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Implementation and validation of a closed form formula for implied volatility

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Abstract

Introduction: Implied volatility is one of the most commonly used estimator to predict future stock market behaviours. Accordingly, millions of option prices are used to compute the implied volatility in the stock market frequently. Traditionally, this is calculated by inverting Black-Scholes option prices with iterative numerical methods but this associates high computational cost. **Objectives:** This research was mainly focused on implementing and validating an explicit closed-form formula for the implied volatility by using market observed call option prices. **Methods:** In order to obtain the explicit formula, the Taylor series of the Black-Schole call option price with respect to the volatility around a pre-determined initial value was obtained using the operator calculus and the Faa' di Bruno's formula. Taylor series of the implied volatility was acquired using the Lagrange inversion theorem. Here, all the coefficients were explicitly determined using known functions and constants. **Findings:** The developed equation was tested using real time market call option prices with the corresponding market listed implied volatilities for options were used as the initial values. Numerical examples illustrate a significant accuracy of the formula. **Novelty :** It is a closed form formula where the coefficients are explicitly determined and free of numerical iterations making it suitable for industrial implementations and adaptation.

Keywords: Implied Volatility; Taylor Series; Faa' di Bruno's Formula; Operator Calculus; Lagrange Inversion Theorem

1 Introduction

According to the Palgrave Dictionary of Economics "Option pricing theory is the most successful theory not only in finance but also in economics"[1]. In 1970 Fischer Black, Myron Scholes, and Robert Merton invented the theory of the pricing of European stock options. Later this was developed as the Black-Scholes-Merton model for option pricing. According to the Black-Scholes-Merton formula, European stock option price depends on strike price, current stock price, time to maturity risk-free rate, dividend rate and the stock price volatility[2]. Black-Scholes formula is rarely used in the original direction because of the observed market volatility fluctuations, but it is frequently used in the opposite direction to quote implied volatility through option prices[3].

As stated, implied volatility can be calculated by inverting Black-Scholes formula, but it's challenging and time consuming. Therefore, it is long believed that there is no exact closed-form formula for the implied volatility. Hence in practice, the implied volatility is typically determined by iterative numerical root finding algorithms such as Newton Raphson method or bisection method[3]. According to[3], numerical root finding methods suffer from divergence issues, slow speed of convergence, and biases which may lead to incorrect interpretations. Therefore, from a practical viewpoint, the calculation of implied volatility is an important part of any financial tool- box and is central to pricing, risk management and model calibration involving market option prices[3]. Practically, millions of real-time option prices are converted to implied volatility at any given time, hence there is a requirement for a quick method to compute implied volatility. Therefore, there is a rising interest on developing productive non-iterative methods to estimate implied volatility to speed up the computational process. So, here are some previous literature that are related to this scenario.

1. An explicit inversion formula of Black-Scholes formula for implied volatility have found in[4] although the formulas are lack of accuracy and numerical methods have suggested to improve the accuracy.
2. A semi-explicit equation for implied volatility has been developed by using a predefined function for option price, in[5]. Furthermore, some uniform bounds for implied volatility was found but no accurate approximation for implied volatility was found in the literature.
3. Some of the literature aimed at developing an asymptotic expansion of the implied volatility, either by using some extreme parameter values (ex; small maturity or large strike) through perturbations of the associated partial differential equations or estimates of semi groups. In certain conditions, asymptotic methods may not converge well outside those extreme parameter limits. Some recent literature[6],[7],[8] can be mentioned as an example for asymptotic expansions.
4. A remarkable breakthrough has been done in[3] by establishing a direct explicit closed-form formula for implied volatility in terms of market observed option prices. The Black-Scholes formula has been inverted by using Taylor series expansion and a closed-form formula was obtained for implied volatility in this literature. This paper followed the same approach but implemented the formula for current data and investigated its accuracy.

So, the above theoretical and practical requirements have encouraged to implement the closed-form formula for the implied volatility in terms of current observed option prices.

The contribution of this research can be listed as follows,

1. This research intends to introduce detailed implementation of a closed-form formula for the market to calculate implied volatility by using market observed call option prices.
2. This implied volatility formula is well defined and it has been derived in terms of known functions and constants.
3. The traditional method of calculating the implied volatility involves an iterative approach which associates with high computational cost. Hence, the constructed equation calculates implied volatility without any time- consuming iterations.
4. Practically, thousands of options are converted to implied volatility in the real-time market circumstances. Thus, the method which comprises a high computational cost is not appropriate for the practical use of calculating implied volatility.

This study is mainly based on[3]. Therefore, it is vital to distinguish this ground study with the present study. Even though both studies have incorporated the same methodology, the past study has used an upper bound in[9] of implied volatility to obtain the initial point of the Taylor series but for this study, a standard listed implied volatility value for a given option from the market has been utilized as the initial point to build the Taylor series of the Black-Scholes formula. Most importantly, the convergence of the method depends on the initial point of the Taylor series, hence it is crucial to define that, to reduce the complexity of the model and make it more appropriate for the market use. Accordingly, this study was conducted to accomplish the objectives of overcoming the disadvantages of traditional methods of calculating implied volatility and to develop a convenient way to find the implied volatility. Furthermore, this study has adjusted Equation (5) and introduced a new initial point for the series (8) in[3].

2 Methodology

2.1 Theoretical Background

2.1.1 Black-Scholes pricing formula

In 1970 Fischer Black, Myron Scholes, and Robert Merton Established the Black-Scholes-Merton theory of pricing European stock options and later it was developed as a formula for option pricing. There are two versions of Black- Scholes formula, one for the call options and rest for the put options [2].

$$c = Se^{-qt}N(d_1) - Ke^{-rt}N(d_2) \tag{1}$$

$$p = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1) \tag{2}$$

where,

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

The variables can be defined as follows.

- S - stock price
- K - strike price
- T - maturity time
- r - risk-free rate
- q - dividend rate
- σ - volatility
- N(x) – cumulative probability distribution function of Standard Normal distribution.

Here, the time to maturity and the strike price can be defined as similar to the section 1 and the risk-free interest rate is the rate of 10-year bonds in the market.

The Equation (1) represents the call option price and the Equation (2) stands for the put option. Here, the call option version of the Black-Scholes formula has been used to derive the closed-form formula of the implied volatility in this research.

2.1.2 Operator Calculus

Operator calculus or Operational Analysis is a technique of solving differential equations by transforming them into an algebraic problem. According to [10], Oliver Heaviside (1850-1925) was a pioneer in promoting operational calculus methods through his research papers. He applied operator calculus in finding the solutions of differential equations, especially in the theory of electricity.

Let $D = \frac{d}{dt}$ be the operator of differentiation. According to Heaviside, D can be defined in an algebraic manner such that, $D^0 = I, D^k = \frac{d^k}{dt^k}$ where $k \in \mathbb{Z}^+$ and I is the identity operator.

Then this method seems to be fine since this satisfies the following rules of calculus.

1. D(cf)(t) = cDf(t) where c is an arbitrary constant
2. D(f+g)(t) = Df(t) + Dg(t)
3. $D^k(D^l f)(t) = D^{k+l} f(t)$ where k, $l \in \mathbb{Z}^+$

According to this definition, derivatives in differential equations can be replaced by operator D and then the relevant differential equation becomes a function of D. Moreover, it can be considered as a polynomial of D. Then one can use algebraic properties to solve differential equation by using above transformation and this is the simple idea behind operator calculus.

2.1.3 Faa' di Bruno's formula, Bell polynomial version (Riordan's formula)

According to the study [11], if f and g are two functions then the nth order derivative of the composition $f(g(t))$ can be defined as,

$$\frac{d^n f(g(t))}{dt^n} = \sum_{k=1}^{n} f^k(g(t)).B_{n,k}(g'(t),\ g''(t),\ldots,g^{(n-k+1)}(t)) \tag{4}$$

where, $B_{n,k}(x_1,x_2,\ldots,x_{n-k+1})$ are the Bell polynomials defined by,

$$B_{n,k}(x_1,x_2,\ldots,x_{n-k+1}) = \sum \frac{n!}{j_1!j_2!\ldots j_{n-k+1}!}\left(\frac{x_1}{1!}\right)^{j_1}\left(\frac{x_2}{2!}\right)^{j_2}\ldots\left(\frac{x_{n-k+1}}{(n-k+1)!}\right)^{j_{n-k+1}} \tag{5}$$

Here, the summation is taken over all combinations of $j_1, j_2, \ldots, j_{n-k+1}$ of non-negative integers such that,

- $j_1 + j_2 + j_3 + \cdots + j_{n-k+1} = k$,
- $j_1 + 2j_2 + 3j_3 + \cdots + (n-k+1)j_{n-k+1} = n$

2.1.4 Lagrange Inversion Theorem

Let f be an analytic function at a point x_0 and $\frac{df(x_0)}{dx} \neq 0$ Then we can express the function f in its Taylor series. Moreover, the inverse function $g = f^{-1}$ can also be performed as a formal power series which has a non-zero radius of convergence[3]. Moreover, if we are given a power series of the form $y = \sum_{n=0}^{\infty} a_n x^n$ then the inverse series has the form of $x = \sum_{n=0}^{\infty} A_n y^n$ where,

$$A_n = \frac{1}{{a_1}^n} \sum_{k=1}^{n-1} (-1)^k \frac{(n+k-1)!}{(n-1)!} B_{n-1,k}(b_1, b_2, \ldots, b_{n-k}), \quad n \geq 2 \tag{6}$$

where,

$$b_k = \frac{a_{k+1}}{(k+1)a_1},\ A_1 = \frac{1}{a_1} \tag{7}$$

Here, the Bell polynomials can be calculated by using Equation (5).

2.1.5 Theorem of radius of convergence

Suppose that function f is analytic at $z_0 \in \mathbb{C}$ with power series expansion
$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ centered at z_0. Then the radius of convergence of the power series is given by[12],

$$R = \sup\{r > 0 : f \text{ extends to be analytic in the disk } |z - z_0| < r\}$$

2.2 Model development

2.2.1 Spatial Derivatives

The higher-order derivatives of the European call option price with respect to the volatility are required to obtain a Taylor series of Black-Scholes call option price with respect to the volatility. Therefore, an explicit equation is needed to calculate these partial derivatives and pursuant to[3] the direct calculation of these derivatives is profound in computation and hardly leads to an explicit formula. Therefore, all these partial derivatives of the call option price with respect to the volatility were linked to spatial derivatives by using operator calculus.

The derivatives of the option price with respect to the log stock price are called the spatial derivatives. So, the following variable transformation has been done in the Black-Scholes call option formula in order to obtain the spatial derivatives.

Denote log stock price as $x = ln(s)$ then the Black-Scholes call option formula can be re expressed as:

$$V(S,T) = U(x,T) = e^{x-qT} N(d_1) - Ke^{-rT} N(d_1) \tag{8}$$

$$d_1 = \frac{x + (r-q)T - lnK + \dfrac{\sigma^2 T}{2}}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T} \tag{9}$$

Here, $V(S,T)$ stands for the non-transforming Black-Scholes formula and $U(x,T)$ denotes the variable transformed version of Black-Scholes formula.

According to[3], there are two desirable properties associated with using the log stock price as the variable:

1. The component of the partial derivative operator corresponding to x commute to each other.
2. Transformation of S to x, the Black-Scholes formula bears no singularity on x.

So, by using above transformation, the following equation was obtained to calculate the spatial derivatives.

For n=1,2,..., the nth order derivative of the European call option price with respect to x is given by,

$$\frac{\partial^n V(s,T)}{\partial x^n} = e^{x-qT} \frac{1}{2}(1 + \mathrm{erf}(g(x))) \frac{e^{x-qT}}{\sqrt{\pi}} \sum_{j=1}^{n} n-1C_j (-1)^{j-1} (\sqrt{2}\sigma\sqrt{T})^{-j} \left(e^{-g(x)^2} H_{j-1}(g(x))\right) \tag{10}$$

$$g(x) = \frac{x - lnK + (r-q)T + {}^{2}T/2}{\sqrt{2}\sqrt{T}} \tag{11}$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \tag{12}$$

Moreover, $H_n(.)$ are called Hermit polynomials given by,

$$H_n(x) = (-1)^n e^{x^2} \left(\frac{d^n}{dx^n} e^{-x^2} \right) \tag{13}$$

The derivation of the Equation (10) is given in [3], appendix A.1.

2.2.2 Higher order partial derivatives of the option price with respect to the volatility

According to [3], using the operator calculus (see 2.1.2), Faa' di Bruno's formula (see 2.1.3) and spatial derivatives, the following equation has been acquired to calculate the partial derivatives of the option price with respect to σ.

For each $n = 1, 2, 3, \ldots,$

$$\frac{\partial^n V(S,T)}{\partial \sigma^n} = \sum_{k=1}^{n} \frac{n!}{(2k-n)!\,(n-k)!} \frac{T^k \sigma^{2k-n}}{2^{n-k}} \sum_{j=0} k^k C_j (-1)^{k-j} \frac{\partial^{k+j} V(S,T)}{\partial x^{k+j}} \tag{14}$$

where, spatial derivatives are given by the Equation (10) .

Although, the major issue in this equation is if the term $2k-n$ becomes negative then the factorial of negative integers cannot be defined in the usual way by using Gamma function, and that was not mentioned in [3]. Hence, a reasonable way of defining the factorial of a negative integer was needed to guarantee the validity of this equation and finally it has been defined as:

$$(-n)! = (-1)^n n! \qquad n \geq 1. \tag{15}$$

Here, the above result was obtained by a recent study [13]. The derivation of the Equation (14) is given in [3], appendix A.2.

2.2.3 Taylor series of the European call option price

The Taylor series of the European call option price with respect to σ around a positive level σ_0 can be expressed as:

$$V(S,T,\sigma) = V(S,T,\sigma_o) + \sum_{n=1}^{\infty} \frac{\frac{\partial^n V(S,T,\sigma_o)}{\partial \sigma^n}}{n!} (\sigma - \sigma_o)^n \tag{16}$$

Here, $V(S,T,\sigma)$ denotes the European call option price and $\frac{\partial^n V(S,T,\sigma_o)}{\partial \sigma^n}$ is given by Equation (14)

Moreover, the radius of convergence of this series is σ_o and it can be obtained using the theorem in the section (2.1.5).

2.2.4 Taylor series of the implied volatility

The Black-Scholes formula can be considered as a function of σ thus, it satisfies the following properties.

Let V (σ) be the Black-Schole function. Then,

- $V(\sigma)$ is analytic everywhere except $\sigma = 0$..
- $\frac{\partial V(\sigma)}{\partial \sigma} \neq 0$ or each i$\sigma > 0$
- V is one to one and V^{-1} exist.

Since $V(\sigma)$ satisfies all the conditions in the Lagrange inversion theorem (see the section 2.1.4) the Taylor series of the implied volatility was obtained using the series (16) and the Lagrange inversion theorem as:

$$\sigma_{implied} = \sigma_o + \sum_{n=1}^{\infty} A_n \left(V_{Market} - V(\sigma_o)\right)^n \tag{17}$$

where,

$$A_n = \frac{1}{V_1^n} \sum_{k=1}^{n-1} (-1)^k \frac{(n+k-1)!}{(n-1)!} B_{n-1,k}(X_1, X_2, \ldots, X_{n-k}), \quad n \geq 2 \tag{18}$$

$$V_k = \frac{\dfrac{\partial^k V(S,T,\sigma_o)}{\partial \sigma^k}}{k!}, \qquad A_1 = \frac{1}{V_1}, \qquad X_k = \frac{V_{k+1}}{(k+1)V_1} \tag{19}$$

Here, $V(\sigma_o) = V(S,T,\sigma_o)$, V_{market} is the market observed European call option price and $\sigma_{implied}$ is the implied volatility. Additionally, these $V_k s$ were acquired using the Equation (14) and these A_n s were expressed according to the section (2.1.4).

The radius of convergence of the series (17) is not given by the Lagrange inversion theorem and according to[3], it cannot be found explicitly. Therefore, a numerical method was determined using Cauchy Hadamard formula to calculate this radius of convergence and it is expressed as follows.

Let $\{R_k\}$ be a sequence of real numbers defined as:

$$R_k = \frac{1}{(|A_k|)^{1/k}}, \quad k \in Z^+ \tag{20}$$

such that, A_k s are the coefficients of the series (18). Then by calculating R_k up to an appropriate number of terms, the radius of convergence R of the series (18) can be obtained approximately, and furthermore, the convergence interval of the series (18) can be expressed as $(V(\sigma_o) - R,\ V(\sigma_o) + R)$.

2.2.5 Validation using error calculation

The Matlab version 2017 was used to conduct all the numerical calculations. The real time trading option in the market was extracted randomly by using "Yahoo Finance". The absolute value of the approximated error between the calculated implied volatility and the true implied volatility can be calculated as follows:

$$|\sigma_{\text{true}} - C.\,\text{I}.\,V| = |\textstyle\sum_{n=2}^{10} A_n (V(\sigma_0) - V_{\text{market}})^n| \tag{21}$$

Here, σ_{true} denotes the true implied volatility. Moreover, this equation can be simply derived using the Equations (16) and (17).

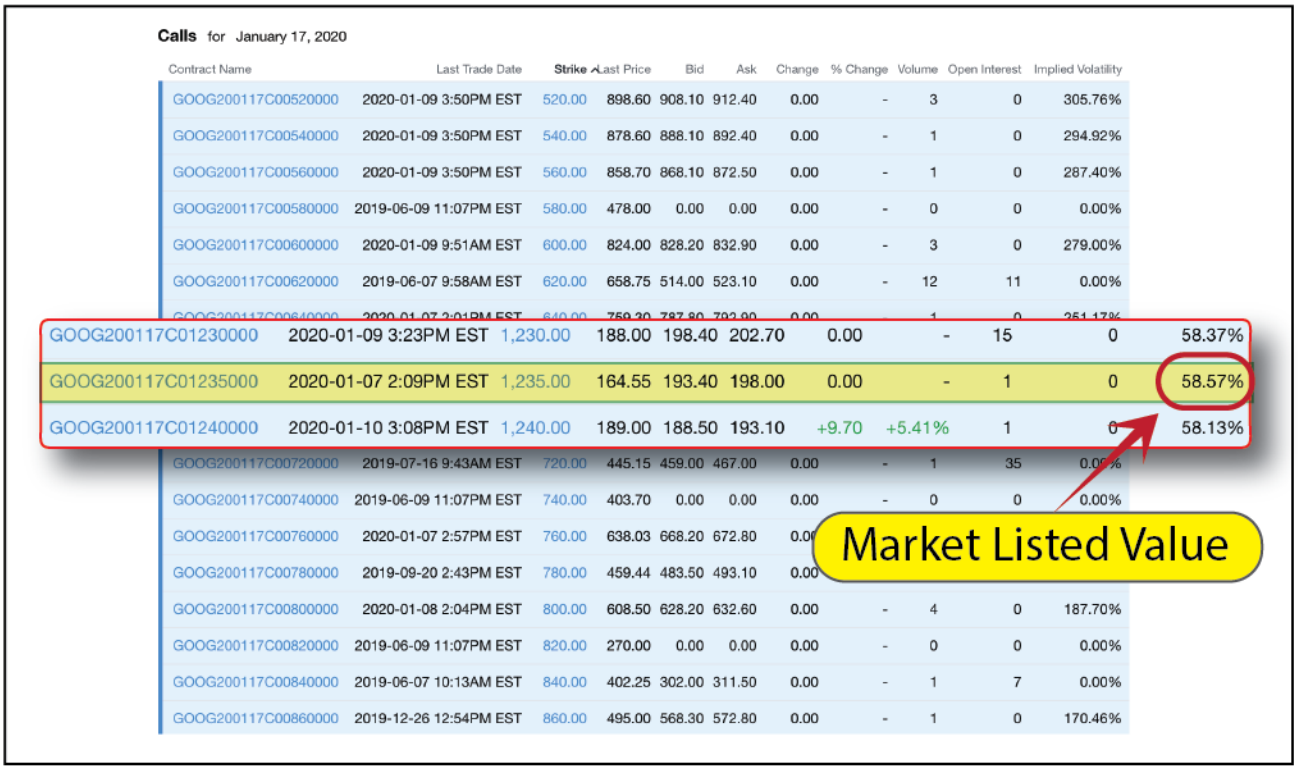
3 Results and Discussion

Implied Volatility is a widely used estimator to forecast the future stock volatility. As per the real circumstance in the stock market, millions of option prices are converted to the implied volatility in every moment. Traditionally, the implied volatility is calculated using options with iterative numerical methods involving high computational cost. Hence, this study was mainly focused on implementing and validating an accurate explicit closed form formula for the implied volatility by using market observed call option prices. In order to achieve the 2^{nd} objective different methodologies were examined thus, the Taylor series method which was introduced in[3] was found to be the most accurate and the apprehensible method for the general public. Hence the same methodology of[3], was followed while adjusting the formulas appropriately.

3.1 Identifying the σ_0 value

According to the previous chapter the convergence of the constructed implied volatility formula depends on the pre-determined positive value (σ_0) which was used for the Taylor series expansions, so when calculating the implied volatility using the developed model, the most important step can be identified as determining this (σ_0) value. The value of the market listed implied volatility for a given option was used as (σ_0) in this study. After studying on Ask and Bid prices of an option it can be assumed that initial implied volatility is calculated using the gap between Ask, Bid prices. Moreover, one can identify this market listed implied volatility value in "Yahoo Finance" according to the following Figure 1.

The computational cost could be reduced using this new (σ_0) value instead of the upper bound value that has been used in[3] for (σ_0).



Calls for January 17, 2020

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
GOOG200117C00520000	2020-01-09 3:50PM EST	520.00	898.60	908.10	912.40	0.00	-	3	0	305.76%
GOOG200117C00540000	2020-01-09 3:50PM EST	540.00	878.60	888.10	892.40	0.00	-	1	0	294.92%
GOOG200117C00560000	2020-01-09 3:50PM EST	560.00	858.70	868.10	872.50	0.00	-	1	0	287.40%
GOOG200117C00580000	2019-06-09 11:07PM EST	580.00	478.00	0.00	0.00	0.00	-	0	0	0.00%
GOOG200117C00600000	2020-01-09 9:51AM EST	600.00	824.00	828.20	832.90	0.00	-	3	0	279.00%
GOOG200117C00620000	2019-06-07 9:58AM EST	620.00	658.75	514.00	523.10	0.00	-	12	11	0.00%
GOOG200117C01230000	2020-01-09 3:23PM EST	1,230.00	188.00	198.40	202.70	0.00	-	15	0	58.37%
GOOG200117C01235000	2020-01-07 2:09PM EST	1,235.00	164.55	193.40	198.00	0.00	-	1	0	58.57%
GOOG200117C01240000	2020-01-10 3:08PM EST	1,240.00	189.00	188.50	193.10	+9.70	+5.41%	1	0	58.13%
GOOG200117C00720000	2019-07-16 9:43AM EST	720.00	445.15	459.00	467.00	0.00	-	1	35	0.00%
GOOG200117C00740000	2019-06-09 11:07PM EST	740.00	403.70	0.00	0.00	0.00	-	0	0	0.00%
GOOG200117C00760000	2020-01-07 2:57PM EST	760.00	638.03	668.20	672.80	0.0				
GOOG200117C00780000	2019-09-20 2:43PM EST	780.00	459.44	483.50	493.10	0.00				
GOOG200117C00800000	2020-01-08 2:04PM EST	800.00	608.50	628.20	632.60	0.00	-	4	0	187.70%
GOOG200117C00820000	2019-06-09 11:07PM EST	820.00	270.00	0.00	0.00	0.00	-	0	0	0.00%
GOOG200117C00840000	2019-06-07 10:13AM EST	840.00	402.25	302.00	311.50	0.00	-	1	7	0.00%
GOOG200117C00860000	2019-12-26 12:54PM EST	860.00	495.00	568.30	572.80	0.00	-	1	0	170.46%

Fig 1. Market listed implied volatility value

3.2 Computational Results

Under this section, the developed explicit formula for the implied volatility has been implemented in order to investigate the convergence and the accuracy. According to the Equation (17) , the implied volatility is given as an infinite sum. In order to convert this equation as a closed form formula, the summation should be taken up to a pre-determined truncation order, hence, n=10 was used as the truncation order. The analysis was mainly divided into 3 categories under option sensitivity, “At the Money”, “In the Money” and the “Out of the Money”.

Randomization was done to represent the strike in the range of 0-1500. The implied volatility for the chosen options were observed by Matlab calculations. Furthermore, respective errors were calculated between the observed values and the true implied volatility values.

Table 1. At the money options

Option name	Strike ($)	Maturity	Current price ($)	Market price ($)	Risk free rate	σ_0	C. I. V	Error
CGC	20	35/252	19.90	1.73	0.017880	0.6509	0.5960	0.0024
CGC	20	260/252	19.90	4.06	0.017880	0.5444	0.4975	0.0024
IBM	135	35/252	134.34	3.69	0.017880	0.2141	0.1936	0.0010
IBM	135	260/252	134.34	9.35	0.017880	0.1810	0.1574	0.0016
FB	210	35/252	208.67	8.50	0.017880	0.3237	0.2884	0.0020
FB	210	260/252	208.67	25.50	0.017880	0.3159	0.2903	0.0011
TSLA	460	34/252	458.09	33.05	0.018130	0.5213	0.4989	4.9339e-04
TSLA	460	259/252	462.36	85.40	0.01807	0.4550	0.4361	4.3345e-04
GOOG	1395	34/252	1395.11	46.90	0.018270	0.2316	0.2212	2.3611e-04

Calculated implied volatility (C.I.V), Canopy Growth Corporation (CGC), International Business Machines Corporation (IBM), Facebook (FB), Tesla, Inc. (TSLA), Alphabet Inc. (GOOG)

Table 2. In the money option

Option name	Strike ($)	Maturity	Current price ($)	Market price ($)	Risk free rate	σ_0	C. I. V	Error
CGC	17.50	35/252	19.90	3.08	0.017880	0.6787	0.5715	0.0036
CGC	17.50	260/252	19.90	4.85	0.017880	0.5151	0.4499	0.0041
IBM	125	35/252	134.34	11.16	0.017880	0.2598	0.2544	1.6660e-05
IBM	125	260/252	134.34	15.59	0.017880	0.1821	0.1651	4.0532e-04
FB	190	35/252	208.67	22.25	0.017880	0.3725	0.3346	1.2505e-04
FB	190	260/252	208.67	36.70	0.017880	0.3455	0.3039	0.0021
TSLA	445	34/252	458.09	40.45	0.018130	0.5251	0.4988	6.4019e-04
TSLA	420	259/252	104.60	85.40	0.01811	0.4655	0.4317	0.0012
GOOG	1370	33/252	1395.11	60.00	0.018270	0.2410	0.2203	8.6243e-04

Table 3. Out of the money options

Option name	Strike ($)	Maturity	Current price ($)	Market price ($)	Risk free rate	σ_0	C. I. V	Error
CGC	22.50	35/252	19.90	0.95	0.017880	0.6851	0.6285	0.0013
CGC	22.50	260/252	19.90	3.34	0.017880	0.5498	0.5183	9.8554e-04
IBM	145	35/252	134.34	0.65	0.017880	0.1946	0.1807	7.0141e-04
IBM	145	260/252	134.34	5.45	0.017880	0.1776	0.1596	7.4456e-04
FB	220	35/252	208.67	4.40	0.017880	0.3106	0.2768	0.0011
FB	230	260/252	208.67	16.75	0.017880	0.3018	0.2787	8.1694e-04
TSLA	475	34/252	458.09	27.23	0.018130	0.5190	0.5067	1.4001e-04
TSLA	500	259/252	463.67	68.87	0.01809	0.4471	0.4272	4.7512e-04
GOOG	1415	34/252	1395.11	37.12	0.018270	0.2263	0.2184	1.4192e-04

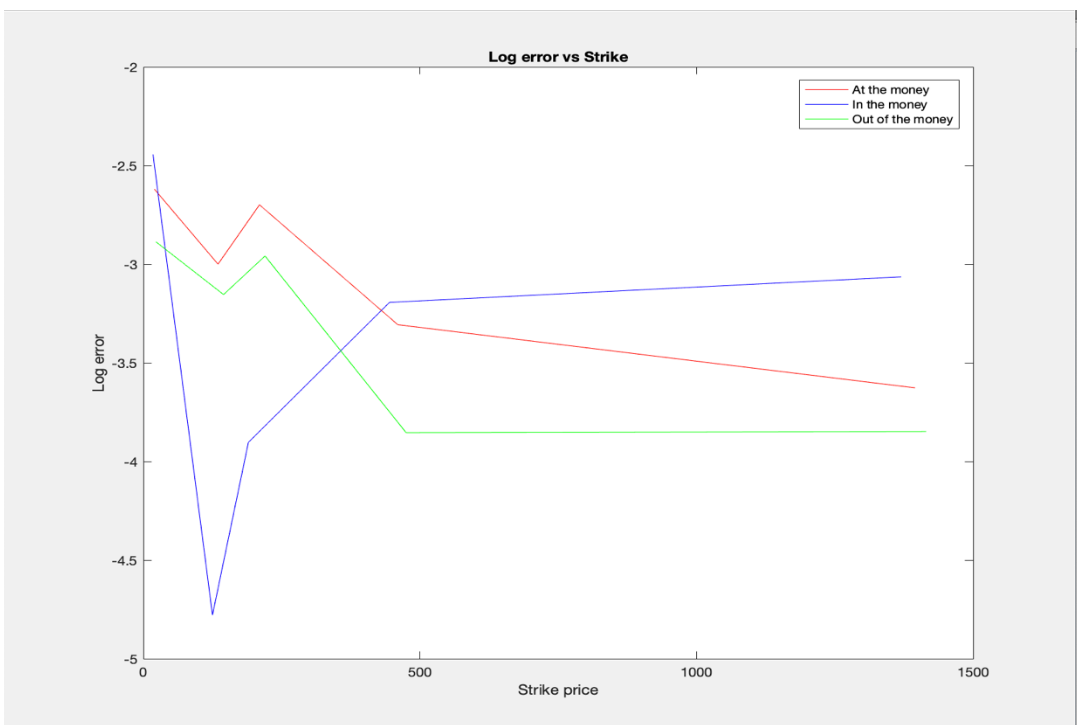


Fig 2. Log error vs strike price

Here, the dividend rate was considered as zero.

Tables 1, 2 and 3 depict the results of Matlab numerical calculations and Figure 2 displays the fluctuation between calculated log error and the strike price. It can be noticed that the minimum calculated error for the implied volatility was found from the "Out of the money" options which was approximately 10^{-4}. The calculated error for the implied volatility for the majority of the cases in all 3 types of options were resulted as approximately between 10^{-3} to 10^{-4}. Most importantly the same accuracy of the model in [3] was obtained using a non-calculated σ_0 value hence, it reduces the computational cost.

Also, it can be introduced some suggestions for future research regarding this study. The development model can be extended for put options to calculate implied volatility within a wider range. Furthermore, a different programming platform can be used for numerical calculations in order to reduce the execution time. Moreover, the error can be reduced increasing the truncation order.

4 Conclusion

The explicit closed form formula was implemented for the implied volatility using Taylor series expansion. The formula contains known functions, constants and, the coefficients of the model were determined accurately and explicitly. The market listed implied volatility value for a given option was utilized as the initial expansion point of the formula. Moreover, it was found that the formula performs exceptionally well for "out of the money" options. Although, overall accuracy produced by the formula is significantly high for all 3 types of options ("At the money", "Out of the money" and "In the money").

References

1) Derman E. My life as a quant: reflections on physics and finance. and others, editor;John Wiley & Sons. 2004.
2) Hull JC. Options, Futures, and Other Derivatives. New Jersey. PrenticeHall. 2012.
3) Xia Y, Cui Z. An exact and explicit implied volatility inversion formula. *International Journal of Financial Engineering*. 2018;5(03). Available from: https://doi.org/10.2139/ssrn.3116440.
4) STEFANICA DAN, RADOIČIĆ R. An explicit implied volatility formula. *International Journal of Theoretical and Applied Finance*. 2017;20(07):1750048–1750048. Available from: https://dx.doi.org/10.1142/s0219024917500480.
5) Tehranchi MR. Uniform bounds for Black–Scholes implied volatility. *SIAM Journal on Financial Mathematics*. 2016;7(1):893–916. Available from: https://dx.doi.org/10.1137/14095248x.
6) Berestycki H, Busca J, Florent I. Asymptotics and calibration of local volatility models. *Quantitative Finance*. 2002;2:61–69. Available from: https://dx.doi.org/10.1088/1469-7688/2/1/305.
7) Forde M, Jacquier A, Mijatović A. Approximate inversion of the Black-Scholes formula using rational functions. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 2008;466:3539–3620. Available from: https://doi.org/10.1098/rspa.2009.0610.
8) Gao K, Lee R. Asymptotics of implied volatility to arbitrary order. *Finance and Stochastics*. 2014;18(2):349–392. Available from: https://dx.doi.org/10.1007/s00780-013-0223-6.
9) Tehranchi MR. Uniform Bounds for Black–Scholes Implied Volatility. *SIAM Journal on Financial Mathematics*. 2016;7(1):893–916. Available from: https://dx.doi.org/10.1137/14095248x.
10) Prudnikov AP, Skòrnik KA. Operational calculus and related topics. and others, editor;CRC Press. 2006.
11) Johnson WP. The curious history of Faà di Bruno's formula. *The American mathematical monthly*. 2002;109:217–234. Available from: https://doi.org/10.2307/2695352.
12) Gamelin TY, Cui Z. An exact and explicit implied volatility inversion formula. *International Journal of Financial Engineering*. 2003;5(03). Available from: https://doi.org/10.2139/ssrn.3116440.
13) Thukral AK. Factorials of real negative and imaginary numbers - A new perspective. *SpringerPlus*. 2014;3(1). Available from: https://dx.doi.org/10.1186/2193-1801-3-658.