-varying boundary conditions[40,41] for the flexible links. Besides that, the change in configuration of the manipulator due to its motion causes the eigenvalues to change.

4 Other approaches for dynamic modelling

Few authors used some different approaches for the dynamic modelling and vibration control of flexible robots. These include the ERLS (Equivalent Rigid Link System) approach, the redundant manipulator or controllable local degrees of freedom approach, the wave-based approach, and Kane's method. Marino and Spong [1986] [42] studied the nonlinear problem for a rigid single link manipulator having joint elasticity. They used feedback linearization, singular perturbation, integral manifold, and composite control for designing the control system. Matsuno et al. [1994] [43] developed the mathematical model of the decoupled bending and torsion vibrations of a flexible beam. This was one of the newest researches in the area of flexible robotics as it considered the torsional modes of vibration also along with the flexural modes of vibration. Until now, only the flexural vibrations were studied by various authors. The vibration suppression was achieved by two rotation motors. The authors also developed a feedback system using a dynamic compensator and implemented the Voigt type internal damping in their system. Elmaraghy et al. [1994] [44] discussed the joints and links flexibility of robots and control of robots by dynamic hybrid force and position control and a model reference adaptive control schemes. Ubertini [2000] [45] performed the numerical analysis of flexible link systems based upon an iterative scheme called Grobner basis. He used a new approach for developing the mathematical model of flexible links. This approach was called Equivalent Rigid Link System, ERLS. Yushu et al. [2008] [46] tried to suppress the vibration in flexible manipulators using controllable local degrees of freedom. Connor [2007] [47] applied wave-based control in lumped flexible robotic systems through position control and active vibration damping. Gao and Yun [2008] [48] improved the dynamic performance of a flexible redundant manipulator through its kinematics redundancy feature. Bian et al. [2009] [49] also suggested improving the motion accuracy of a flexible manipulator by suppressing the vibration using the kinematic redundancy feature. Zimmert and Sawodny [2010] [50] developed a spring-damper-based mathematical model and used a two-degree of freedom control approach for the active damping of the bending oscillation of a forklift's mast using the flatness technique. They used a feed-forward control strategy. Kermani [2010] [51] modelled the flexible link as Timoshenko beam and studied the effect of an actuator on the natural frequencies and mode shapes. Bian et al. [2011] [52] studied issues about reduction of vibration and improvement of mobility for the flexible redundant manipulator. They derived the dynamic model of the flexible manipulator using Kane's method. Vidoni et al. [2013] [53] developed a dynamic model of 3-D robots undergoing large displacements and small elastic deformations using the approach of ERLS. Gasparetto et al. [2013] [54] used FEM along with ERLS in modelling a flexible link.

5 Control strategies for accurate tip positioning

Many researchers adopted the approach of 'model-based control' for accurate tip positioning of the flexible manipulator. Morgul [1992] [55] performed the dynamic boundary force and torque control of an EB beam. He successfully showed exponential decay of vibrations using Lyapunov control. Hillsley and Yurkovich [1993] [56] achieved the endpoint position control of a Two-Link Flexible mechanism using input shaping control and acceleration feedback. Luca and Siciliano [1993] [57] designed a Lyapunov controller based on proportional-derivative (PD) feedback and feedforward control schemes for the flexible robots under the influence of gravity. Khorrami et al. [1995] [58] controlled the tip vibrations of flexible manipulators under varying payload conditions using adaptive control and input preshaping. Lu et al. [1996] [59] designed and implemented the controller and sensors based on a reduced-order model obtained through assumed mode analysis of flexible arm. Tso et al. [2003] [60] minimized tip deflection using a non-linear Lyapunov-type controller designed for trajectory control. Sun et al. [2004] [61] performed the active vibration control of a flexible link through the placement of segmented piezoelectric actuators. They used a PD-based feedback control scheme and command shaping together. The researchers also performed the dynamic modelling using Lagrangian dynamics and performed the stability analysis by virtual joint model. Monje et al. [2007] [62] studied the control of single link, lightweight flexible manipulators in the presence of payload using PD and fractional-order derivative controller. Pereira et al. [2008] [63] implemented integral resonant control (IRC) scheme for controlling the vibrations of a cantilever beam. Many authors used new control approaches that included integral resonant control, sliding mode control, adaptive control schemes, fuzzy-based control methods, etc. for vibration suppression and optimal trajectory control. Chaoui et al. [2009] [64] proposed a control strategy based on artificial neural networks (ANNs) while Korayem et al. [2009] [65] developed an algorithm to improve the maximum load-carrying capacity (MLCC) of flexible robots. They used a sliding mode controller and non-linear observer in their control system. Pereira et al. [2009] [66] also proposed a new control approach for a single-link flexible manipulator, based on Integral Resonant Control (IRC) scheme and thus addressed the two main problems which complicate the design of a control system viz. the high order of the system and the non-minimum phase dynamics that exist between the tip position and the input. Mamani et al. [2009] [67] applied a sliding mode control (SMC) design methodology based upon a unit vector approach and a state estimator based on polynomial approximations. Liu et al. [2010] [68] presented an FBFN-based (Fuzzy Basis Function Network) adaptive control algorithm for flexible manipulators. Diaz et al. [2010] [69]

dealt with active vibration cancellation in flexible manipulators using a robust input shaper. Takahashi and Sasaki [2010][70] investigated a feedback controller for tip angular position control of a single-link flexible robot arm. They considered the passivity of the flexible robot arm based on a distributed parameter system while designing the controller. Macchelli and Melchiorri [2011][71] developed a procedure for improving the transient response of a boundary-controlled non-linear flexible elastic beam through a feed-forward action. The boundary conditions were time-varying. Bossi et al. [2011][72] proposed the use of model predictive control (MPC) to control a fast mechanical system having a flexible arm with limited oscillations during the manoeuver. Chaoui and Sicard [2012][73] proposed a control structure based upon ANN that compensated the nonlinear friction terms and flexibility and accurately tracked the desired trajectory. It could be used to improve the static and dynamic performance of electromechanical systems. Kulakov et al. [2015][74] provided a new approach of force-torque control for service robots that involve contact with the environment. In their approach, they successfully transformed the joint control forces into artificial potential forces.

6 Optimization techniques

Various optimization schemes were used by the researchers to achieve the optimal control of flexible manipulators. Command shaping control strategy for vibration reduction was used by Alam et al. [2007][75]. They used particle swarm optimization (PSO) for achieving the optimal control. Zebin and Alam [2010][76] used FEM to derive a dynamic model of the Two-Link Flexible manipulator and developed a Fuzzy logic controller (FLC) optimized through a Genetic algorithm (GA) for input tracking and tip vibration control of the flexible manipulator. Yin et al. [2011][77] made use of the Nelder-Mead simplex (NM) algorithm and proposed a decomposed dynamic control (DDC) scheme that searched for the desired trajectory considering nonlinearity for a flexible manipulator. Abe and Komuro [2011][78] presented an energy-saving, open-loop control technique for a single-link flexible manipulator with point-to-point motion using ANN having Vector Evaluated Particle Swarm Optimization (VEPSO) as a learning algorithm. Deif et al. [2011][79] proposed a modified genetic algorithm (MGA) to optimize the feedback gains of a PD controller used for the control of position and vibration of a flexible arm. Loudini [2013][80] discussed the precise control of the end-point position of a planar single-link flexible manipulator using PD-type Mamdani FLC and GA optimized FLC. He modelled the flexible link using Timoshenko beam theory and considered Kelvin-Voigt damping in the system. The efficiency of the flexible manipulator is greatly influenced by the presence of residual vibrations. In one research, these vibrations are controlled using internal frictional dampers[81]. The optimization of frictional dampers was performed using PSO and some other techniques.

7 Conclusions from literature survey

In this section, conclusions from the literature survey will be provided. Most of the papers deal with the planar single-link flexible robotic arms with small 3D motions. For such links, a linear model is sufficient to describe the dynamic characteristics. A lot of research is going on, on non-linear models of flexible arms. The simple case of non-linearity in flexible arms is that of a two-link case which was described in a few papers. Furthermore, links having revolute joints were studied a lot. The establishment of an accurate mathematical model for a flexible manipulator can be done in the floating frame of reference, corotational frame of reference, and inertial frame of reference[82]. Table 1 describes these three frames of reference.

Table 1. Various types of reference frames used for mathematical modelling of flexible manipulators

S. No.	Type of reference frame	Description	Few latest references
1.	Floating frame of reference	● Each flexible link is assigned a different frame called the floating frame. ● The small elastic deformations are superimposed on large rigid body motion.	[Bajodah and Hodges, 2019][83]; [Xu et al, 2020][84]; [Vlase et al, 2020][85]; [Zhang et al, 2019][86]
2.	Corotational frame of reference	● Each finite element is assigned a corotational frame. ● The motion of the finite element is divided into two parts- rigid motion and elastic deformation. ● It applies to structures undergoing large displacements and small deformations.	[Ghorbani et al, 2019][87]; [Meier et al, 2019][88]; [Huang et al, 2018][89]; [Chhang et al, 2017][90]
3.	Inertial frame of reference	● The motion of all the flexible links is expressed concerning the global frame of reference. ● It applies to systems undergoing both large rotations and deformations.	[Gaonkar and Kulkarni, 2017][91]; [Warminski et al, 2021][92]; [Kloda et al, 2020][93]; [Guillot et al, 2020][94]; [Abdeljawad et al, 2020][95]; [Acar and Feeny, 2018][96]

Most of the authors focused upon the design of controllers according to the accurate dynamic model of the flexible arm. Researchers used both the AMM and FEM to model the flexible arm without compromising the accuracy. Some researchers tried to use intelligent control techniques like fuzzy logic, neural networks, and genetic algorithms for designing robust controllers as these do not require complicated mathematical modeling. There are two types of vibration control schemes: feedforward and feedback. It is also found that to describe the active control of vibrations, the state-space model can accurately represent the system without sacrificing its simplicity. Trajectory control using forward and inverse dynamics methods [Judd and Falkenburg, 1985][7], [Dado and Soni, 1986][19] and effect of gravity [Luca and Siciliano, 1993][134] on the motion of flexible links were studied in one or two papers. Few authors recommended the use of nonlinear beam theory ([Simo and Vu-Quoc, 1987][23], [Hu and Ulsoy, 1994][28]) to account for geometric nonlinearities and a nonlinear controller ([Marino and Spong, 1986][35], [Khorrami et al., 1995][135]) for effentrol of flexible manipulators. To improve the dynamic performance of flexible manipulators, few optimization techniques were also proposed. But, the most attractive one is the use of the 3kinematic redundancy feature for minimizing joint torques and vibration suppression. To deal with the nonlinear behaviour associated with nonlinear elasticity and large deflection effects, wave-based control may be suitable. In a rare case, both the bending and torsional vibrations of a single flexible link were controlled by using two electric motors. Mishra and Singh [2019][21] tried to achieve the vibration control of a Two-Link Flexible manipulator using both passive and active damping control methods. They considered both the bending and torsional vibrations of the flexible links. All these authors considered the small deformation of the flexible links. Banerjee and Lemak [1991][97] used a different approach for modelling the flexible links. They used Kane's method in association with AMM for obtaining the equations of motion of a Flexible multibody system. They introduced the concept of dynamic stiffness into their dynamic model and proposed that to obtain correct simulation results it is necessary to incorporate this. Besides that, Hu and Zhang [2015][98] also used this method for obtaining the equations of motion. The literature also gives a comparison between AMM and FEM ([Meghdari and Ghassempouri, 1994][99], [Theodore and Ghosal, 1995][100] and provides information about stability analysis of flexible manipulators. The proper placement of actuators made up of smart materials on flexible links plays a crucial role in damping the vibrations [Sun et al., 2004][61]. This approach is known as active vibration control. A survey related to active vibration control is provided in Part-2 of this review. The effect of axial foreshortening on bending vibrations, the effect of the actuator on vibration mode shapes, and the question of using the number of modes for accurately representing the vibratory motion of a flexible link are highlighted. A summary of the work done by various researchers in the area of flexible robotics is given below in Table 2.

Table 2. Breakthroughs in the field of flexible robotics since 1975

S. No.	Year	Breakthroughs
1	1975-1980	Starting years of research on flexible robots; Focus on the single flexible link; Dynamic modelling of flexible robots; Frequency domain and time domain analysis; Use of Lagrangian approach; Use of feedback control schemes
2	1981-1986	Dynamic modelling using FEM; Feedback control of vibrations; Inverse and forward dynamics; Consideration of joint flexibility, Development of other control methods- singular perturbation, composite control, etc., Consideration of the effect of gravity, Use of Newton-Euler approach in dynamic modelling
3	1986-1993	Use of nonlinear beam theory, Work starts on the nonlinear controller, Implementation of damping in dynamic modeling, Development of control strategy using input shaping, Dynamic modelling of multi-link flexible manipulators, Consideration of active damping using LQR (Linear Quadratic Regulator), Consideration of dynamic boundary conditions, Research on stability conditions, Feedforward control for gravity compensation
4	1994-2000	Hybrid force and position control, Adaptive control, Bending-torsion vibrations of a single flexible link, Comparison of AMM-FEM, Introduction of flexible link with prismatic joint, Stability characteristics considering the effect of damping and tip mass, Adaptive nonlinear control, Effect of payload variations, Effect of geometric nonlinearities, Concept of ERLS for dynamic modelling
5	2001-2008	Nonlinear Lyapunov control, Robustness and stability issues in model-based control, Use of smart materials for active vibration damping, Fractional-order controllers, Optimization techniques for controllers for vibration reduction, Redundant manipulators, Integral resonant control
6	2009-2015	Control strategy based on intelligent techniques, Sliding mode control, Nonlinear observer, Intelligent control techniques, Robust input shaping, Spring-damper based mathematical model, Study of transient response, Study of the effect of an actuator on vibration modes
7	2015 onwards	Compatibility equations for the flexible links, bifurcation studies, nonlinear analysis, trajectory control of flexible manipulator, vision-based control, particle-swarm optimization technique, sliding mode control, fuzzy logic control, artificial neural networks, model-predictive control, etc.

From the literature survey, it is found that different design methods were used by various researchers for preparing the mathematical model of the flexible manipulators. These are provided in Table 3.

Table 3. Design methods used by various researchers for modelling the flexible manipulators

S.No.	Design Method	Few researchers	Description
1	Euler-Bernoulli beam theory	[Book et al., 1975] [7], [DU et al., 1996] [16], [Macchelli and Melchiorri, 2011] [71], [Zimmert et al., 2012] [101], [Wang and Gao, 2016] [102]	It is a linear beam theory. Shear effects are not included. The theory holds good for long and slender beams.
2	Spring-damper system/ Lumped-parameter model	[Zimmert and Sawodney, 2010] [50]; [Oetinger et al, 2016] [103]; [Giorgio and Vescovo, 2018] [104]	The flexible structure is supposed to be composed of mass, spring, and damper units.
3	Timoshenko beam theory	[Naganathan and Soni, 1986] [25], [Kermani, 2010] [51], [Loudini, 2013] [80], [Wang and Gao, 2016] [102]	It considers the effect of shear deformation. It is suitable for beams with low aspect ratios. The theory predicts the transient response of beams very well.
4	Lagrangian-Finite Element Method	[Sunada and Dubowsky, 1981] [23], [Usoro et al., 1986] [26], [Bakr and Shabana, 1986] [12], [Bayo, 1987] [27], [Chedmail et al., 1991] [30], [Gaultier and Cleghorn, 1992] [31], [Stylianou and Tabarrok, 1994] [34], [Zebin and Alam, 2010] [76], [Mishra and Singh, 2019] [105]	It makes use of Lagrangian dynamics and discretization using the finite element method for finding out the governing equations of motion. It is an energy-based approach.
5	Controllable local degrees of freedom/ Redundant manipulator	[Gao et al., 2008] [48], [Bian et al., 2009] [49], [Bian et al., 2011] [52]	There is a main chain and a branch chain. The main chain consists of flexible links while the branch chain consists of rigid links. The branched-chain makes the manipulator kinematically redundant. By controlling the motion of the branch chain, the vibration of the main chain can be suppressed actively.
6	Wave-based approach	[Coleman, 1998] [106], [Connor, 2007] [47]; [Halevi, 2005] [107]; [Connor, 2011] [108]; [Connor and Habibi, 2015] [109]	This approach is based on the concept of mechanical waves. It is considered that the motion of the actuator launches mechanical waves into the system.
7	Equivalent Rigid Link System	[Ubertini, 2000] [45], [Vidoni et al., 2013] [53], [Gasparetto et al., 2013] [54], [Vidoni et al, 2017] [110]	It is suitable for flexible manipulators undergoing large displacements and small elastic deformations. It facilitates the kinematic decoupling of the compatibility equations at joints and the equations formed during the ERLS approach.
8	Lagrangian-Assumed modes method	[Oakley and Cannon, 1989] [13], [Luca and Siciliano, 1991] [14], [Li and Sankar, 1993] [15], [Mayo et al., 1995] [111], [Lu et al., 1996] [59], [Theodore and Ghosal, 1997] [17], [Ata et al., 2012] [19], [Loudini, 2013] [80], [Chen and Shan, 2020] [113], [Yang et al., 2017] [113]	In this method, Lagrangian dynamics is used to formulate the equations of motion. It is an energy-based approach and hence easy to use because energy is a scalar quantity. The vibratory motion of the flexible links is explained with the help of the assumed modes method.
9	Hamilton's principle	[Gaultier and Cleghorn, 1992] [31], [Najafi and Dehgolan, 2017] [114]	It is an energy-based approach that makes use of the total mechanical energy of a system to derive the equations of motion. Along with the equations of motion, the boundary conditions are also obtained during the derivation of equations of motion.

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Table 3 continued

10	Newton-Euler- FEM	[Naganathan and Soni, 1986][25], [Mohan, 2006][115], [Scaglioni et al., 2017][116]	The Newton-Euler approach is based upon Newton's second law of motion. FEM is used to find out the elastic deformation of the flexible links. This method can be easily implemented in computer programs.
11	Decoupled Natural Orthogonal Complement	[Mohan, 2006][115]; [Mohan and Saha, 2007][117]; [Saha et al, 2013][118]	It helps in obtaining the generalized inertia matrix required for recursive forward dynamic analysis of manipulators. It decomposes a given matrix into an upper block triangular and a block diagonal matrix. This helps in achieving good computational efficiency and numerical stability.
12	Kane's Method	Banerjee and Lemak [1991][97], Hu and Zhang [2015][98], [Jia et al, 2017][119]	This method is more convenient than the Lagrangian dynamics. It makes use of the concepts of generalized active forces and generalized inertial forces.

Meghdari and Ghassempouri [1994][99] performed a comparison of Lagrangian AMM and FEM. Theodore and Ghosal [1995][100] also did a comparison of the assumed modes method and finite element models but for flexible multi-link manipulators. The conclusion is that less number of mathematical operations is required for computation of inertia matrix in case of finite element model as compared to assumed modes model but with a large number of differential equations of motion. The use of FEM is recommended for manipulator links with complex geometries and having flexibilities and the use of AMM for manipulator links with uniform geometries and numerical simulation. The AMM has a faster convergence than FEM. Furthermore, the FEM overestimates the natural frequencies of the structure being considered.

Various control approaches were used by the researchers to achieve the tip position control of flexible manipulators. These approaches are tabulated in Table 4.

Table 4. Control approaches used by various researchers for control of flexible manipulators

S. No.	Control Approach	Few researchers	Description
1	Feedback control	[Book et al., 1975][7], [Luh et al., 1980][8], [Book and Majette, 1983][9], [Takahashi and Sasaki, 2010][70], [Zimmert et al., 2012][101], [Chaoui and Sicard, 2012][73]	This type of control employs a comparison between the controlled variable and the reference variable. The error thus generated is minimized by using a suitable controller. Any type of control system that utilizes feedback can be considered as feedback control.
2	Feedforward control	[Zimmert and Sawodney, 2010][50], [Macchelli and Melchiorri, 2011][71], [Chaoui and Sicard, 2012][73], Lismonde et al. [2019][120]	In feedforward control, the adjustment of the controlled variable is not based on error. It is based upon the knowledge of the process being controlled gained through either the mathematical model or the measurements. Feedforward control is based on a command signal from an external source.
3	Integral resonant control	[Pereira et al., 2008][63], [Pereira et al., 2009][66], [Bian et al, 2018][121]	This control scheme is based upon the fact that the positioning bandwidth of vibrating systems like beams can be improved significantly if the lower dominant modes of vibration are shifted to higher resonating frequencies.
4	Artificial neural network/ Adaptive control	[Khorrami et al., 1995][58], [Chaoui, 2009][64], [Liu et al., 2010][68], [Abe, 2011][122], [Chaoui and Sicard, 2012][73], [Rahmani and Mohammed, 2019][123]	An adaptive control system can modify its mode of operation and takes care of uncertainties that may adversely affect the controlled variable. It suitably modifies the control gains such that the system performance remains within the specified limits. Most of the time, the artificial neural network is used to predict any kind of uncertainty.

Continued on next page

	Table 4 continued		
5	Input shaping	[Hillsley and Yurkovich, 1993] [56], [Alam and Tokhi, 2007] [75], [Diaz et al., 2010] [69], [Chen and Huang, 2017] [124]	It is an open-loop control technique that utilizes a command signal for the reduction of vibrations of digital control systems. The command signal chosen is such that the vibration induced by the previous part of the signal is cancelled by the vibration induced by the latter part.
6	Sliding mode control	[Korayem et al., 2009] [65], [Mamani et al., 2009] [67], [Lochan et al., 2017] [125], [Yanmin et al., 2015] [126], [Yang and Tan, 2018] [127], [Aghababa and Saif, 2021] [128], [Homaeinezhad et al., 2021] [129]	It is a nonlinear control technique that makes use of discontinuous control signals for controlling the output of a nonlinear system. It is a kind of variable structure control.
7	Fuzzy logic control	[Liu et al., 2010] [68], [Zebin and Alam, 2010] [76], [Loudini, 2013] [80], [Zhang et al., 2020] [130]	In a fuzzy logic control, the control laws are specified with the help of linguistics. Fuzzy sets are used to establish these laws.
8	Model predictive control	[Bossi et al., 2011] [72], [Guaraci Bastos Jr., 2019] [131], [Schnelle and Eberhard, 2017] [132], [Ettefagh et al, 2019] [133], [Guechi et al., 2018] [134]	Model predictive control is used in process control. It satisfies a given set of constraints required to be fulfilled for the process being controlled. It makes use of the technique of system identification and can predict future events.
9	PD control	[Monje et al., 2007] [62], [Deif et al., 2011] [79], [Loudini, 2013] [80], [Yang et al., 2019] [135]	It is the most popular proportional-derivative control available in the literature. By incorporating integral action, the control technique gets modified to the famous proportional-derivative-integral control technique.
10	Wave-based control	[Connor et al., 2009] [136]	This type of control technique is based on the concept of generating mechanical waves. It is considered that the motion of the actuator launches mechanical waves into the system that may be used for control of vibrations.
11	Lyapunov control	[Morgul et al., 1992] [55], [Tso et al., 2003] [60]	It makes use of a Lyapunov function that is used to test the stability of a dynamical system.
12	Genetic Algorithm	[Zebin and Alam, 2010] [76], [Loudini, 2013] [80], [Obe et al, 2020] [137], [Sangdani et al, 2018] [138], [Wu et al, 2020] [139]	It is an optimization technique that is based on Darwin's theory of natural selection.
13	Singular Perturbation	[Yang et al., 2018] [140], [Yang et al., 2017] [113], [Hu and Zhang, 2015] [98], [Liu and He, 2018] [141]	It can be understood with the help of the following equation: $\varepsilon x^2 - 5x + 6 = 0$ where $\varepsilon \to 0$. If $\varepsilon = 1$ then the above equation has two roots, viz. $x = 2$ and $x = 3$, but when $\varepsilon = 0$ then the above equation has only one root and that is $x = 6/5$. When the parameter ε is very small, then the roots of the above quadratic equation are obtained using singular perturbation theory.

All these control approaches require the application of proper control and sensor system [142]. The current trends show the use of sliding mode control and command shaping in vibration control of flexible manipulators. The performances of the control systems used for flexible manipulators were optimized using various optimization techniques like Genetic Algorithm, Particle Swarm Optimization, Vector Evaluated Particle Swarm Optimization [143], Nelder-Mead Simplex Algorithm [144], and Fuzzy Basis Function Network adaptive control algorithm [145].

The present survey on flexible link manipulators identified few research gaps for manipulators having revolute joints. More research is required on flexible manipulators with prismatic joints. This may involve modelling the phenomenon of friction present within the system. Besides that, the effect of torsional vibrations [105,146] on positional accuracy must be studied. It is also found that the flexible manipulators exhibit nonlinear frequency that depends upon vibration amplitudes and mass ratios [Chen et al., 2019] [147]. This further increases the positional inaccuracy. Thus, it is necessary to damp the vibrations of links of a flexible manipulator so that positional accuracy can be maintained. This becomes significant for space robots used for performing on-orbit assembly missions [148]. Various other open-problems [149] related to the area of flexible multibody systems include the choice of a suitable reference frame (Table 2), definition of exact boundary conditions at the location where the

two successive flexible links are joined together, inclusion of a sufficient number of vibration modes into the dynamic model, convergence issue during finite element analysis of flexible links, the effect of centrifugal stiffening, and various non-linear terms present in the dynamics equation of the flexible manipulator that involves coupling between rigid and elastic motions. Besides that, certain modelling issues need to be addressed. These are explained in the following paragraphs.

The governing equations of motion of the flexible link manipulators can be described using the following matrix equation[105,150,151].

$$\left[\begin{array}{cc} (M_{rr})_{N\times N} & (M_{rf})_{N\times n} \\ (M_{fr})_{n\times N} & (M_{ff})_{n\times n} \end{array}\right]_{(N+n)\times(N+n)} \left\{\begin{array}{c} (\ddot{q}_r)_{N\times 1} \\ (\ddot{q}_f)_{n\times 1} \end{array}\right\}_{(N+n)\times 1} + \left[\begin{array}{c} (H_r)_{N\times 1} \\ (H_f)_{n\times 1} \end{array}\right]_{(N+n)\times 1} + \left[\begin{array}{c} (G_r)_{N\times 1} \\ (G_f)_{n\times 1} \end{array}\right]_{(N+n)\times 1} = \left\{\begin{array}{c} \tau_{N\times 1} \\ 0_{n\times 1} \end{array}\right\}_{(N+n)\times 1}$$

In the above matrix,

M_{rr} = mass matrix corresponding to rigid degrees of freedom
M_{ff} = mass matrix corresponding to flexible degrees of freedom
M_{rf} = row matrix representing coupling between rigid and flexible motions of the flexible manipulator
M_{fr} = column matrix representing coupling between flexible and rigid motions of the flexible manipulator
q_r = rigid degrees of freedom of the flexible manipulator representing the motions of the joints
q_f = flexible degrees of freedom of the flexible manipulator representing the elastic motions of the flexible links
H_r = centrifugal and Coriolis matrix corresponding to rigid motion
H_f = centrifugal and Coriolis matrix corresponding to flexible motion
G_r = gravity matrix corresponding to rigid motion
G_f = gravity matrix corresponding to flexible motion
τ = Joint torque vector
N = number of rigid degrees of freedom; it is equal to the number of joints
n = number of elastic degrees of freedom.

The matrix equation-3 is obtained by using integration. From equation-3, the equation of motion of joints of the flexible link manipulator can be expressed as follows.

$$(M_{rr})_{N\times N}(\ddot{q}_r)_{N\times 1} + (H_r)_{N\times 1} + (G_r)_{N\times 1} = (\tau)_{N\times 1} - (M_{rf})_{N\times n}(\ddot{q}_f)_{n\times 1}$$

Similarly, the equations of motion of the flexible links can be obtained as follows.

$$(M_{ff})_{n\times n}(\ddot{q}_f)_{n\times 1} + (H_f)_{n\times 1} + (G_f)_{n\times 1} = (0)_{n\times 1} - (M_{fr})_{n\times N}(\ddot{q}_r)_{N\times 1}$$

Equation-3b is an integro-partial differentiation equation that describes the vibratory motion of the flexible links. It is difficult to solve this equation. The solution of this equation is rarely found in the literature. Most of the researchers have solved the vibratory motion of the flexible links by solving the partial-differentiation equation using the assumed-modes method. In the case of lateral vibrations of the flexible links, the vibration of the flexible links has been described by the Euler-Bernoulli beam equation. Thus, the accuracy of the solution depends upon the number of 'assumed modes' incorporated into the equation. To describe the behavior of the flexible manipulator exactly, it is required to solve equation-3 as it is.

Literature also describes the use of the finite element method for obtaining the oscillatory behavior of flexible links. While using FEM, the order of the mass matrix increases proportionally to the number of nodes[151]. If a flexible link is divided into n finite elements having two nodes each then the maximum number of elastic degrees of freedom will be **6 x (n+1)**. Thus, the order of the inertia matrix described in equation-3 will be **N + 6 x (n+1)**. Now equation-3b will not be an integro-partial differentiation equation. It will then consist of a summation of various terms. The accuracy of the solution will now depend upon the number of finite elements (n) used. If n→∞, then equation-3b will become an integral equation as per the definition of 'integration' described using the ab-initio method. Thus, it is very cumbersome to solve equation-3 using FEM. Furthermore, results will not be accurate also. To improve the accuracy of results, a new error function must be defined as a minimization problem.

In equation-3, coupling terms M_{rf} and M_{fr} are present within the inertia matrix. These terms represent the coupling between rigid and elastic motions. It is required to study the effect of these terms[105] on simulation.

It is crucial to find out the exact boundary conditions of a flexible manipulator. Besides satisfying the geometric constraints, the boundary conditions must also satisfy the compatibility equations at the location where the two successive links are joined together. While using the assumed modes method, it is essential to correctly determine the boundary conditions. Various

researchers have described the types of boundary conditions to be used. In the present work, a new method for finding out the boundary conditions is proposed by using equation-3. The boundary equation can be represented as follows.

$$\left[M_{ff}\right]^*_{n\times n}\left(\ddot{q}_f\right)^*\}_{n\times 1} + \left[H_f\right]^*_{n\times 1} + \left(G_f\right)^*_{n\times 1} = (0)_{n\times 1} - \left(M_{fr}\right)^*_{n\times N}\left(\ddot{q}_r\right)\}_{N\times 1}$$

In equation-3c, the subscript '*' represents the motion of the point where the previous and the successive flexible links are joined together; $q_f{}^*$ represents the elastic motion at the tip of the previous link and the starting point of the successive link. Such a boundary condition is called the dynamic boundary condition. The exact solution of this equation will give information about the exact mode shapes of the flexible links.

Recent trends show that sliding mode control, variable structure control, model-predictive control, vision-based control[152] and, reinforcement learning control[153] offer some promising results in control of flexible manipulators. Through this review paper, the authors feel that awareness and interest will be created among the readers, and they will get motivation for doing further research in the area of flexible robotics.

References

1) Dwivedy SK, Eberhard P. Dynamic analysis of flexible manipulators, a literature review. *Mechanism and Machine Theory*. 2006;41(7):749–777. Available from: https://dx.doi.org/10.1016/j.mechmachtheory.2006.01.014. doi:10.1016/j.mechmachtheory.2006.01.014.

2) Benosman M, Vey GL. Control of flexible manipulators: A survey. *Robotica*. 2004;22(5):533–545. Available from: https://dx.doi.org/10.1017/s0263574703005642. doi:10.1017/s0263574703005642.

3) Lochan K, Roy BK, Subudhi B. A review on two-link flexible manipulators. *Annual Reviews in Control*. 2016;42:346–367. Available from: https://dx.doi.org/10.1016/j.arcontrol.2016.09.019. doi:10.1016/j.arcontrol.2016.09.019.

4) Jing Z, Xu Q, Huang J. A review on kinematic analysis and dynamic stable control of space flexible manipulators. *Aerospace Systems*. 2019;2(1):1–14. Available from: https://dx.doi.org/10.1007/s42401-018-00024-4. doi:10.1007/s42401-018-00024-4.

5) Tokhi MO, Azad AKM. Flexible Robot Manipulators: Modelling, Simulation and Control. 2nd ed. and others, editor;Institution of Engineering and Technology. 2017. doi:10.1049/PBCE086E.

6) Li X, Zhang L, Jiang B, Fang J, Zheng Y. Research trends in China for macro-micro motion platform for microelectronics manufacturing industry. *Journal of Advanced Mechanical Design, Systems, and Manufacturing*. 2021;15(3):1–14. Available from: https://dx.doi.org/10.1299/jamdsm.2021jamdsm0032. doi:10.1299/jamdsm.2021jamdsm0032.

7) Book WJ, Maizza-Neto O, Whitney DE. Feedback Control of Two Beam, Two Joint Systems With Distributed Flexibility. *Journal of Dynamic Systems, Measurement, and Control*. 1975;97(4):424–431. Available from: https://dx.doi.org/10.1115/1.3426959. doi:10.1115/1.3426959.

8) Luh J, Walker M, Paul R. Resolved-acceleration control of mechanical manipulators. *IEEE Transactions on Automatic Control*. 1980;25(3):468–474. Available from: https://dx.doi.org/10.1109/tac.1980.1102367. doi:10.1109/tac.1980.1102367.

9) Book WJ, Majette M. Controller Design for Flexible, Distributed Parameter Mechanical Arms Via Combined State Space and Frequency Domain Techniques. *Journal of Dynamic Systems, Measurement, and Control*. 1983;105(4):245–254. Available from: https://dx.doi.org/10.1115/1.3140666. doi:10.1115/1.3140666.

10) Judd RP, Falkenburg D. Dynamics of nonrigid articulated robot linkages. *IEEE Transactions on Automatic Control*. 1985;30(5):499–502. Available from: https://dx.doi.org/10.1109/tac.1985.1103978. doi:10.1109/tac.1985.1103978.

11) Kanoh H, Tzafestas S, Lee HG, Kalat J. Modelling and control of flexible robot arms. In: and others, editor. Proceedings of the IEEE Conference on Decision and Control. IEEE. 1986. doi:10.1109/CDC.1986.267312.

12) Bakr EM, Shabana AA. Geometrically nonlinear analysis of multibody systems. *Computers & Structures*. 1986;23(6):739–751. Available from: https://dx.doi.org/10.1016/0045-7949(86)90242-7. doi:10.1016/0045-7949(86)90242-7.

13) Oakley CM, Cannon RH. End-Point Control of a Two-Link Manipulator with a Very Flexible Forearm: Issues and Experiments. In: and others, editor. 1989 American Control Conference. IEEE. 1989;p. 1381–1388. doi:10.23919/ACC.1989.4790405.

14) Luca AD, Siciliano B. Closed-form dynamic model of planar multilink lightweight robots. *IEEE Transactions on Systems, Man, and Cybernetics*. 1991;21(4):826–839. Available from: https://dx.doi.org/10.1109/21.108300. doi:10.1109/21.108300.

15) Li CJ, Sankar TS. Systematic methods for efficient modeling and dynamics computation of flexible robot manipulators. *IEEE Transactions on Systems, Man, and Cybernetics*. 1993;23(1):77–95. Available from: https://dx.doi.org/10.1109/21.214769. doi:10.1109/21.214769.

16) Du H, Lim MK, Liew KM. A nonlinear finite element model for dynamics of flexible manipulators. *Mechanism and Machine Theory*. 1996;31(8):1109–1119. Available from: https://dx.doi.org/10.1016/0094-114x(96)84602-7. doi:10.1016/0094-114x(96)84602-7.

17) Theodore RJ, Ghosal A. Modeling of flexible-link manipulators with prismatic joints. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*. 1997;27(2):296–305. Available from: https://dx.doi.org/10.1109/3477.558822. doi:10.1109/3477.558822.

18) Yüksel Ş, Aksoy TM. Flexural vibrations of a rotating beam subjected to different base excitations. *Gazi University Journal of Science*. 2009;22(1):33–40. Available from: https://dergipark.org.tr/en/pub/gujs/issue/7388/96768.

19) Ata AA, Fares WF, Sa'adeh MY. Dynamic Analysis of a Two-link Flexible Manipulator Subject to Different Sets of Conditions. *Procedia Engineering*. 2012;41:1253–1260. Available from: https://dx.doi.org/10.1016/j.proeng.2012.07.308. doi:10.1016/j.proeng.2012.07.308.

20) Zhang F, Yuan Z. The Study of Dynamic Modeling and Multivariable Feedback Control for Flexible Manipulators with Friction Effect and Terminal Load. *Sensors*. 2021;21(4):1522–1522. Available from: https://dx.doi.org/10.3390/s21041522. doi:10.3390/s21041522.

21) Mishra N, Singh SP. Hybrid vibration control of a Two-Link Flexible manipulator. *SN Applied Sciences*. 2019;1(7). Available from: https://dx.doi.org/10.1007/s42452-019-0691-1. doi:10.1007/s42452-019-0691-1.

22) Kumar P, Pratiher B. Influences of generic payload and constraint force on modal analysis and dynamic responses of flexible manipulator. *Mechanics Based Design of Structures and Machines*. 2020;0(0):1–19. Available from: https://dx.doi.org/10.1080/15397734.2020.1766980. doi:10.1080/15397734.2020.1766980.

23) Sunada W, Dubowsky S. The Application of Finite Element Methods to the Dynamic Analysis of Flexible Spatial and Co-Planar Linkage Systems. *Journal of Mechanical Design*. 1981;103(3):643–651. Available from: https://dx.doi.org/10.1115/1.3254965. doi:10.1115/1.3254965.

24) Dado M, Soni AH. A generalized approach for forward and inverse dynamics of elastic manipulators. In: and others, editor. 1986 IEEE International Conference on Robotics and Automation. Institute of Electrical and Electronics Engineers. 1986.

25) Naganathan G, Soni AH. Non-linear flexibility studies for spatial manipulators. In: and others, editor. 1986 IEEE International Conference on Robotics and Automation. Institute of Electrical and Electronics Engineers. 1986;p. 359–64. doi:10.1109/ROBOT.1986.1087719.

26) Usoro PB, Nadira R, Mahil SS. A Finite Element/Lagrange Approach to Modeling Lightweight Flexible Manipulators. *Journal of Dynamic Systems, Measurement, and Control*. 1986;108(3):198–205. Available from: https://dx.doi.org/10.1115/1.3143768. doi:10.1115/1.3143768.

27) Bayo E. A finite-element approach to control the end-point motion of a single-link flexible robot. *Journal of Robotic Systems*. 1987;4(1):63–75. Available from: https://dx.doi.org/10.1002/rob.4620040106. doi:10.1002/rob.4620040106.

28) Simo JC, Vu-Quoc L. The role of non-linear theories in transient dynamic analysis of flexible structures. *Journal of Sound and Vibration*. 1987;119(3):487–508. Available from: https://dx.doi.org/10.1016/0022-460x(87)90410-x. doi:10.1016/0022-460x(87)90410-x.

29) Tzou HS, Wan GC. Distributed structural dynamics control of flexible manipulators —I. Structural dynamics and distributed viscoelastic actuator. *Computers & Structures*. 1990;35(6):669–677. Available from: https://dx.doi.org/10.1016/0045-7949(90)90412-u. doi:10.1016/0045-7949(90)90412-u.

30) Chedmail P, Aoustin Y, Chevallereau C. Modelling and control of flexible robots. *International Journal for Numerical Methods in Engineering*. 1991;32(8):1595–1619. Available from: https://dx.doi.org/10.1002/nme.1620320806. doi:10.1002/nme.1620320806.

31) Gaultier PE, Cleghorn WL. A spatially translating and rotating beam finite element for modeling flexible manipulators. *Mechanism and Machine Theory*. 1992;27(4):415–433. Available from: https://dx.doi.org/10.1016/0094-114x(92)90033-e. doi:10.1016/0094-114x(92)90033-e.

32) Alberts TE, Xia H, Chen Y. Dynamic Analysis to Evaluate Viscoelastic Passive Damping Augmentation for the Space Shuttle Remote Manipulator System. *Journal of Dynamic Systems, Measurement, and Control*. 1992;114(3):468–475. Available from: https://dx.doi.org/10.1115/1.2897370. doi:10.1115/1.2897370.

33) Hu FL, Ulsoy AG. Dynamic Modeling of Constrained Flexible Robot Arms for Controller Design. *Journal of Dynamic Systems, Measurement, and Control*. 1994;116(1):56–65. Available from: https://dx.doi.org/10.1115/1.2900681. doi:10.1115/1.2900681.

34) Stylianou M, Tabarrok B. Finite Element Analysis Of An Axially Moving Beam, Part II: Stability Analysis. *Journal of Sound and Vibration*. 1994;178(4):455–481. Available from: https://dx.doi.org/10.1006/jsvi.1994.1498. doi:10.1006/jsvi.1994.1498.

35) Theodore RJ, Ghosal A. Robust control of multilink flexible manipulators. *Mechanism and Machine Theory*. 2003;38(4):367–377. Available from: https://dx.doi.org/10.1016/s0094-114x(02)00125-8. doi:10.1016/s0094-114x(02)00125-8.

36) Fotouhi R. Dynamic analysis of very flexible beams. *Journal of Sound and Vibration*. 2007;305(3):521–533. Available from: https://dx.doi.org/10.1016/j.jsv.2007.01.032. doi:10.1016/j.jsv.2007.01.032.

37) Grazioso S, Sonneville V, Gironimo GD, Bauchau O, Siciliano B. A nonlinear finite element formalism for modelling flexible and soft manipulators. *2016 IEEE International Conference on Simulation, Modeling, and Programming for Autonomous Robots (SIMPAR)*. 2016. doi:10.1109/SIMPAR.2016.7862394.

38) Bamdad M, Feyzollahzadeh M. Computational efficient discrete time transfer matrix method for large deformation analysis of flexible manipulators. *Mechanics Based Design of Structures and Machines*. 2020;0(0):1–23. Available from: https://dx.doi.org/10.1080/15397734.2020.1830800. doi:10.1080/15397734.2020.1830800.

39) Tirupathi RC, Ashok DB. Introduction to Finite Elements in Engineering. and others, editor;New Delhi. PHI Learning Private Limited. 2012.

40) Kim YW. Dynamic analysis of Timoshenko beam subjected to support motions. *Journal of Mechanical Science and Technology*. 2016;30(9):4167–4176. Available from: http://link.springer.com/10.1007/s12206-016-0828-8.

41) Omolofe B, Adedowole A. Response characteristics of non-uniform beam with time-dependent boundary conditions and under the actions of travelling distributed masses. *Journal of Applied Mathematics and Computational Mechanics*. 2017;16(2):77–99. Available from: https://dx.doi.org/10.17512/jamcm.2017.2.07. doi:10.17512/jamcm.2017.2.07.

42) Marino R, Spong MW. Nonlinear control techniques for flexible joint manipulators: A single link case study. In: and others, editor. 1986 IEEE International Conference on Robotics and Automation. Institute of Electrical and Electronics Engineers. 1986;p. 1030–1036. doi:10.1109/ROBOT.1986.1087613.

43) Matsuno F, Murachi T, Sakawa Y. Feedback control of decoupled bending and torsional vibrations of flexible beams. *Journal of Robotic Systems*. 1994;11(5):341–353. Available from: https://dx.doi.org/10.1002/rob.4620110502. doi:10.1002/rob.4620110502.

44) ElMaraghy WH, ElMaraghy HA, Zaki A, Massoud A. Design and Control of Robots with Flexibilities. *CIRP Annals*. 1994;43(1):359–362. Available from: https://dx.doi.org/10.1016/s0007-8506(07)62231-8. doi:10.1016/s0007-8506(07)62231-8.

45) Ubertini F. A contribution to the analysis of flexible link systems. *International Journal of Solids and Structures*. 2000;37(7):969–990. Available from: https://dx.doi.org/10.1016/s0020-7683(99)00106-7. doi:10.1016/s0020-7683(99)00106-7.

46) Yushu B, Zhihui G, Chao Y. Vibration Reduction of Open-chain Flexible Manipulators by Optimizing Independent Motions of Branch Links. *Chinese Journal of Aeronautics*. 2008;21(1):79–85. Available from: https://dx.doi.org/10.1016/s1000-9361(08)60011-4. doi:10.1016/s1000-9361(08)60011-4.

47) Connor WJO. Wave-based analysis and control of lump-modeled flexible robots. *IEEE Transactions on Robotics*. 2007;23(2):342–52. doi:10.1109/TRO.2007.895061.

48) Gao Z, Yun C, Bian Y. Minimizing joint-torques of the flexible redundant manipulator on the premise of vibration suppression. *International Journal of Intelligent Computing and Cybernetics*. 2008;1(4):634–645. Available from: https://dx.doi.org/10.1108/17563780810919168. doi:10.1108/17563780810919168.

49) Bian Y, Gao Z, Yun C. Motion control of the flexible manipulator via controllable local degrees of freedom. *Nonlinear Dynamics*. 2009;55(4):373–384. Available from: https://dx.doi.org/10.1007/s11071-008-9370-2. doi:10.1007/s11071-008-9370-2.

50) Zimmert N, Sawodny O. Active damping control for bending oscillations of a forklift mast using flatness based techniques. In: and others, editor. Proceedings of the 2010 American Control Conference. IEEE. 2010;p. 1538–43. doi:10.1109/ACC.2010.5531299.

51) Kermani MR. Analytic Modal Analyses of a Partially Strengthened Timoshenko Beam. *IEEE Transactions on Control Systems Technology*. 2010;18(4):850–858. Available from: https://doi.org/10.1007/s11071-010-9897-x.

52) Bian Y, Gao Z, Yun C. Study on vibration reduction and mobility improvement for the flexible manipulator via redundancy resolution. *Nonlinear Dynamics*. 2011;65(4):359–368. Available from: https://dx.doi.org/10.1007/s11071-010-9897-x. doi:10.1007/s11071-010-9897-x.

53) Vidoni R, Gasparetto A, Giovagnoni M. Design and implementation of an ERLS-based 3-D dynamic formulation for flexible-link robots. *Robotics and Computer-Integrated Manufacturing*. 2013;29(2):273–282. Available from: https://dx.doi.org/10.1016/j.rcim.2012.07.008. doi:10.1016/j.rcim.2012.07.008.

54) Gasparetto A, Moosavi AK, Boscariol P, Giovagnoni M. Experimental Validation of a Dynamic Model for Lightweight Robots. *International Journal of Advanced Robotic Systems*. 2013;10(3):182–182. Available from: https://dx.doi.org/10.5772/55955. doi:10.5772/55955.

55) Omer M. Dynamic boundary control of a Euler-Bernoulli beam. *IEEE Transactions on Automatic Control*. 1992;37(5):639–681. Available from: https://ieeexplore.ieee.org/document/135504.

56) Hillsley KL, Yurkovich S. Vibration control of a two-link flexible robot arm. *Dynamics and Control*. 1993;3(3):261–280. Available from: https://dx.doi.org/10.1007/bf01972699. doi:10.1007/bf01972699.

57) de Luca A, Siciliano B. Regulation of flexible arms under gravity. *IEEE Transactions on Robotics and Automation*. 1993;9(4):463–467. Available from: https://dx.doi.org/10.1109/70.246057. doi:10.1109/70.246057.

58) Khorrami F, Jain S, Tzes A. Experimental results on adaptive nonlinear control and input preshaping for multi-link flexible manipulators. *Automatica*. 1995;31(1):83–97. Available from: https://dx.doi.org/10.1016/0005-1098(94)00070-y. doi:10.1016/0005-1098(94)00070-y.

59) Lu J, Shafiq M, Yahagi T. Vibration control of flexible robotic arms using robust model matching control. In: Proceedings of 4th IEEE International Workshop on Advanced Motion Control - AMC '96 - MIE. IEEE. 1996. doi:10.1109/AMC.1996.509327.

60) Tso SK, Yang TW, Xu WL, Sun ZQ. Vibration control for a flexible-link robot arm with deflection feedback. *International Journal of Non-Linear Mechanics*. 2003;38(1):51–62. Available from: https://dx.doi.org/10.1016/s0020-7462(01)00040-3. doi:10.1016/s0020-7462(01)00040-3.

61) Sun D, Mills JK, Shan J, Tso SK. A PZT actuator control of a single-link flexible manipulator based on linear velocity feedback and actuator placement. *Mechatronics*. 2004;14(4):381–401. Available from: https://dx.doi.org/10.1016/s0957-4158(03)00066-7. doi:10.1016/s0957-4158(03)00066-7.

62) Monje CA, Ramos F, Vinagre BM, Feliu V. Tip position control of a lightweight flexible manipulator using a fractional order controller. *IET Control Theory & Applications*. 2007;1(5):1451–1460. Available from: https://dx.doi.org/10.1049/iet-cta:20060477. doi:10.1049/iet-cta:20060477.

63) Pereira E, Moheimani SOR, Aphale SS. Analog implementation of an integral resonant control scheme. *Smart Materials and Structures*. 2008;17(6):067001–067001. Available from: https://dx.doi.org/10.1088/0964-1726/17/6/067001. doi:10.1088/0964-1726/17/6/067001.

64) Chaoui H, Sicard P, Gueaieb W. ANN-Based Adaptive Control of Robotic Manipulators With Friction and Joint Elasticity. *IEEE Transactions on Industrial Electronics*. 2009;56(8):3174–3187. Available from: https://dx.doi.org/10.1109/tie.2009.2024657. doi:10.1109/tie.2009.2024657.

65) Korayem MH, Haghighi R, Korayem AH, Nikoobin A, Alamdari A. Determining maximum load carrying capacity of planar flexible-link robot: closed-loop approach. *Robotica*. 2010;28(7):959–973. Available from: https://dx.doi.org/10.1017/s0263574709990798. doi:10.1017/s0263574709990798.

66) Pereira E, Aphale SS, Feliu V, Moheimani SOR. A hybrid control strategy for vibration damping and precise tip-positioning of a single-link flexible manipulator. In: and others, editor. 2009 IEEE International Conference on Mechatronics. IEEE. 2009. doi:10.1109/ICMECH.2009.4957128.

67) Mamani G, Silva JMAD, Feliu-Batlle V. Least squares state estimator based sliding mode control of a very lightweight single-link flexible robot arm. *2009 IEEE International Conference on Mechatronics*. 2009. doi:10.1109/ICMECH.2009.4957205.

68) Liu G, Zhen WD, Du T, Yang J. Adaptive Control of Robotic Dynamic Systems Based on Fuzzy Basis Function Networks. *2010 International Conference on Artificial Intelligence and Computational Intelligence*. 2010;2:263–269. doi:10.1109/AICI.2010.177.

69) Díaz IM, Pereira E, Feliu V, Cela JJL. Concurrent Design of Multimode Input Shapers and Link Dynamics for Flexible Manipulators. *IEEE/ASME Transactions on Mechatronics*. 2010;15(4):646–651. Available from: https://dx.doi.org/10.1109/tmech.2009.2031434. doi:10.1109/tmech.2009.2031434.

70) Takahashi K, Sasaki M. Remarks on tip angular position control of single-link flexible robot arm using modified Lyapunov function. *2010 IEEE International Conference on Mechatronics and Automation*. 2010;p. 1024–1033. doi:10.1109/ICMA.2010.5588105.

71) Macchelli A, Melchiorri C. A formal method for improving the transient behaviour of a non-linear flexible link. *Mathematical and Computer Modelling of Dynamical Systems*. 2011;17(1):3–18. Available from: https://dx.doi.org/10.1080/13873954.2010.537508. doi:10.1080/13873954.2010.537508.

72) Bossi L, Rottenbacher C, Mimmi G, Magni L. Multivariable predictive control for vibrating structures: An application. *Control Engineering Practice*. 2011;19(10):1087–1098. Available from: https://dx.doi.org/10.1016/j.conengprac.2011.05.003. doi:10.1016/j.conengprac.2011.05.003.

73) Chaoui H, Sicard P. Adaptive neural network control of flexible-joint robotic manipulators with friction and disturbance. In: and others, editor. IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society. IEEE. 2012;p. 2644–2653. doi:10.1109/IECON.2012.6389159.

74) Kulakov F, Alferov GV, Efimova P, Chernakova S, Shymanchuk D. Modeling and control of robot manipulators with the constraints at the moving objects. *2015 International Conference "Stability and Control Processes" in Memory of VI Zubov (SCP)*. 2015;p. 102–105. doi:10.1109/SCP.2015.7342075.

75) Alam MS, Tokhi MO. Design of command shaper using gain-delay units and particle swarm optimization for vibration control of flexible systems. *International Journal of Acoustics and Vibration*. 2007;12(3):99–108. Available from: https://www.researchgate.net/publication/273120977_Design_of_Command_Shaper_using_Gain-delay_Units_and_Particle_Swarm_Optimisation_Algorithm_for_Vibration_Control_of_Flexible_Systems.

76) Zebin T, Alam MS. Dynamic modeling and fuzzy logic control of a two-link flexible manipulator using genetic optimization techniques. *2010 13th International Conference on Computer and Information Technology (ICCIT)*. 2010;p. 418–441. doi:10.1109/ICCITECHN.2010.5723894.

77) Yin H, Kobayashi Y, Hoshino Y, Emaru T. Decomposed Dynamic Control for Flexible Manipulator in Consideration of Nonlinearity -Flexible Dynamic Control-. *Journal of System Design and Dynamics*. 2011;5(2):219–230. Available from: https://dx.doi.org/10.1299/jsdd.5.219. doi:10.1299/jsdd.5.219.

78) Abe A, Komuro K. An energy saving open-loop control technique for flexible manipulators. *2011 IEEE International Conference on Mechatronics and Automation*. 2011;p. 416–421. doi:10.1109/ICMA.2011.5985694.

79) Deif S, Tawfik M, Kamal HA. Vibration and Position Control of a Flexible Manipulator using a PD- tuned Controller with Modified Genetic Algorithm. In: and others, editor. CCTA 2011;vol. 15-17. 2011. Available from: https://www.researchgate.net/publication/231618342_Vibration_and_Position_Control_of_a_Flexible_Manipulator_using_a_PD-_tuned_Controller_with_Modified_Genetic_Algorithm.

80) Loudini M. Modelling and Intelligent Control of an Elastic Link Robot Manipulator. *International Journal of Advanced Robotic Systems*. 2013;10(1):81–81. Available from: https://dx.doi.org/10.5772/51102. doi:10.5772/51102.

81) Biglari H, Golmohammadi M, Hayati S, Hemmati S. Vibration reduction of a flexible robot link using a frictional damper. *Journal of Vibration and Control*. 2021;27(9-10):985–997. Available from: https://dx.doi.org/10.1177/1077546320936092. doi:10.1177/1077546320936092.

82) Wasfy TM, Noor AK. Computational strategies for flexible multibody systems. *Applied Mechanics Reviews*. 2003;56(6):553–613. Available from: https://dx.doi.org/10.1115/1.1590354. doi:10.1115/1.1590354.

83) Bajodah AH, Hodges DH. Canonical Kane's Equations of Motion for Discrete Dynamical Systems. *AIAA Journal*. 2019;57(10):4226–4240. Available from: https://dx.doi.org/10.2514/1.j057603. doi:10.2514/1.j057603.

84) Xu Y, Teng Z, Yao J, Zhou Y, Zhao Y. Elastodynamic analysis of a novel motion-decoupling forging manipulator. *Mechanism and Machine Theory*. 2020;147:103771–103771. Available from: https://dx.doi.org/10.1016/j.mechmachtheory.2019.103771. doi:10.1016/j.mechmachtheory.2019.103771.

85) Vlase S, Negrean I, Marin M, Năstac S. Kane's Method-Based Simulation and Modeling Robots with Elastic Elements, Using Finite Element Method. *Mathematics*. 2020;8(5):805–805. Available from: https://dx.doi.org/10.3390/math8050805. doi:10.3390/math8050805.

86) Zhang Z, Wang T, Shabana AA. Development and implementation of geometrically accurate reduced-order models: Convergence properties of planar beams. *Journal of Sound and Vibration*. 2019;439:457–478. Available from: https://dx.doi.org/10.1016/j.jsv.2018.06.005. doi:10.1016/j.jsv.2018.06.005.

87) Ghorbani H, Tarvirdizadeh B, Alipour K, Hadi A. Near-time-optimal motion control for flexible-link systems using absolute nodal coordinates formulation. *Mechanism and Machine Theory*. 2019;140:686–710. Available from: https://dx.doi.org/10.1016/j.mechmachtheory.2019.06.032. doi:10.1016/j.mechmachtheory.2019.06.032.

88) Meier C, Popp A, Wall WA. Geometrically Exact Finite Element Formulations for Slender Beams: Kirchhoff–Love Theory Versus Simo–Reissner Theory. *Archives of Computational Methods in Engineering*. 2019;26(1):163–243.

89) Huang CC, Fujii F, Hsiao KM. An explicit algorithm for geometrically nonlinear transient analysis of spatial beams using a corotational total Lagrangian finite element formulation. *Computers & Structures*. 2018;200:68–85. Available from: https://dx.doi.org/10.1016/j.compstruc.2018.01.011. doi:10.1016/j.compstruc.2018.01.011.

90) Chhang S, Sansour C, Hjiaj M, Battini JM. An energy-momentum co-rotational formulation for nonlinear dynamics of planar beams. *Computers & Structures*. 2017;187:50–63. Available from: https://dx.doi.org/10.1016/j.compstruc.2017.03.021. doi:10.1016/j.compstruc.2017.03.021.

91) Gaonkar AK, Kulkarni SS. Model order reduction for dynamic simulation of slender beams undergoing large rotations. *Computational Mechanics*. 2017;59(5):809–829. Available from: https://dx.doi.org/10.1007/s00466-017-1374-7. doi:10.1007/s00466-017-1374-7.

92) Warminski J, Kloda L, Latalski J, Mitura A, Kowalczuk M. Nonlinear vibrations and time delay control of an extensible slowly rotating beam. *Nonlinear Dynamics*. 2021;103(4):3255–3281. Available from: https://dx.doi.org/10.1007/s11071-020-06079-3. doi:10.1007/s11071-020-06079-3.

93) Kloda L, Lenci S, Warminski J. Hardening vs. softening dichotomy of a hinged-simply supported beam with one end axial linear spring: Experimental and numerical studies. *International Journal of Mechanical Sciences*. 2020;178:105588–105588. Available from: https://dx.doi.org/10.1016/j.ijmecsci.2020.105588. doi:10.1016/j.ijmecsci.2020.105588.

94) Guillot V, Givois A, Colin M, Thomas O, Savadkoohi AT, Lamarque CH. Theoretical and experimental investigation of a 1:3 internal resonance in a beam with piezoelectric patches. *Journal of Vibration and Control*. 2020;26(13-14):1119–1132. Available from: https://dx.doi.org/10.1177/1077546320910536. doi:10.1177/1077546320910536.

95) Abdeljawad T, Mahariq I, Kavyanpoor M, Ghalandari M, Nabipour N. Identification of nonlinear normal modes for a highly flexible beam. *Alexandria Engineering Journal*. 2020;59(4):2419–2427. Available from: https://dx.doi.org/10.1016/j.aej.2020.03.004. doi:10.1016/j.aej.2020.03.004.

96) Acar GD, Feeny BF. Bend-bend-twist vibrations of a wind turbine blade. *Wind Energy*. 2018;21(1):15–28. Available from: https://dx.doi.org/10.1002/we.2141. doi:10.1002/we.2141.

97) Banerjee AK, Lemak ME. Multi-flexible body dynamics capturing motion-induced stiffness. *Journal of Applied Mechanics*. 1991;58(3):766–75.

98) Hu Q, Zhang J. Maneuver and vibration control of flexible manipulators using variable-speed control moment gyros. *Acta Astronautica*. 2015;113:105–119. Available from: https://dx.doi.org/10.1016/j.actaastro.2015.03.026. doi:10.1016/j.actaastro.2015.03.026.

99) Meghdari G. Dynamics of flexible manipulators. *Journal of Engineering, Islamic Republic of Iran*. 1994;7:19–32.

100) Theodore RJ, Ghosal A. Comparison of the Assumed Modes and Finite Element Models for Flexible Multilink Manipulators. *The International Journal of Robotics Research*. 1995;14(2):91–111. Available from: https://dx.doi.org/10.1177/027836499501400201. doi:10.1177/027836499501400201.

101) Zimmert N, Pertsch A, Sawodny O. 2-DOF Control of a Fire-Rescue Turntable Ladder. *IEEE Transactions on Control Systems Technology*. 2012;20(2):438–452. Available from: https://dx.doi.org/10.1109/tcst.2011.2116021. doi:10.1109/tcst.2011.2116021.

102) Wang FY, Gao Y. On Frequency Sensitivity and Mode Orthogonality of Flexible Robotic Manipulators. *IEEE/CAA Journal of Automatica Sinica*. 2016;3(4):394–397.

103) Oetinger D, Knierim KL, Sawodny O. Parameter identification and controller design for high-rack feeder systems and fork lifts. *2016 IEEE International Conference on Automation Science and Engineering (CASE)*. 2016;p. 509–523.

104) Giorgio I, Vescovo DD. Non-Linear Lumped-Parameter Modeling of Planar Multi-Link Manipulators with Highly Flexible Arms. *Robotics*. 2018;7(4):60–60. Available from: https://dx.doi.org/10.3390/robotics7040060. doi:10.3390/robotics7040060.

105) Mishra N, Singh SP. Dynamic Analysis of Two-Link Flexible Manipulator Using Fem Undergoing Bending-Torsional Vibrations. *ACTA TECHNICA NAPOCENSIS - Series: APPLIED MATHEMATICS, MECHANICS, and ENGINEERING*. 2019;62(3):431–479. Available from: https://atna-mam.utcluj.ro/index.php/Acta/article/view/1223.

106) Coleman MP, Vibration, Of, Manipulator SLF. *Journal of Sound and Vibration*. 1998;212(1):109–129.

107) Halevi Y. Control of Flexible Structures Governed by the Wave Equation Using Infinite Dimensional Transfer Functions. *Journal of Dynamic Systems, Measurement, and Control*. 2005;127(4):579–588. Available from: https://dx.doi.org/10.1115/1.2098895. doi:10.1115/1.2098895.

108) Connor O, J W. Wave-like modelling of cascaded, lumped, flexible systems with an arbitrarily moving boundary. *Journal of Sound and Vibration*. 2011;330(13):3070–83.

109) O'connor WJ, Habibi H. Wave-based control of under-actuated flexible structures with strong external disturbing forces. *International Journal of Control*. 2015;88(9):1818–1847.

110) Renato V, Lorenzo S, Gasparetto A, Giovagnoni M. Comparison of Model Order Reduction Techniques for Flexible Multibody Dynamics using an Equivalent Rigid-Link System Approach. In: Proceedings of 8th ECCOMAS Thematic Conference on Multibody Dynamics. 2017. Available from: https://core.ac.uk/download/84481461.pdf.

111) Mayo J, Dominguez J, Shabana AA. Geometrically Nonlinear Formulations of Beams in Flexible Multibody Dynamics. *Journal of Vibration and Acoustics*. 1995;117(4):501–509. Available from: https://dx.doi.org/10.1115/1.2874490. doi:10.1115/1.2874490.

112) Chen T, Shan J. Distributed Control of Multiple Flexible Manipulators With Unknown Disturbances and Dead-Zone Input. *IEEE Transactions on Industrial Electronics*. 2020;67(11):9937–9947. Available from: https://dx.doi.org/10.1109/tie.2019.2955417. doi:10.1109/tie.2019.2955417.

113) Yang X, Ge SS, He W. Dynamic modelling and adaptive robust tracking control of a space robot with two-link flexible manipulators under unknown disturbances. *International Journal of Control*. 2018;91(4):969–988. Available from: https://dx.doi.org/10.1080/00207179.2017.1300837. doi:10.1080/00207179.2017.1300837.

114) Najafi M, Dehgolan FR. Non-Linear Vibration and Stability Analysis of an Axially Moving Beam with Rotating-Prismatic Joint. *Aerospace, Industrial, Mechatronic and Manufacturing Engineering*. 2017;11(4):780–789.

115) Mohan A. 2006.

116) Scaglioni B, Bascetta L, Baur M, Ferretti G. Closed-form control oriented model of highly flexible manipulators. *Applied Mathematical Modelling*. 2017;52:174–185. Available from: https://dx.doi.org/10.1016/j.apm.2017.07.034. doi:10.1016/j.apm.2017.07.034.

117) Mohan A, Saha SK. A recursive, numerically stable, and efficient simulation algorithm for serial robots. *Multibody System Dynamics*. 2007;17(4):291–319. Available from: https://dx.doi.org/10.1007/s11044-007-9044-8. doi:10.1007/s11044-007-9044-8.

118) Saha SK, Shah SV, Nandihal PV. Evolution of the DeNOC-based dynamic modelling for multibody systems. *Mechanical Sciences*. 2013;4(1):1–20. Available from: https://dx.doi.org/10.5194/ms-4-1-2013. doi:10.5194/ms-4-1-2013.

119) Jia S, Jia Y, Xu S, Hu Q. Maneuver and Active Vibration Suppression of Free-flying Space Robot. *IEEE Transactions on Aerospace and Electronic Systems*. 2017;1(1):1–1. Available from: https://dx.doi.org/10.1109/taes.2017.2775780. doi:10.1109/taes.2017.2775780.

120) Lismonde A, Sonneville V, Brüls O. A geometric optimization method for the trajectory planning of flexible manipulators. *Multibody System Dynamics*. 2019;47(4):347–362. Available from: https://dx.doi.org/10.1007/s11044-019-09695-z. doi:10.1007/s11044-019-09695-z.

121) Bian Y, Gao Z, Lv X, Fan M. Theoretical and experimental study on vibration control of flexible manipulator based on internal resonance. *Journal of Vibration and Control*. 2018;24(15):3321–3337. Available from: https://dx.doi.org/10.1177/1077546317704792. doi:10.1177/1077546317704792.

122) Abe A. Trajectory planning for flexible Cartesian robot manipulator by using artificial neural network: numerical simulation and experimental verification. *Robotica*. 2011;29(5):797–804.

123) Rahmani B, Belkheiri M. Adaptive neural network output feedback control for flexible multi-link robotic manipulators. *International Journal of Control*. 2019;92(10):2324–2338. Available from: https://dx.doi.org/10.1080/00207179.2018.1436774. doi:10.1080/00207179.2018.1436774.

124) Chen B, Huang J. Decreasing infinite-mode vibrations in single-link flexible manipulators by a continuous function. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*. 2017;231(6):436–446. Available from: https://dx.doi.org/10.1177/0959651817708489. doi:10.1177/0959651817708489.

125) Lochan K, Roy BK, Subudhi B. SMC Controlled Chaotic Trajectory Tracking of Two-Link Flexible Manipulator with PID Sliding Surface. *IFAC-PapersOnLine*. 2016;49(1):219–224. Available from: https://dx.doi.org/10.1016/j.ifacol.2016.03.056. doi:10.1016/j.ifacol.2016.03.056.

126) Wang Y, Cao Y, Xia H. Optimized continuous nonsingular terminal sliding mode control of uncertain flexible manipulators. *34th Chinese Control Conference*. 2015;p. 3392–3397.

127) Yang HJ, Tan M. Sliding Mode Control for Flexible-link Manipulators Based on Adaptive Neural Networks. *International Journal of Automation and Computing*. 2018;15(2):239–248. Available from: https://dx.doi.org/10.1007/s11633-018-1122-2. doi:10.1007/s11633-018-1122-2.

128) Aghababa MP, Saif M. 2021. Available from: https://doi.org/10.1177/09596518211014328.

129) Homaeinezhad MR, Yaqubi S, Gholyan HM. Simultaneous path-following and vibration control for uncertain nonlinear flexible mechanical systems without dependency on oscillatory mathematical model. *Multibody System Dynamics*. 2021;51(3):279–303. Available from: https://dx.doi.org/10.1007/s11044-020-09757-7. doi:10.1007/s11044-020-09757-7.

130) Zhang C, Yang T, Sun N, Fang Y. An Adaptive Fuzzy Control Method of Single-Link Flexible Manipulators with Input Dead-Zones. *International Journal of Fuzzy Systems*. 2020;22(8):2521–2533. Available from: https://dx.doi.org/10.1007/s40815-020-00962-2. doi:10.1007/s40815-020-00962-2.

131) Bastos G. A stable reentry trajectory for flexible manipulators. *International Journal of Control*. 2019;0(0):1–12. Available from: https://doi.org/10.1080/00207179.2019.1644538.

132) Schnelle F, Eberhard P. Adaptive nonlinear model predictive control design of a flexible-link manipulator with uncertain parameters. *Acta Mechanica Sinica*. 2017;33(3):529–542. Available from: https://dx.doi.org/10.1007/s10409-017-0669-4. doi:10.1007/s10409-017-0669-4.

133) Ettefagh MH, Naraghi M, Towhidkhah F. Position Control of a Flexible Joint via Explicit Model Predictive Control: An Experimental Implementation. *Emerging Science Journal*. 2019;3(3):146–156. Available from: https://dx.doi.org/10.28991/esj-2019-01177. doi:10.28991/esj-2019-01177.

134) Guechi EHH, Bouzoualegh S, Messikh L, Blazic S. Model predictive control of a two-link robot arm. In: and others, editor. 2018 International Conference on Advanced Systems and Electric Technologies (IC_ASET). IEEE. 2018;p. 409–14. doi:10.1109/ASET.2018.8379891.

135) Yang X, Ge SS, Liu J. Dynamics and Noncollocated Model-Free Position Control for a Space Robot with Multi-Link Flexible Manipulators. *Asian Journal of Control*. 2019;21(2):714–724. Available from: https://dx.doi.org/10.1002/asjc.1772. doi:10.1002/asjc.1772.

136) O'Connor WJ, de la Flor FR, McKeown DJ, Feliu V. Wave-based control of non-linear flexible mechanical systems. *Nonlinear Dynamics*. 2009;57(1-2):113–123. Available from: https://dx.doi.org/10.1007/s11071-008-9425-4. doi:10.1007/s11071-008-9425-4.

137) Obe OO. Genetic Algorithm Based Optimal Trajectories Planning for Robot Manipulators on Assigned Paths. *International Journal of Emerging Trends in Engineering Research*. 2020;8(8):4888–93.

138) Sangdani MH, Tavakolpour-Saleh AR, Lotfavar A. Genetic algorithm-based optimal computed torque control of a vision-based tracker robot: Simulation and experiment. *Engineering Applications of Artificial Intelligence*. 2018;67:24–38. Available from: https://dx.doi.org/10.1016/j.engappai.2017.09.014. doi:10.1016/j.engappai.2017.09.014.

139) Wu D, Zhang W, Qin M, Xie B. Interval Search Genetic Algorithm Based on Trajectory to Solve Inverse Kinematics of Redundant Manipulators and Its Application. *2020 IEEE International Conference on Robotics and Automation (ICRA)*. 2020;p. 7088–94.

140) Yang C, Xu Y, Zhou L, Sun Y. Model-Free Composite Control of Flexible Manipulators Based on Adaptive Dynamic Programming. *Complexity*. 2018;2018:1–9. Available from: https://dx.doi.org/10.1155/2018/9720309. doi:10.1155/2018/9720309.

141) Liu J, He W. Boundary Control for Flexible Manipulator Using Singular Perturbation. In: Distributed Parameter Modeling and Boundary Control of Flexible Manipulators. Springer Singapore. 2018;p. 27–43. doi:10.1007/978-981-10-8300-6_4.

142) Kiang CT, Spowage A, Yoong CK. Review of Control and Sensor System of Flexible Manipulator. *Journal of Intelligent & Robotic Systems*. 2015;77(1):187–213. Available from: https://dx.doi.org/10.1007/s10846-014-0071-4. doi:10.1007/s10846-014-0071-4.

143) Riekert Z, Helbig M. Solving Inverse Kinematics with Vector Evaluated Particle Swarm Optimization. *Lecture Notes in Computer Science*. 2017;p. 225–237.

144) Shah P, Agashe S. Design of Controller for a Higher Order System Without Using Model Reduction Methods. *Progress in Fractional Differentiation and Applications*. 2017;3:289–304. Available from: https://dx.doi.org/10.18576/pfda/030405. doi:10.18576/pfda/030405.

145) Khan MU, Kara T. Adaptive Control of a Two-Link Flexible Manipulator Using a Type-2 Neural Fuzzy System. *Arabian Journal for Science and Engineering*. 2020;45(3):1949–1960. Available from: https://dx.doi.org/10.1007/s13369-020-04341-9. doi:10.1007/s13369-020-04341-9.

146) Kalyoncu M, Botsalı FM. Lateral and Torsional Vibration Analysis of Elastic Robot Manipulators With Prismatic Joint. *Engineering Technology Conference on Energy, Parts A and B*. 2002;p. 949–56.

147) Chen B, Huang J, Ji JC. Control of flexible single-link manipulators having Duffing oscillator dynamics. *Mechanical Systems and Signal Processing*. 2019;121:44–57. Available from: https://dx.doi.org/10.1016/j.ymssp.2018.11.014. doi:10.1016/j.ymssp.2018.11.014.

148) Chen G, Liu D, Wang Y, Jia Q, Liu X. Contact Force Minimization for Space Flexible Manipulators Based on Effective Mass. *Journal of Guidance, Control, and Dynamics*. 2019;42(8):1870–1877. Available from: https://dx.doi.org/10.2514/1.g003987. doi:10.2514/1.g003987.

149) Madeleine P. Some Open Problems in Dynamic Analysis of Flexible Multibody Systems. *Multibody System Dynamics*. 2001;5:315–349. Available from: https://link.springer.com/article/10.1023/A:1011427015995.

150) Mishra N, Singh SP, Nakra BC. Dynamic modelling of two link flexible manipulator using Lagrangian assumed modes method. *Global Journal of Multidisciplinary Studies*. 2015;4:93–105.

151) Mishra N, Singh SP. Dynamic modeling of a two-link flexible manipulator using the Lagrangian finite elements method. In: Mahajan, Modi, Patel, editors. Technology Drivers: Engine for Growth. CRC Press. 2018;p. 167–172.

152) Hussein MT. A review on vision-based control of flexible manipulators. *Advanced Robotics*. 2015;29(24):1575–1585. Available from: https://doi.org/10.1080/01691864.2015.1078743.

153) He W, Gao H, Zhou C, Yang C, Li Z. Reinforcement Learning Control of a Flexible Two-Link Manipulator: An Experimental Investigation. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. 2021;51(12):7326–7336. Available from: https://dx.doi.org/10.1109/tsmc.2020.2975232. doi:10.1109/tsmc.2020.2975232.