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RESEARCH ARTICLE

Direct Product of GK Algebra

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Abstract

Objectives: To find the direct product of an algebraic structure namely as GK algebra. **Methods/Findings**: We derive some important results in which direct product of two GK algebra is again GK algebra as a particular case and also, derive the general case of the same then after investigate the direct product of kernel of GK algebra.

Keywords: Direct Product; Kernel; isomorphism; Homomorphism; GK algebra

1 Introduction

BCK-algebras and BCI-algebras are abridged to two B-algebras. The BCK algebra was coined in 1966 by the Japanese mathematicians, Y. Imai and K. Iseki (1). Two B-algebras are created from two different provenances. In 2007, the new algebraic structure which is said to be BF algebra, was explored by Andrze J Walendziak (2) which is a generalization of BCI/BCK/B-algebras. In 2008, the generalization of B algebra called as BG algebra initiated by Kim & Kim (3). In 2009, another algebra which is generalization of BE algebra and dual BCK/BCI/BCH algebras, namely CI algebra was initiated by Biao long Meng (4).

Direct product plays an important role in algebraic structures. In 2019, Slamet Widianto, Sri Gemawati, Kartini (5–7) were discussed about the Direct product of BG algebra. Likewise, many authors have discussed this topic in their work. Motivated by these, in this paper we discuss about direct product of GK algebra and obtain its some interesting results. In 2018, we introduced the new algebraic structure namely GK algebra (8) and discussed about its characteristics and investigated some results. In this paper we discuss about the direct product of GK algebra and investigate its properties.

2 Direct product of GK algebra

2.1 Definition

Let $(M,\circledast,1_M)$ and $(N,\circledast,1_N)$ be GK algebras. Direct product $M \times N$ is defined as a structure $M \times N = (M \times N; \otimes; (1_M; 1_N))$, where $M \times N$ is the set $\{(m,n)/m \in M, n \in N\}$

and $\otimes$ is given by

$$(m_1,n_1)\otimes(m_2,n_2)=(m_1 \circledast m_2, n_1 \circledast n_2)$$

This shows that the direct product of two sets of GK algebra M and N is denoted by $M\times N$, which each (m,n) is an ordered pair.

2.2 Theorem

Direct product of any two GK algebras is again a GK algebra.

Proof:

Let M and N be GK algebras, let $m_1,m_2 \in M \; and \; n_1,n_2 \in N$

We know that $M\times N = (M\times N; \otimes ;(1_M; 1_N))$

Since $1_M \in M, 1_N \in N$

This implies that $(1_M,1_N)\in M\times N$

$\therefore \; M\times N \; is \; non-empty.$

Now let us prove it is GK algebra.

Let $m_1,m_2 \in M \; and \; n_1,n_2 \in N$

1. $(m_1,n_1)\otimes(m_1,n_1)=(m_1\circledast m_1, n_1\circledast n_1)$
   = $(1_M,1_N)$ by definition of GK algebra
2. $(m_1,n_1)\otimes(1_M,1_N)=(m_1\circledast 1_M, n_1\circledast 1_N)$
   = (m_1,n_1) by definition GK algebra
3. If $(m_1,n_1)\otimes(m_2,n_2)=(1_M,1_N)$ and $(m_2,n_2)\otimes(m_1,n_1)=(1_M,1_N)$
   then $(m_1\circledast m_2, n_1\circledast n_2)=(1_M,1_N)$
   $\implies m_1\circledast m_2 = 1_M \; and \; n_1\circledast n_2 = 1_N$
   $\implies m_1 = m_2 \; and \; n_1 = n_2$ by definition GK algebra.
4. $[(m_2,n_2)\otimes(m_3,n_3)]\otimes[(m_1,n_1)\otimes(m_3,n_3)]$
   $\implies (m_2\circledast m_3, n_2\circledast n_3)\otimes(m_1\circledast m_3, n_1\circledast n_3)$
   $\implies \{[(m_2\circledast m_3)\circledast(m_1\circledast m_3)]\circledast[(n_2\circledast n_3)\circledast(n_1\circledast n_3)]\}$
   $\implies (m_2\circledast m_1, n_2\circledast n_1)$
   $\implies (m_2,n_2)\otimes(m_1,n_1).$
5. $[(m_1,n_1)\otimes(m_2,n_2)]\otimes[(1_M,1_N)\otimes(m_2,n_2)]$
   $\implies [(m_1\circledast m_2),(n_1\circledast n_2)]\otimes[(1_M\circledast m_2),(1_N\circledast n_2)]$
   $\implies ((m_1\circledast m_2)\circledast(1_M\circledast m_2)],[(n_1\circledast n_2)\circledast(1_N\circledast n_2)]$
   $\implies (m_1\circledast 1_M, n_1\circledast 1_N)$
   $\implies (m_1,n_1)$

Hence $M\times N$ is a GK algebra.

2.3 Theorem

Let $\{M_i\,/(M_i;\circledast;1) \; : i=1,2,3\ldots.n\} \; and \; \{N_i\,/(N_i;\circledast;1) \; : i=1,2,3\ldots.n\}$ be the family of GK algebras and let $\zeta_i: \; M_i \longrightarrow N_i, \; i=1,2,3\ldots..n$ be the set of isomorphism.

If $\zeta \; from \prod_1^n M_i \longrightarrow \prod_1^n N_i$ given by $\zeta\,(m_i),(i=1,2,3\ldots n)=\zeta_i\,(m_i)\,,\; i=1,2,3\ldots n$, then ζ is also an isomorphism.

Proof :

Let $\{M_i\,/(M_i;\circledast;1) \; : i=1,2,3\ldots.n\} \; and \; \{N_i\,/(N_i;\circledast;1) \; : i=1,2,3\ldots.n\}$ be the family of GK algebras and let $\zeta_i: \; M_i \longrightarrow N_i, \; i=1,2,3\ldots..n$ be the set of isomorphism.

Let $\zeta \; from \prod_1^n M_i \longrightarrow \prod_1^n N_i$ given by $\zeta\,(m_i),(i=1,2,3\ldots n)=\zeta_i\,(m_i)\,,\; i=1,2,3\ldots n.$

We have to prove ζ is an isomorphism.

If $(m_i,n_i)\in \prod_1^n M_i$ then $\zeta\,[(m_1,m_2,\ldots..m_n)\otimes(n_1,n_2,\ldots\ldots n_n)]$

$=\zeta\,[m_1\circledast n_1, m_2\circledast n_2\ldots\ldots m_n\circledast n_n]$

$=(\zeta_1(m_1\circledast n_1),\zeta_2(m_2\circledast n_2)\ldots\ldots\zeta_n(m_n\circledast n_n))$

$=((\zeta_1(m_1)\circledast\zeta_1(n_1)),\;(\zeta_2(m_2)\circledast\zeta_2(n_2))\;\ldots\ldots(\zeta_n(m_n)\circledast\zeta_n(n_n))$

$=(\zeta_1(m_1),\;\zeta_2(m_2),\ldots\ldots.\;\zeta_n(m_n)] \;\otimes\; (\zeta_1(n_1),\;\zeta_2(n_2),\ldots\ldots.\;\zeta_n(n_n)]$

$=\zeta\,(m_1,m_2,\ldots..m_n) \;\otimes\; \zeta\,(n_1,n_2,\ldots\ldots n_n)$

This implies that ζ is a homomorphism.
We have to prove ζ is onto, we have ζ_i is onto, where i=1,2,3….n.
Let $(n_1, n_2, \ldots\ldots n_n) \in N_1 \times N_2 \times \ldots. \times N_n$
$\Longrightarrow Since\ \zeta\ is\ onto,\ n_i \in N_i,\ there\ exists\ m_i \in M_i\ such\ that\ \ \zeta_i(m_i) = n_i\ for\ i = 1,2,3\ldots n$
$\Longrightarrow (n_1, n_2, \ldots\ldots n_n) = [(\zeta_1(m_1),\ \zeta_2(m_2), \ldots\ldots.\ \zeta_n(m_n)] =\ \zeta(m_1, m_2, \ldots..m_n)$
$\Longrightarrow\ \zeta\ is\ onto.$
Now, to prove $\zeta\ is\ 1-1.$
$\zeta(m_1, m_2, \ldots..m_n) =\ \zeta(n_1, n_2, \ldots\ldots n_n)$
$[(\zeta_1(m_1),\ \zeta_2(m_2), \ldots\ldots.\ \zeta_n(m_n)]\ =\ [(\zeta_1(n_1),\ \zeta_2(n_2), \ldots\ldots.\ \zeta_n(n_n)]$
$\Longrightarrow\ \zeta_i(m_i) = \zeta_i(n_i)$
$\Longrightarrow\ m_i = n_i$, where i=1,2,3….n , since ζ_i is 1-1.
$\Longrightarrow (m_1, m_2, \ldots..m_n) = (\ n_1, n_2, \ldots\ldots n_n)$
$\Longrightarrow\ \zeta\ is\ 1-1.$
Hence ζ is an isomorphism.

2.4 Theorem

Let $M_i, N_i,\ i = 1,2$ be GK algebras. consider the mapping $\zeta_1 : M_1 \longrightarrow N_1\ and$
$\zeta_2 : M_2 \longrightarrow N_2$ where ζ_1, ζ_2 are homomorphisms. If the map $\zeta : M_1 \times M_2 \longrightarrow N_1 \times N_2$ given by
$\zeta(m_1, m_2) = (\zeta_1(m_1), \zeta_2(m_2))$, then

1. ζ is a homomorphism.
2. Ker $\zeta = ker\zeta_1 \times ker\zeta_2$.

Proof:
Let us consider the mapping $\zeta_1 : M_1 \longrightarrow N_1\ and\ \ \zeta_2 : M_2 \longrightarrow N_2$ where ζ_1, ζ_2 are homomorphisms.
If the map $\zeta : M_1 \times M_2 \longrightarrow N_1 \times N_2$ given by $\zeta(m_1, n_1) = (\zeta_1(m_1), \zeta_2(n_1))$,
for $m_1, m_2 \in M_1\ and\ n_1, n_2 \in M_2$ then

- $\zeta[(m_1, n_1) \otimes (m_2, n_2)] = \zeta(m_1 \circledast m_2, n_1 \circledast n_2)$
  = $(\zeta_1(m_1 \circledast m_2), \zeta_2(n_1 \circledast n_2))$
  = $(\zeta_1(m_1) \circledast \zeta_1(m_2), \zeta_2(n_1) \circledast \zeta_2(n_2))$
  = $(\zeta_1(m_1), \zeta_2(n_1))\ \otimes (\zeta_1(m_2), \zeta_2(n_2))$
  = $\zeta_1(m_1, n_1) \otimes \zeta_2(m_2, n_2)$

Therefore ζ is a homomorphism.

- Let $(m, n) \in \ker\zeta \Leftrightarrow \zeta(m, n) = (1_{M_1}, 1_{M_2})$
  $\Longleftrightarrow (\ \zeta_1(m), \zeta_2(n)) = (1_{M_1}, 1_{M_2})$
  $\Longleftrightarrow\ \zeta_1(m) = 1_{M_1}, \zeta_2(n) = 1_{M_2}$
  $\Longleftrightarrow m \in ker\zeta_1\ , n \in ker\zeta_2$
  $\Longleftrightarrow\ \ (m, n)\ \in\ \ ker\zeta_1 \times ker\zeta_2.$

Hence Ker $\zeta = ker\zeta_1 \times ker\zeta_2$.

3 Conclusion

In this article we discussed about the concept of the direct product of GK algebra. We derived the finite form of direct product of GK algebra is isomorphism and also, we investigated and applied the concept of direct product of GK algebra in GK homomorphism and GK kernel, then obtained interesting results.

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